Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

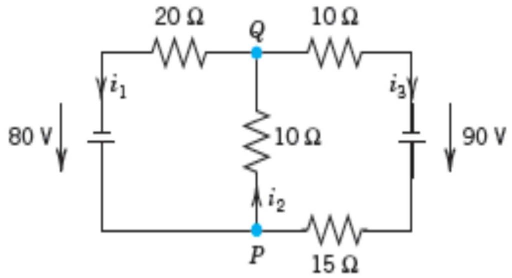
End Semester Examination, December 2018

Programme Name	**: B.Sc. (Hons.) Mathematics**	**Semester**	**: I (ODD-2018-19)**
Course Name	**: Algebra**	**Time**	**: 03 hrs**
Course Code	**: MATH 1032**	**Max. Marks**	**: 100**
Nos. of page(s)	**: 03**		

Instructions: Attempt all question from Sections A, B and C.

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q1.	Find the polar representation of $z = -1 + i\sqrt{3}$ and determine its extended argument.	**[4]**	**CO1**
Q2.	Prove that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n is an equivalence relation on Z.	**[4]**	**CO2**
Q3.	Compute the following product using the polar representation of a complex number $(\frac{1}{2} - i\frac{\sqrt{3}}{2})(-3 + 3i)(2\sqrt{3} + 2i)$.	**[4]**	**CO1**
Q4.	Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$ show that f is bijective.	**[4]**	**CO2**
Q5.	Find the linear transformation $T: R^2 \to R^3$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$.	**[4]**	**CO5**
	SECTION B **(Q6-Q8 are compulsory and Q9-Q10 have internal choice)**		
Q6.	Find $\lvert z \rvert$ and $\arg z$ for the following $z = \frac{(2\sqrt{3}+2i)^8}{(1-i)^6} + \frac{(1+i)^6}{(2\sqrt{3}-2i)^8}$	**[8]**	**CO1**
Q7.	If $T: V_3(R) \to V_3(R)$ be a linear transformation defined by $T(a, b, c) = (3a, a - b, 2a + b + c)$ for all $a, b, c \in R$. Prove that T is invertible and find T^{-1}.	**[8]**	**CO5**

Q8.	Calculate i_1, i_2 and i_3 for the following system  Node P: $i_1 - i_2 + i_3 = 0$ Node Q: $-i_1 + i_2 - i_3 = 0$ Right loop: $10i_2 + 25i_3 = 90$ Left loop: $20i_1 + 10i_2 = 80$	**[8]**	**CO4**
Q9.	Prove that the composition of functions is associative i.e. if f, g, h are three functions such that $(fog)oh$ and $fo(goh)$ exists, then $(fog)oh = fo(goh)$ **OR** Let $f: A \to B$ and $g: B \to A$ are two functions such that $gof = I_A$ and $fog = I_B$. Then f and g are bijections and $g = f^{-1}$.	**[8]**	**CO3**
Q10.	If the system of equations $x = cy + bz,\ y = az + cx,\ z = bx + ay$ have non-trivial solutions, prove that $a^2 + b^2 + c^2 + 2abc = 1$ and the solutions are $x: y: z = \sqrt{1 - a^2} : \sqrt{1 - b^2} : \sqrt{1 - c^2}$. **OR** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$.	**[8]**	**CO4**
SECTION-C **(Q11 is compulsory and Q12(A) and Q12(B) have internal choice)**			
Q11 (A).	Solve the following simultaneous linear congruences: $x \equiv 1 (mod\ 3),\ x \equiv 2 (mod\ 4), x \equiv 3 (mod\ 5)$.	**[10]**	**CO2**
Q11 (B).	By using the Euclidean algorithm, find the greatest common divisor d of the numbers 1109 and 4999 and then find the integers x and y to satisfy $d = 1109x + 4999y$.	**[10]**	**CO2**

Q12 (A).	State and prove Cayley Hamilton Theorem. **OR** Let $T: V \rightarrow W$ be a linear transformation and V be a finite dimensional vector space. Then show that $Rank\ (T) + Nullity\ (T) = dimension\ V.$	**[10]**	**CO3**
Q12 (B).	Define the algebraic and geometric multiplicities of a matrix. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence specify their multiplicities. **OR** Investigate for what values of λ and μ the equations $x + 2y + z = 8$ $2x + 2y + 2z = 13$ $3x + 4y + \lambda z = \mu$ have (i) no solution, (ii) unique solution and (iii) many solutions. Also find the solutions in case of (ii) and (iii).	**[10]**	**CO4**

Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name : B.Sc. (Hons.) Mathematics **Semester : I (ODD-2018-19)**
Course Name : Algebra **Time : 03 hrs**
Course Code : MATH 1032 **Max. Marks : 100**
Nos. of page(s) : 03

Instructions: Attempt all question from Sections A, B and C.

