Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2018

Programme Name: B.Tech. (CSE), All Specializations **Semester : III**
Course Name : Discrete Mathematical Structures **Time : 03 hrs**
Course Code : CSEG2006 **Max. Marks : 100**
Nos. of page(s) : 3
Instructions: Attempt all the questions.

SECTION A

S. No.		Marks	CO
Q1.	Determine whether the function T: R^2->R^2 is linear transformation. If yes, prove it; if not, provide a counterexample to one of the properties: $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}$	**[4]**	**CO3**
Q2.	Discuss the necessary and sufficient condition for Euler circuit with suitable example.	**[4]**	**CO4**
Q3.	Apply Havel-Hakimi algorithm and determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence. a) 3,3,3,3,2 b) 1,1,1,1,1	**[4]**	**CO4**
Q4.	Consider the following relation R on Z, determine whether the relation is reflexive, symmetric or transitive, and specify the equivalence classes if R is an equivalence relation on Z: (a, b) €R if a + b is odd	**[4]**	**CO1**
Q5.	Prove that if n is an integer and 3n+2 is even, then n is even using a proof by contradiction.	**[4]**	**CO1**

SECTION B

Q6.	Let $V=\mathbf{R}^2$, and let $u,v\in$V such that $u = (u_1, u_2)$ and $v = (v_1, v_2)$. Define addition component-wise, that is $u+v = (u_1+v_1, u_2+v_2)$ and define scalar multiplication by scalar a to be $au = (au_1, 0)$. Determine whether or not this set under these operations is a vector space. **or** Sketch the image of the rectangle with vertices (0, 0), (1, 0), (1, 2), and (0, 2) under (a) A reflection about the x-axis (b) A reflection about the y-axis (c) A compression of factor k=1/4 in the y-direction (d) An expansion of factor k=2 in the x-direction (e) A shear of factor k=3 in the x-direction	**[8]**	**CO3** **CO3**

(f) A shear of factor k=2 in the y-direction

Q7. Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry Wij in the matrix W below is the weight of the edge {i, j}. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T? **CO5**

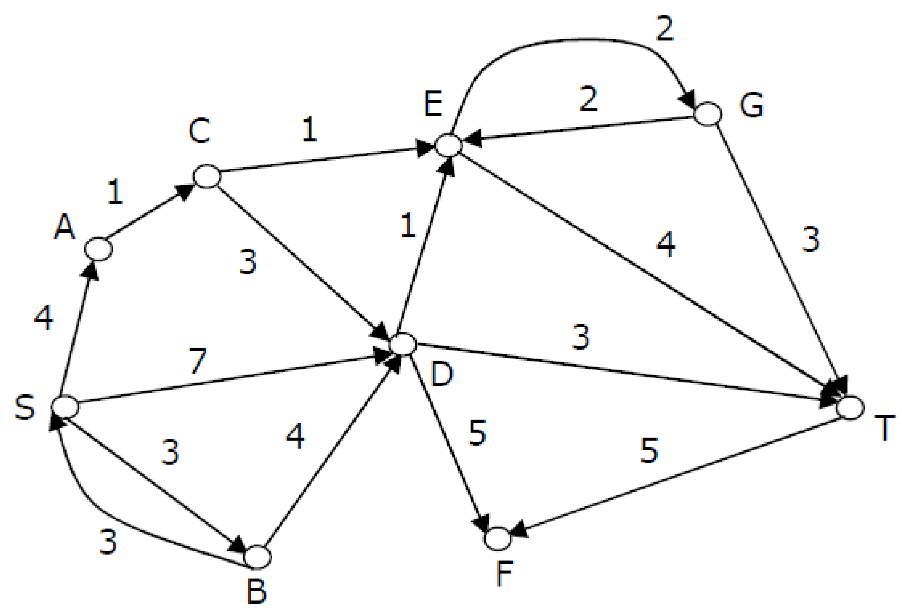
$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

or

CO5

Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijstra?s shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.

[8]



Q8. Calculate the chromatic polynomial of cyclic graph C_4 using deletion-contraction (decomposition) theorem. **[8]** **CO5**

Q9. Solve the following recurrence relation. **[8]** **CO2**

$a_n + 4a_{n-1} + 4\,a_{n-2} = 2^n + n^2 + 5$

Q10. (i) Given any two sets A and B, is it true that the set A-B is always equal to the intersection of A and the complement of B. If true, prove it. If not, give a counter **[8]** **CO1** example.

(ii) Apply the distributive laws of set operations to prove that the set $(X \cup Y)-Z$ is always equal to the set $(X-Z) \cup (Y-Z)$ for any sets X, Y, and Z.

