Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Course: Mathematics I **Semester: I**

Program: B. Tech. (All SOE) **Time 03 hrs.**

Course Code: MATH 1026 **Max. Marks: 100**

Instructions:

SECTION A

(All Questions are compulsory)

S. No.		**Marks**	**CO**
Q 1	If x, y and z are linearly independent vectors then show that $(x + y)$, $(y + z)$ and $(z + x)$ are linearly independent vectors.	**4**	**CO1**
Q 2	Determine whether $u = (x^2 + xz - xy)$, $v = (x + y - z)$ and $w = (x - y + z)$ are functionally independent or not.	**4**	**CO2**
Q 3	If a is a constant vector and $r = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ then verify that $curl\,(a \times r) = 2\,a$.	**4**	**CO3**
Q 4	Evaluate $\iint_R \sqrt{x^2+y^2}\,dxdy$ by changing to polar coordinates where R is the region in x-y plane bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.	**4**	**CO2**
Q 5	Find the approximate value of $\tan 46^0$ using Taylor's series (using first four terms of the expansion).	**4**	**CO4**

SECTION B

(All Questions are compulsory, Q 9 has internal choices)

Q 6	Evaluate $\oint 2y^3dx + x^3dy + zdz$ where C is the trace of the cone $z = \sqrt{x^2+y^2}$ intersected by plane $z = 4$ and S is the surface of the cone below $z = 4$.	**10**	**CO3**
Q 7	Find the Fourier series expansion of the periodic function $f(x) = x, -\pi \le x \le \pi, f(x + 2\pi) = f(x)$.	**10**	**CO4**
Q 8	Find the Fourier cosine series of the function $$f(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 4, & 2 \le x \le 4 \end{cases}$$	**10**	**CO4**
Q 9	Use the divergence theorem to evaluate $\iint_S (v \bullet n)dA$, where $v = x^2z\,\boldsymbol{i} + y\,\boldsymbol{j} - xz^2\,\boldsymbol{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. **OR**	**10**	**CO3**

	Show that $\oint_C \frac{\partial u}{\partial n} ds = \iint_R \nabla^2 u\, dx\, dy$ where ∇^2 is the Laplace operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and n is the unit outward normal to C.		
SECTION-C **(All Questions are compulsory, Q 11 A and Q 11 B have internal choices)**			
Q 10 A	If $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ always satisfies the matrix equation $A^3 - A^2 + A = kI$, then find the value of constant k. Hence find A^5.	**10**	**CO1**
Q 10 B	A solid fills the region between two concentric spheres of radii 4 cm and 6 cm. with constant density k, Find the total mass of the solid.	**10**	**CO2**
Q11A	Evaluate $\oint_c f(x, y)dx + g(x, y)dy$ where $f(x, y) = e^{-x} \sin y$, $g(x, y) = e^{-x} \cos y$ and C is the square with vertices at $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$ and $(0, \pi/2)$. **OR** If $A = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$, evaluate $\iiint_V A\, dv$ where V is the region bounded by the surface $x = 0, y = 0, x = 3, y = 4, z = x^2, z = 4$.	**10**	**CO3**
Q 11 B	Find the Fourier series expansion of $f(x) = \begin{cases} 2 + x & -2 \le x \le 0 \\ 2 - x & 2 < x \le 4 \end{cases}$ and $f(x + 4) = f(x)$ **OR** Show that $a^x = 1 + x \log a + \frac{(x \log a)^2}{2!} + \frac{(x \log a)^3}{3!} + ...$ where $-\infty < x < \infty$.	**10**	**CO4**