S. No.		Marks	CO
	SECTION A **(Attempt all questions)**		
Q1.	Find the cube root of the following complex number $z = \frac{1}{2} - i\frac{\sqrt{3}}{2}$	**[4]**	**CO1**
Q2.	Show that the relation R on the set $A = x \in Z: 0 \leq x \leq 12\}$ given by $R = \{(a,b): \|a - b\|$ is a multiple of $4\}$ is an equivalence relation.	**[4]**	**CO2**
Q3.	Find the polar representation of $z = 2 + 2i$ and determine its extended argument.	**[4]**	**CO1**
Q4.	Show that the function $f: R - \{3\} \to R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is a bijection.	**[4]**	**CO2**
Q5.	Find the linear transformation $T: R^2 \to R^2$ such that $T(1,2) = (3,0)$ and $T(2,1) = (1,2)$.	**[4]**	**CO5**
	SECTION B **(Q6-Q8 are compulsory and Q9-Q10 have internal choice)**		
Q6.	Find $\|z\|$ and $\arg z$ for the following $z = \frac{(-1+i)^4}{(\sqrt{3}-i)^{10}} + \frac{1}{(2\sqrt{3}+2i)^4}$	**[8]**	**CO1**
Q7.	If $T: V_3(R) \to V_3(R)$ be a linear transformation defined by $T(a,b,c) = (3a, a-b, 2a+b+c)$ for all $a,b,c \in R$. Prove that T is invertible and find T^{-1}.	**[8]**	**CO5**
Q8.	The manufacturing of an automobile requires painting, drying and polishing. The Rome Motor Company produces three types of cars: the Centurion, the Tribune, and the Senator. Each Centurion requires 8 hours for painting, 2 hours for drying and 1 hour for polishing. A Tribune needs 10 hours for painting, 3 hours of drying and 2 hours for polishing. It takes 16 hours of painting, 5 hours of drying and 3 hours of polishing to prepare a Senator. If the company uses 240 hours for painting, 69 hours for drying and 41 hours for polishing in a given month, how	**[8]**	**CO4**

	many of each type of cars are produced?		
Q9.	If $f: A \to B$ and $g: B \to A$ are two bijections, then show that $gof: A \to C$ is a bijection and $(gof)^{-1} = f^{-1}og^{-1}$. **OR** If $f: A \to B$ and $g: B \to A$ be two functions such that $gof = I_A$. Then show that f is an injection and g is a surjection.	**[8]**	**CO3**
Q10.	If the system of equations $x = cy + bz,\ y = az + cx,\ z = bx + ay$ have non-trivial solutions, prove that $a^2 + b^2 + c^2 + 2abc = 1$ and the solutions are $x: y: z = \sqrt{1-a^2}: \sqrt{1-b^2}: \sqrt{1-c^2}$. **OR** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$.	**[8]**	**CO4**
SECTION-C **(Q11 is compulsory and Q12(A) and Q12(B) have internal choice)**			
Q11 (A).	For the following pair of integers a and b, find the integers q and r such that $a = bq + r$ and $0 \leq r < b$ $a = -278, b = 12$.	**[10]**	**CO2**
Q11 (B).	Solve the following simultaneous linear congruences: $x \equiv 2(mod\ 3),\ x \equiv 3(mod\ 7), x \equiv 4(mod\ 8)$.	**[10]**	**CO2**
Q12 (A).	Let $T: V \to W$ be a linear transformation and V be a finite dimensional vector space. Then show that $Rank\ (T) + Nullity\ (T) = dimension\ V$. **OR** Show that the characteristic roots of a unitary matrix are of unit modulus.	**[10]**	**CO3**
Q12 (B).	Define the characteristic equation and find the same for the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$. Show that this matrix has less than three linearly independent eigen vectors. Also find them.	**[10]**	**CO4**

	OR Show that the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ satisfies its own characteristic equation and hence find A^{-1}.		