Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, July 2020

Programme Name: B. Sc. (Hons) Mathematics **Semester :** IV
Course Name : Riemann Integration & Series of functions **Time : 03 hrs**
Course Code : MATH 2014 **Max. Marks : 100**
Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 18 multiple choice questions and 7 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 2:00 PM to 5:00 PM on 6th July 2020. The due time for PART A is 5:00 PM on 6th July 2020. After the due time, the PART A will not be available.

S. No.		**Marks**	**CO**
Q1 (i)	Approximation of the definite integral $\int_1^4 x^2\, dx$ with the Riemann sum by dividing [1, 4] into equal subintervals: a) 21 b) 63 c) 4 d) 64	**2**	**CO1**
Q1 (ii)	Upper Darboux sum for the function $f(x) = \begin{cases} 1, & x \in Q \\ -1 & x \notin \text{Q} \end{cases}$ on the interval [0, 1] is: a) 1 b) -1 c) 0 d) Q	**2**	**CO1**
Q1 (iii)	A bounded function $f: [a, b] \to$ R is Riemann integrable on $[a, b]$ iff for each $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that a) $U(P, f) - L(P, f) < \varepsilon$ b) $U(P, f) - L(P, f) > \varepsilon$ c) $U(P, f) + L(P, f) < \varepsilon$ d) $U(P, f) + L(P, f) > \varepsilon$	**2**	**CO1**
Q1 (iv)	Determine the value of integral $\int_0^1 x \ln(x)\, dx$: a) -1/4 b) 0 c) 1	**2**	**CO2**

	d) Divergent		
Q1 (v)	The given integral $\int_1^\infty \frac{1}{x^2}\, dx$ converges to a) 1 b) -1 c) 0 d) 2	**2**	**CO2**
Q1 (vi)	Which of the following is not true about $S_n = \frac{1}{n}$? a) The sequence converges to 0. b) $\lim_{n\to\infty} \sum_{i=1}^{n} S_i = L$, for some finite L. c) The series $\sum {S_n}^2$ converges. d) The series $\sum (-1)^n S_n$ converges	**2**	**CO3**
Q1 (vii)	Let $f_n: R \to R$ by $f_n(x) = \frac{Sinnx}{n}$. Then a) $f_n \to 0$ pointwise on R. b) $f_n \to \pi$ pointwise on R. c) $f_n \to 1$ pointwise on R. d) $f_n \to \pi/2$ pointwise on R.	**2**	**CO3**
Q1 (viii)	Let $f_n: R \to R$ by $f_n(x) = \left(1 + \frac{x}{n}\right)^n$. Then a) $f_n \to e^x$ pointwise on R. b) $f_n \to x$ pointwise on R. c) $f_n \to 1$ pointwise on R. d) $f_n \to 0$ pointwise on R.	**2**	**CO3**
Q1 (ix)	The uniform limit of a sequence of real-valued bounded functions defined on a set is a) bounded b) unbounded c) not defined d) None of the above	**2**	**CO3**
Q1 (x)	The given series $\sum_{n=1}^{\infty} \frac{n!^2}{2n!}$ converges to a) ¼ b) ½ c) 1 d) 0	**2**	**CO3**

Q1 (xi)	The geometric series $\sum_{n=0}^{\infty}(x)^n$ has radius of convergence a) 1 b) -1 c) 0 d) Infinity	**2**	**CO4**
Q1 (xii)	The power series $\sum_{n=1}^{\infty}\frac{1}{n}(x)^n$ has radius of convergence a) 1 b) 2 c) 0 d) None of these	**2**	**CO4**
Q1 (xiii)	The power series $\sum_{n=0}^{\infty}\frac{1}{n!}(x)^n$ has radius of convergence a) ∞ b) 1 c) -1 d) 2	**2**	**CO4**
Q1 (xiv)	The power series $\sum_{n=0}^{\infty} n!\,(x)^n$ has radius of convergence a) 0 b) 1 c) -1 d) ∞	**2**	**CO4**
Q1 (xv)	If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n$ is equal to a) 0 b) 1 c) Infinity d) None of these	**2**	**CO4**
Q1 (xvi)	Select all Riemann integrable functions: a) Continuous function on $[a, b]$ b) A bounded function on $[a, b]$ which is continuous except at finitely many points in $[a, b]$	**3**	**CO1**

