Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

Programme Name: B.Tech. (ADE, Mechatronics, Mechanical) **Semester : III**
Course Name : Mathematics III **Time : 03 hrs.**
Course Code : MATH2008 **Max. Marks : 100**
Nos. of page(s) : 2
Instructions: All questions are compulsory.

SECTION A

S. No.		Marks	CO
Q 1	Let a random variable X follows a standard normal distribution. If $P(X \geq a) = 0.2$, where a is a positive real number. Find $P(X \leq a)$ and $P(X \leq -a)$.	**04**	**CO3**
Q 2	Probability density function $f(x)$ of a random variable X is given as follows $f(x) = 2e^{-2x}$ if $x > 0$ and 0 otherwise. Find $P(2 < X < 4)$.	**04**	**CO3**
Q 3	The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = 1/3$. What is the probability that the repair time exceeds 3 hours?	**04**	**CO3**
Q 4	The number of traffic accidents in a city in 10 randomly chosen days in a year is 4, 0, 6, 5, 2, 1, 2, 0, 4, 3. Use these data to estimate (maximum likelihood estimate) the proportion of days that had no accidents that year.	**04**	**CO4**
Q 5	If the second and third central moments of a data are 1.987 and 0.019, respectively. Compute the coefficient of skewness and classify the skewness present in the data.	**04**	**CO4**

SECTION B

Q 6	Solve the following partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y)$	**10**	**CO1**
Q 7	Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$, where $u(x, 0) = 6e^{-3x}$	**10**	**CO2**

<table>
<tr><td>Q 8</td><td>A sample of 50 units was considered for intensive inspection, and the units were grouped as having 0, 1, 2, 3 or 4 defectives. The frequency table of observations on the random variable X for number of defective units is given as
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Frequency</td><td>4</td><td>21</td><td>10</td><td>13</td><td>2</td></tr>
</table>
Test at 1% level of significance whether this data can be considered to follow a binomial distribution. Use $\chi^2_{0.01,\ 2} = 9.21$.</td><td>10</td><td>CO4</td></tr>
<tr><td>Q 9</td><td>If a sample of 900 units has a mean width of 3.5 cm and standard deviation of 2.61 cm then test whether this sample has come from a large population of mean width 3.25 cm and standard deviation 2.61 cm. Use $z_{0.025} = 1.96$.
OR
Test the significance of difference between the means of the samples, drawn from two normal populations with the same standard deviation using the following data:
<table>
<tr><td></td><td>Size</td><td>Mean</td><td>Standard deviation</td></tr>
<tr><td>Sample 1</td><td>100</td><td>61</td><td>4</td></tr>
<tr><td>Sample 2</td><td>200</td><td>63</td><td>6</td></tr>
</table>
Use $z_{0.025} = 1.96$.</td><td>10</td><td>CO4</td></tr>
<tr><td colspan="4">SECTION-C</td></tr>
<tr><td>Q 10 A</td><td>Three card players play a series of matches. The probability that player A will win any game is 20%, the probability that player B will win is 30%, and the probability that player C will win is 50%. If they play 6 games, what is the probability that player A will win 1 game, player B will win 2 games, and player C will win 3?</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 10 B</td><td>In a bolt factory, machine A, B and C produce 25%, 35% and 40%, respectively, of the total product. Of their output, 5%, 4% and 2%, respectively, are defective. A bolt is chosen at random from the product and found to be defective. What is the probability that the bolt was produced by machine A.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 11</td><td>If X and Y are the marks obtained by 8 students in two subjects, calculate the two regression lines and the correlation coefficient.
<table>
<tr><td>X</td><td>65</td><td>66</td><td>67</td><td>67</td><td>68</td><td>69</td><td>70</td><td>72</td></tr>
<tr><td>Y</td><td>67</td><td>68</td><td>65</td><td>68</td><td>72</td><td>72</td><td>69</td><td>71</td></tr>
</table>
OR
The following are scores of two batsmen A and B in a series of innings:
<table>
<tr><td>A</td><td>12</td><td>115</td><td>6</td><td>73</td><td>7</td><td>19</td><td>119</td><td>36</td><td>84</td><td>29</td></tr>
<tr><td>B</td><td>47</td><td>12</td><td>16</td><td>42</td><td>4</td><td>51</td><td>37</td><td>48</td><td>13</td><td>0</td></tr>
</table>
Who is the better score getter and who is more consistent? Test for the skewness present in the scores of A and B.</td><td>20</td><td>CO4</td></tr>
</table>