Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2018

Course: Mathematics-I **Semester: I**
Programme: B. Tech. – ASE, ASE & AVI, ADE, Mechanical and Mechatronics
Time: 03 hrs. **Max. Marks: 100**
Course Code- MATH-1011 **No. of pares: 02**

SECTION A

S. No.		Marks	CO
Q 1	Show that the following equations are not consistent: $$2x+6y=-11$$ $$6x+20y-6z=-3$$ $$6y-18z=-1$$	**4**	**CO1**
Q 2	For which value of b the rank of the following matrix is 2: $$A=\begin{bmatrix}1 & 5 & 4\\ 0 & 3 & 2\\ b & 13 & 10\end{bmatrix}$$	**4**	**CO1**
Q 3	Discuss the convergence of the following series by D'Alembert's Ratio test: $$1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+...$$	**4**	**CO3**
Q 4	If $V=f(2x-3y,3y-4z,4z-2x)$ compute the value of $6V_x+4V_y+3V_z$.	**4**	**CO4**
Q 5	Find a unit vector normal to the surface $x^2y+2xz=4$ at the point $(2,-2,3)$.	**4**	**CO4**

SECTION B

Q 6	If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, find two non singular matrices P and Q such that $PAQ=I$.	**10**	**CO1**
Q 7	Using beta and gamma functions evaluate $$\int_0^1 \sqrt{\frac{x^3}{1-x^3}}\,dx$$	**10**	**CO2**
Q 8	Expand $\log\sin x$ in powers of (x-2) up to third derivative term.	**10**	**CO2**
Q 9	Find the Fourier series expansion for $f(x)=e^{-x}$, $0<x<2\pi$. **OR** Discuss the convergence of the following series:	**10**	**CO3**

	$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\dots$$		
	SECTION-C		
Q10 (A)	State the values of x for which the following series converge: $$\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+\dots$$	**10**	**CO3**
Q10(B)	Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.	**10**	**CO4**
Q11(A)	Find a Fouries cosine series of $x\sin x$ in the interval $(0,\pi)$. **OR** Examine the convergence of the following series : $$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4)+\dots$$	**10**	**CO3**
Q11(B)	A vector field is given by $\vec{A}=\left(x^2+xy^2\right)\hat{i}+\left(y^2+x^2y\right)\hat{j}$. Show that the field is irrotational and find the scaler potential. **OR** If $x^x y^y z^z=c$ show that $x=y=z,\ \frac{\partial^2 z}{\partial x\partial y}=-\left(x\log ex\right)^{-1}$	**10**	**CO4**

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SECTION A

S. No.		Marks	CO
Q 1	Find all possible values of k such that the equations $x+y+z=1, 2x+y+4z=k, 4x+y+10z=k^2$ has a solution.	**4**	**CO1**
Q 2	Find the eigen values of $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$. Hence find the eigenvalues of A^{25}.	**4**	**CO1**
Q 3	Discuss the convergence of the following series : $$\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\ldots$$	**4**	**CO3**
Q 4	If $z=e^{ax+by} f(ax-by)$, show that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz$.	**4**	**CO4**
Q 5	Find $div\,\vec{F}$ and $curl\,\vec{F}$ where $\vec{F}=grad\left(x^3+y^3+z^3-3xyz\right)$.	**4**	**CO4**

SECTION B

S. No.		Marks	CO
Q 6	If $A=\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, find A^n (n is a positive integer) using Cayley-Hamilton's theorem.	**10**	**CO1**
Q 7	Using beta and gamma functions evaluate $$\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}}dx$$	**10**	**CO2**
Q 8	Expand $\log(x+a)$ in powers of x up to fourth derivative term.	**10**	**CO2**
Q 9	Find the Fourier series expansion for $f(x)=x\sin x$, $0<x<2\pi$. **OR** Discuss the convergence of the following series:	**10**	**CO3**

	$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots$$		
SECTION-C			
Q10 (A)	Discuss the convergence of the series of $\log(1+x)$.	**10**	**CO3**
Q10(B)	Given that $x+y+z=a$, find the maximum value of $x^m y^n z^p$.	**10**	**CO4**
Q11(A)	Expand the following function as the Fourier series of sine terms: $$f(x)=\begin{cases}\frac{1}{4}-x\,,\ if\ 0<x<1/2\\ x-\frac{3}{4},\ if\ 1/2<x<1\end{cases}$$ **OR** Discuss the convergence of the following series: $$x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\ldots$$	**10**	**CO3**
Q11(B)	Show that $\vec{V}=2xyz\,\hat{i}+(x^2z+2y)\hat{j}+x^2y\,\hat{k}$ is irrotational and find a scaler function $u(x,y,z)$ such that $\vec{V}=grad(u)$. **OR** If $u=x^y$, show that $\frac{\partial^3 u}{\partial x^2\partial y}=\frac{\partial^3 u}{\partial x\partial y\partial x}$.	**10**	**CO4**