Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme Name: B.Tech Civil Engineering (Spl. Infrastr) **Semester : II**
Course Name : Mathematics II **Time : 03 hrs**
Course Code : MATH 1012 **Max. Marks : 100**
Nos. of page(s) : 2

SECTION A
(All questions are compulsory)

S. No.		Marks	CO
Q 1	Find the solution of the differential equation $x^2 y'' + 2xy' - 2y = 0$.	4	CO 1
Q 2	Find the general solution of the equation $2yz\frac{\partial z}{\partial x} + zx\frac{\partial z}{\partial y} = 3xy$.	4	CO 2
Q 3	Show that $\tan^{-1} z = -\frac{i}{2}\log\left(\frac{1+iz}{1-iz}\right), z \neq \pm i$	4	CO 3
Q 4	Compute the residues at the singular points of $f(z)$ where $f(z) = \frac{z}{(z+1)(z-2)}$	4	CO 4
Q 5	Derive the Cauchy-Riemann equations in polar form.	4	CO 3

SECTION B
(All questions are compulsory, Question 9 has internal choices)

Q 6	Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region (i) $\lvert z \rvert < 1$ (ii) $1 < \lvert z \rvert < 2$	10	CO 1
Q 7	Find the general and singular solution of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + z = x\left(\frac{\partial z}{\partial y}\right)^2$.	10	CO 2
Q 8	Show that (i) if $f(z)$ is analytic and $Re\, f(z)$ = constant then $f(z)$ is constant. (ii) If $f(z)$ is analytic and $Im\, f(z)$= constant then $f(z)$ is constant.	10	CO 3

Q 9	Using the residues evaluate $I=\int_0^{2\pi}\sin^4\theta d\theta$.	**10**	**CO 4**
	OR		
Q 9	Evaluate $\int_0^{\infty}\frac{x}{(x^2+a^2)^2}dx$ (using the residues)	**10**	**CO 4**
SECTION C **(All questions are compulsory, Question 11 has internal choices)**			
Q 10 (A)	Evaluate $\int_{-\infty}^{\infty}\frac{x^2+2}{(x^2+1)(x^2+4)}dx$	**10**	**CO 4**
Q 10 (B)	Find the general solution of equation $9y'''+3y''-5y'+y=42e^x+64e^{x/3}$.	**10**	**CO 1**
Q 11 (A)	Show that $u(x, y) = 2x+ y^3 -3x^2y$ is harmonic. Find its harmonic conjugate function $v(x, y)$ and the corresponding analytic function $f(z)$.	**10**	**CO 3**
Q 11 (B)	Evaluate the integral $\int_C (x+y^2-ixy)dz$ where $C: z=z(t)=\begin{cases} t-2i, & 1<t\le 2 \\ 2-i(4-t), & 2<t\le 3 \end{cases}$.	**10**	**CO 3**
	OR		
Q 11 (A)	Evaluate $\oint_C \frac{dz}{z(z+2)}$, where C is any rectangle containing the points z = 0 and z = -2 inside it.	**10**	**CO 3**
Q 11 (B)	Obtain the Taylor series expansion of $f(z)=\frac{1}{z^2+(1+2i)z+2i}$ about z =0.	**10**	**CO 3**

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SECTION A
(All questions are compulsory)

S. No.		Marks	CO
Q 1	Find the solution of the differential equation $4x^3y''' - x^2y'' + 2(xy' - y) = 0$.	4	CO 1
Q 2	Find the general solution of the equation $2x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3x^2y$.	4	CO 2
Q 3	Show that $\tan^{-1}z = \frac{i}{2}\log\left(\frac{i+z}{i-z}\right), z \neq \pm i$	4	CO 3
Q 4	Compute the residues at the singular points of $f(z)$ where $f(z) = \frac{z^2}{z^2 - 2z + 2}$	4	CO 4
Q 5	Derive the Cauchy-Riemann equations.	4	CO 3

SECTION B
(All questions are compulsory, Question 9 has internal choices)

Q 6	Expand $f(z) = \frac{z}{(z+1)(z-2)}$ in the region (i) $\lvert z\rvert < 1/2$ (ii) $1 < \lvert z\rvert < 3$	10	CO 1
Q 7	Find the general and singular solution of $x\{(\sin x + \cos x)^2 - \sin 2x\}\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + z = x\left(\frac{\partial z}{\partial y}\right)^2$.	10	CO 2

Q 8	Show that $f(z)=\bar{z}$ is continuous at the origin but not differentiable.	**10**	**CO 3**
Q 9	Use the residue theorem to evaluate $\oint_C \frac{e^z-1}{z(z-1)(z-2)}dz$ where (i) $C:\lvert z\rvert=1/2$ (ii) $C:\lvert z\rvert=2$	**10**	**CO 4**
	OR		
Q 9	Evaluate $\int_0^{\infty} \frac{x\sin x}{(x^2+a^2)^2}dx$ (using the residues)	**10**	**CO 4**
	SECTION C **(All questions are compulsory, Question 11 has internal choices)**		
Q 10 (A)	Evaluate $\int_0^{\infty} \frac{x^2-1}{(x^2+1)(x^2+4)}dx$	**10**	**CO 4**
Q 10 (B)	Find the general solution of $3y''' + 6y'' - 10\{ y' + 2y = 8e^{x} + 2e^{x/3}$¿ .	**10**	**CO 1**
Q 11 (A)	Show that $u(x, y) = 2x(\sec^2 x - \tan^2 x) + y^3 - (4\sec^2 x - 4\tan^2 x - 1)x^2 y$ is harmonic. Find its harmonic conjugate function $v(x, y)$ and the corresponding analytic function $f(z)$.	**10**	**CO 3**
Q 11 (B)	Evaluate the integral $\int_C \left(x+y^2-i\left[(x+y)^2-(x-y)^2\right]\right)dz$ where $C: z=z(t)=\begin{cases} t-2i, & 1<t\le 2 \\ 2-i(4-t), & 2<t\le 3 \end{cases}$..	**10**	**CO 3**
	OR		
Q 11 (A)	If $0 < r < R$, evaluate the integral $I=\oint_C \frac{R+z}{z(R-z)}dz$, where $C:\lvert z\rvert=r$.	**10**	**CO 3**
Q 11 (B)	Obtain the first three terms of the Laurent series expansion of $f(z)=\frac{1}{e^z-1}$ about the point $z = 0$ valid in the region $0<\lvert z\rvert<2\pi$.	**10**	**CO 3**