	c) A monotonic function on $[a, b]$ d) Differentiable function on $[a, b]$		
Q1 (xvii)	Let $[a, b]$ be a given interval. A partition P on $[a, b]$ is a finite set of points x_0, x_1, x_2 such that $a = x_0 \leq x_1 \leq x_2 \ldots \ldots . \leq x_n = b$. Let $f(x)$ be real valued function on $[a, b]$, there exist real numbers m and M such that $m \leq f(x) \leq M$. For all $x \in [a, b]$ a) $m(b-a) \leq L(P, f)$ b) $L(P, f) \leq U(P, f)$ c) $M(b-a) \leq L(P, f)$ d) $m(b-a) \leq M(b-a)$	**3**	**CO1**
Q1 (xviii)	Select all that apply for the integral $\int_a^b \frac{1}{(x-a)^p}\, dx$ a) Converges if $p < 1$. b) Diverges if $p < 1$. c) Converges if $p \geq 1$. d) Diverges if $p \geq 1$.	**3**	**CO2**
Q1 (xix)	Let $\lim_{x \to \infty} x^p f(x) = A$. Then a) $\int_a^\infty f(x)dx$ converges if $p > 1$ and A is finite. b) $\int_a^\infty f(x)dx$ diverges if $p \leq 1$ and $A \neq 0$ (A may be infinite). c) $\int_a^\infty f(x)dx$ converges if $p > 1$. d) $\int_a^\infty f(x)dx$ diverges if $p \leq 1$.	**3**	**CO2**
Q1 (xx)	Which of the following is true about $S_n = \frac{1}{n}$? a) The sequence converges to 0. b) $\lim_{n \to \infty} \sum_{i=1}^{n} S_i = L$, for some finite L. c)The series $\sum {S_n}^2$ converges. d) The series $\sum (-1)^n S_n$ converges	**3**	**CO3**
Q1 (xxi)	Suppose that $\langle u_n \rangle$ and $\langle M_n \rangle$ are sequence of real numbers, with $0 \leq u_n \leq M_n$ for each positive integer n. If $\sum_{n=0}^{\infty} M_n$ converges, then a) $\sum_{n=0}^{\infty} u_n$ converges b) $\sum_{n=0}^{\infty} u_n$ diverges c) $\sum_{n=0}^{\infty} u_n$ oscillates d) None of the above	**3**	**CO3**
Q1 (xxii)	If $\langle f_n \rangle$ and $\langle g_n \rangle$ are sequence of bounded functions and $f_n \to f$ and $g_n \to g$ on a set E, then a) $\{f_n + g_n\}$ converges uniformly on E. b) $\{f_n g_n\}$ converges uniformly on E. c) $\{f_n + g_n\}$ diverges.	**3**	**CO3**

	d) $\{f_n g_n\}$ diverges.		
Q1 (xxiii)	The radius of convergence R of the power series $\sum_{n=0}^{\infty} a_n (x-c)^n$ is given by $R = \frac{1}{\limsup_{n\to\infty} \lvert a_n \rvert^{1/n}}$ where a) $R = 0$ if $limsup$ diverges to ∞ b) $R = \infty$ if $limsup$ is 0 c) $R = 1$ if $limsup$ diverges to ∞ d) $R = c$ if $limsup$ diverges to ∞	**3**	**CO4**
Q1 (xxiv)	Suppose that the power series $\sum_{n=0}^{\infty} a_n (x-c)^n$ has radius of convergence R. Then the power series $\sum_{n=0}^{\infty} n\, a_n (x-c)^{n-1}$ has radius of convergence a) R b) 1 c) R^2 d) 0	**3**	**CO4**
Q1 (xxv)	The exponential function has radius of convergence a) Infinity b) 1 c) 2 d) 0	**3**	**CO4**

PART B

The link for PART B will be available from 2:00 PM on 6th July 2020 to 2:00 PM on 7th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.

Q2	Show that the function f defined as follows: $f(x) = \frac{1}{2^n}$, when $\frac{1}{2^{n+1}} < x < \frac{1}{2^n}$, $(n = 0, 1, 2, \dots)$, $f(0) = 0$, is integrable on [0, 1], although it has an infinite number of points of discontinuity.	**8**	**CO1**

Q3	Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in any interval $[0, b]$, $b > 0$.	**8**	**CO2**
Q4	Find the radius of convergence of the series $$x + \frac{1}{2^2}x^2 + \frac{2!}{3^3}x^3 + \frac{3!}{4^4}x^4 + \cdots \quad .$$	**8**	**CO3**
Q5	If a function f is continuous on $[a, b]$, then there exists a number ξ in $[a, b]$ such that $\int_a^b f\,dx = f(\xi)(b-a)$.	**8**	**CO4**
Q6	If a function is monotonic on $[a, b]$, then it is integrable on $[a, b]$.	**8**	**CO4**