Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Course: Mathematics-I (MATH-1002) **Semester: I**

Programme: All SOCS Branches **Time: 03 hrs.**

No. of pages: 2 **Max. Marks: 100**

Instructions: All sections are compulsory

SECTION A

Attempt all Questions

S. No.		Marks	CO
Q 1	Show that the contrapositive and conditional propositions are logically equivalent.	**4**	**CO2**
Q2	Find the values of k, for which the rank of matrix $A=\begin{bmatrix} 2 & 4 & 1 \\ k & 2 & 4 \\ 1 & 2 & k \end{bmatrix}$ is 2.	**4**	**CO3**
Q3	Let $G=\{-1,\ 1,-i,\ i\}$ be a cyclic group under multiplication. Find the all generators of G.	**4**	**CO4**
Q4	Evaluate $\int_0^1\int_0^1\int_0^1 xyz\,dz\,dy\,dx$	**4**	**CO1**
Q5	If $A=\begin{bmatrix} 4 & -5 \\ 1 & -2 \end{bmatrix}$, then find the Eigen values of matrix $B=A^3+2A+I$.	**4**	**CO3**

SECTION B

Attempt all Questions

Q6	For what values of k, the given system of linear equations $3x+y+z=0$ $x+(k-2)y+2z=0,\ 2x+y+(k-3)z=0$ has non trivial solution. Also find the solution for $k=4$.	**10**	**CO3**
Q7	Find principal disjunctive normal and principal conjunctive normal forms of $A\cong(p\wedge q)\vee(\sim p\wedge r)\vee(q\wedge r)$.	**10**	**CO2**
Q8	Consider the following argument: If Roli has completed B.Tech, then she is assured of a good job. If Roli is assured of a good job, then she is happy. Roli is not happy. ______________________________ Therefore, Roli has not completed B.Tech. Is the given argument valid?	**10**	**CO2**
Q9	Show that the set $G=\{x+y\sqrt{3};\ x,y\in\mathbb{Q}\}$ is an abelian group with respect to addition. **OR** Show that the set $G=\{0,1,2,3,4,5\}$ is an abelian group with respect to addition modulo 5.	**10**	**CO4**

SECTION-C **Attempt all Questions**			
Q10(A)	If two operations $\bullet$ and $\circ$ on the set $\mathbb{Z}$ of integers are defined as: $a \bullet b = a + b - 1$ and $a \circ b = a + b - ab$. Prove that $(\mathbb{Z}, \bullet, \circ)$ is commutative ring with unity element.	**10**	**CO4**
Q10(B)	Change the order of integration of $I = \int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dxdy}{\sqrt{y^4 - a^2x^2}}$ and hence evaluate.	**10**	**CO1**
Q11(A)	Let $\mathbb{R}^+$ be the multiplicative group of all positive real numbers and $\mathbb{R}$ be the additive group of all real numbers. Show that the mapping $f : \mathbb{R}^+ \to \mathbb{R}$ defined as $f(x) = \log(x)\ \forall x \in \mathbb{R}^+$ is an isomorphism. **OR** Let the set $G = \{1, 2, 3, 4, 5, 6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7. Find the order of each element with explanation.	**10**	**CO4**
Q11(B)	If $u = x + y + z,\ v = x^3 + y^3 + z^3 - 3xyz$ and $w = x^2 + y^2 + z^2 - xy - yz - zx$. Check whether u, v, w are functionally related or not. If so, find the relation between them. **OR** If $u = r^n$, where $r^2 = x^2 + y^2 + z^2$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = n(n+1)r^{n-2}$.	**10**	**CO1**

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Name of the School (Please tick, symbol is given)	**:**	**SOE**		**SOCS**	✓	**SOP**	
Programme	**:**	**B.Tech (All SOCS Branches)**					
Semester	**:**	**I**					
Name of the Course	**:**	**Mathematics-I**					
Course Code	**:**	**MATH-1002**					
Name of Question Paper Setter	**:**	**Dr Pradeep Malik**					
Employee Code	**:**	**40001183**					
Mobile & Extension	**:**	**8979426020**					

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Course: Mathematics-I (MATH-1002) **Semester: I**
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No. of pages: 2 **Max. Marks: 100**
Instructions: All sections are compulsory

SECTION A
Attempt all Questions

S. No.		Marks	CO
Q 1	Investigate the pair of propositions $p \Leftrightarrow q$ and $(p \Rightarrow q) \wedge (q \Rightarrow p)$ are logically equivalent or not?	**4**	**CO2**
Q2	For what values of k, the given system of linear equations $3x+y+z=0$ $x+ky+z=0,\ x+y+kz=0$ has non trivial solution.	**4**	**CO3**
Q3	Find the order of each element of Klein's group $G=\{e,a,b,ab\}$, where $a^2=b^2=e,\ ab=ba$.	**4**	**CO4**
Q4	Find the n^{th} derivative of the function $y=e^{2x}\sin 3x\cos x$.	**4**	**CO1**
Q5	Let $\lambda_1 = 2+i\sqrt{3}$ and $\lambda_2 = 4$ be the Eigen values of the matrix A of order 3×3. Then find the determinant and trace of the matrix A.	**4**	**CO3**

SECTION B
Attempt all Questions

S. No.		Marks	CO
Q6	Verify Caley-Hamilton theorem of the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence, find the matrix B is represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$.	**10**	**CO3**
Q7	Find the principal disjunctive normal and principal conjunctive normal forms of $A \cong (p \wedge \sim (q \wedge r)) \vee (p \Rightarrow q)$.	**10**	**CO2**
Q8	Consider the following argument: If a student knows Mathematics then he does well in Computer Science. If a student does well in Computer Science, he gets handsome salary in a reputed company. A student is getting handsome salary in a reputed company. ———————————————— Therefore, he knows Mathematics. Is the above argument valid?	**10**	**CO2**
Q9	The set **G** of all rational numbers other than -1 with the composition defined as $a \bullet b = a+b+ab$. Is **G** an abelian group?	**10**	**CO4**

	OR Show that the set $G = \{1, 2, 3, 4, 5, 6\}$ is an abelian group of order 6 with respect to multiplication modulo 7.		
	SECTION-C **Attempt all Questions**		
Q10(A)	If $G = \{a + b\sqrt{-5} : a, b \in \mathbb{Z}\}$. Prove that G is a commutative ring with unity element under the usual addition and multiplication of complex numbers.	**10**	**CO4**
Q10(B)	Find the volume bounded by the elliptic paraboloids $z = 18 - x^2 - 9y^2$ & $z = x^2 + 9y^2$.	**10**	**CO1**
Q11(A)	Define subgroup and let set $H = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : a \neq 0; a, b \in \mathbb{R} \right\}$ be a subset of the multiplicative group G of 2×2 non-singular matrices over $\mathbb{R}$. Is the given set H a subgroup of G? **OR** Define the cyclic group and let the set $G = \{-1, 1, -i, i\}$ is a group with respect to multiplication. Find the all generators and show that G is a cyclic group.	**10**	**CO4**
Q11(B)	Evaluate $\iiint \frac{dxdydz}{\sqrt{1 - x^2 - y^2 - z^2}}$ over the positive octant of sphere $x^2 + y^2 + z^2 = 1$. **OR** A rectangular box, open at the top, is to have a volume of 32 cubic feet. Determine the dimension of the box requiring least material for its construction.	**10**	**CO1**