Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: M. Tech. CFD **Semester : I**
Course Name : Computational Gas Dynamics **Time : 03 hrs.**
Course Code : ASEG 7020 **Max. Marks: 100**
Nos. of page(s) : 03
Instructions: Assume any missing data appropriately.

SECTION A

S. No.		Marks	CO
Q 1	Define Riemann problems for scalar conservation equation and Euler equations.	4	CO1
Q 2	The Riemann problem for a system of N equations is equivalent to N Riemann problems for linear advection equations. Prove this statement.	4	CO1
Q 3	Construct a first-order upwind method for the linear advection equation using flux splitting.	4	CO3
Q 4	Evaluate the shock capturing ability of the conservative finite volume methods.	4	CO3
Q 5	Find the conservative numerical flux $f^{n}_{i+1/2}$ of Godunov's first-order upwind method.	4	CO3

SECTION B

Q 6	Deduce the eigenvalues of the Jacobian Matrix ***A***, for the one dimensional Euler Equations given by $$\frac{\partial \mathbf{u}}{\partial t} + A\frac{\partial \mathbf{u}}{\partial x} = 0$$	10	CO1
Q 7	Consider a normal shock wave moving inside a one-dimensional shock tube with pressures p_L and p_R on its left and right sides respectively. Find an expression for the jump in internal energy across the discontinuity assuming the shock moving through a perfect gas. ***OR*** A shock wave across which the pressure ratio is 1.15 moves down a duct into still air at a pressure of 50 kPa and a temperature of 30^{0}C. Find the temperature and velocity of the air behind the shock wave.	10	CO1
Q 8	Assume that $u \neq 0$, $u \neq a$ and $u \neq -a$. Find an expression for the primitive	10	CO2

	variables it terms of the components (f_1, f_2, f_3) of the conservative flux vector **f** given by $$\mathbf{f} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (\rho e_T + p)u \end{bmatrix} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho h_T u \end{bmatrix}$$		
Q 9	Apply Roe's scheme to the following system of equation $U_t + E_x = 0$ where $$U = \begin{bmatrix} u \\ v \end{bmatrix} \qquad E = \begin{bmatrix} cu \\ cv \end{bmatrix}$$ Thus, find the Roe averaged Jacobian matrix $[\lvert A \rvert]$.	**10**	**CO3**
	SECTION C		
Q 10	Find the exact solution to the Riemann problem for the Euler equations at t = 0.01 s if p_L = 100,000 N/m^2, ρ_L = 1 kg/m^3 , u_L= 100 m/s and p_R = 10,000 N/m^2, ρ_R = 0.125 kg/m^3, u_R= -50 m/s. Assume γ =1.4 and R = 287 N.m/kg.K. ***OR*** Compute the left and right eigenvectors of the Jacobian matrix for the one dimensional Euler Equations.	**20**	**CO2**
Q 11	The flux-vector splitting method of Steger and Warming splits the system of equations $U_t + E_x = 0$ into the following form: $U_t + E_x^{+¿+E_x^{-¿=0¿}¿}$ If this method is applied to the system of equations $$U = \begin{bmatrix} u \\ v \end{bmatrix} \qquad E = \begin{bmatrix} cu \\ cv \end{bmatrix}$$ where c is a constant, evaluate the following quantities: a. [A] b. [λ^+], [λ^-] c. [A^+], [A^-]	**20**	**CO4**

	d. [E^+], [E^-]		

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Course Code :	**ASEG 7020**	**Max. Marks: 100**
Nos. of page(s) :	**03**	
Instructions:	**Assume any missing data appropriately.**	

SECTION A

S. No.		Marks	CO
Q 1	Discuss the condition on wave speeds for the occurrence of an expansion wave in a one-dimensional space.	4	CO1
Q 2	Find the conservative numerical flux $f^n_{i+1/2}$ of the Roe's first order upwind method.	4	CO3
Q 4	Using the exact Riemann solver, write expressions for the fluxes at the cell interface $A\boldsymbol{u}(x=0)$ in terms of left and right states for various wave speeds.	4	CO2
Q 4	Project a first order upwind method for the linear advection equation using wave speed splitting.	4	CO3
Q 5	The unsteady Euler Equations have a full wave description. Justify	4	CO1

SECTION B

Q 6	For a Roe's approximate Riemann problem for the Euler problem is given as $\frac{\partial u}{\partial t}+A_{RL}\frac{\partial u}{\partial x}=0$ where $u(x,0)=\begin{cases} u_L x<0 \\ u_R x>0 \end{cases}$ Calculate the Roe-average velocity at the cell interface in terms of the velocities and densities on the left and right of the cell interface.	10	CO3
Q 7	Show that, for an isothermal flow, the one dimensional unsteady Euler equations can be written as $\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0$	10	CO1

	$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\left(\rho\left(u^2+a^2\right)\right)=0$ ***OR*** Consider the one dimensional Euler Equations $$\frac{\partial \mathbf{u}}{\partial t}+A\frac{\partial \mathbf{u}}{\partial x}=0$$ Show that the Jacobian Matrix A is diagonalizable, i.e. $Q_A^{-1} A\, Q_A=\Lambda$.		
Q 8	Prove that Van Leer's flux vector splitting satisfies $d\boldsymbol{f}^{+}/d\boldsymbol{u} \geq 0$ and $d\boldsymbol{f}^{-}/d\boldsymbol{u} \leq 0$.	**10**	**CO4**
Q 9	For a steady state adiabatic flow, assuming $s=const.$, and $u+2a/(\gamma-1)=const.$, derive an expression for the velocity u, speed of sound a and pressure p in the expansion fan centered on $(x, t)=(0,0)$, which connects two steady uniform flows $\boldsymbol{u}_L$ and $\boldsymbol{u}_R$, as a function of space and time.	**10**	**CO2**
	SECTION-C		
Q 10	Find the solution to Roe's approximate Riemann problem at t= 0.01 s if p_L= 100, 000 N/m^2, ρ_L=1 kg/m^3, u_L=100 m/s and p_R=10,000 N/m^2, ρ_R = 0.125 kg/m^3, u_R=-50 m/s.	**20**	**CO3**
Q 11	The 1-D unsteady Euler equations are given by $U_t+[A]U_x=0$ where $$U=[\rho, u, p]^T$$ and $$[A]=\begin{bmatrix} u & \rho & 0 \\ 0 & u & \frac{1}{\rho} \\ 0 & \rho a^2 & u \end{bmatrix}$$ Find the eigenvalues and the left eigenvectors for this system of equations. ***OR***	**20**	**CO4**

	Apply Roe's scheme to the following system of equations, $U_t + [A] U_x = 0$ where $$U = [\rho, u, p]^T$$ and $$[A] = \begin{bmatrix} u & \rho & 0 \\ 0 & u & \frac{1}{\rho} \\ 0 & \rho a^2 & u \end{bmatrix}$$ and thus evaluate the Roe-averaged Jacobian matrix $[\lvert A \rvert]=[T][\lvert \Lambda \rvert][T]^{-1}$, if $0 < u < a$.		