SECTION-C

Q11. Determine whether $T_1 o T_2 = T_2 o T_1$ **CO3**

(a) T_1: $R^2 \rightarrow R^2$ is the orthogonal projection on the y-axis, and T_2:$R^2 \rightarrow R^2$ is the orthogonal projection on the x-axis.

(b) T_1: $R^2 \rightarrow R^2$ is the clockwise rotation through an angle α_1, and T_2:$R^2 \rightarrow R^2$ is the anticlockwise rotation through an angle α_2. **[20]**

(c) T_1: $R^2 \rightarrow R^2$ is the orthogonal projection on the y-axis, and T_2:$R^2 \rightarrow R^2$ is the rotation through an angle α.

Q12. Prove the following:

(a) The graph K_5 is not planar. **CO5**

(b) The graph $K_{3,3}$ is not planar.

(c) Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to 6. **[20]**

(d) The graph K_4 is planar.

or

Prove the Euler's theorem for connected planar graph with n vertices, e edges and r regions. **CO5**

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Programme	:	**B.Tech. (CSE)**					
Semester	:	**III**					
Name of the Course	:	**Discrete Mathematical Structures**					
Course Code	:	**CSEG2006**					
Name of Question Paper Setter	:	**Dr. Shamik Tiwari**					
Employee Code	:	**40001641**					
Mobile & Extension	:	**9509041102**					

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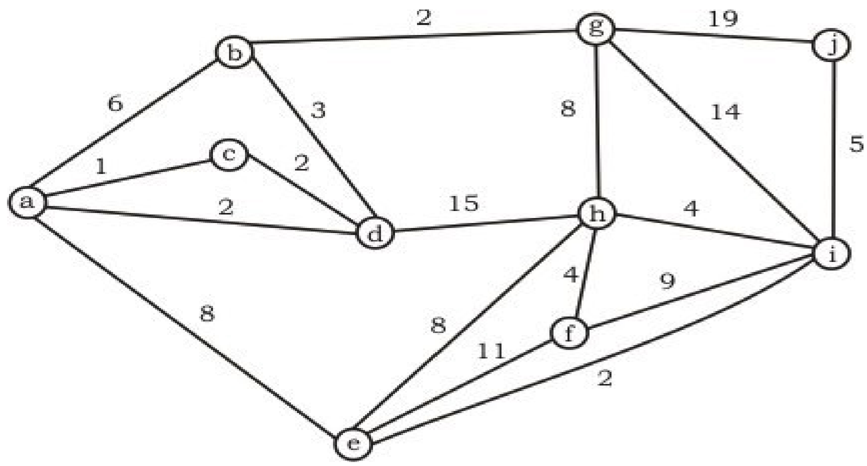
Instructions: Attempt all the questions.

SECTION A

		Marks	CO
Q1.	Determine whether the function T: R^2->R^2 is linear transformation. If yes, prove it; if not, provide a counterexample to one of the properties: $$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$$	[4]	CO3
Q2.	A sequence $d_1, d_2, \ldots, d_n$ is called graphic if it is the degree sequence of a simple graph. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence. a) 5, 4, 3, 2, 1, 0 b) 6, 5, 4, 3, 2, 1	[4]	CO4
Q3.	Prove undirected graph has even number of vertices with odd degree.	[4]	CO4
Q4.	Consider the following relation R on Z, determine whether the relation is reflexive, symmetric or transitive, and specify the equivalence classes if R is an equivalence relation on Z: (a, b) €R if a + b is even	[4]	CO1
Q5.	Prove that$\sqrt{2}$ is irrational by giving a proof by contradiction.	[4]	CO1

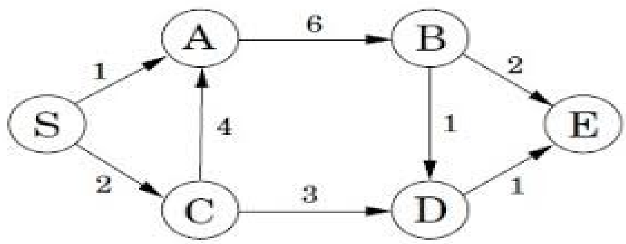
SECTION B

Q6.	Let $V=\mathbf{R}^2$, and let $\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\in$V such that $\boldsymbol{u} = (\boldsymbol{u}_1, \boldsymbol{u}_2)$ and $\boldsymbol{v} = (\boldsymbol{v}_1, \boldsymbol{v}_2)$. Define addition as $\boldsymbol{u}+\boldsymbol{v} = (\boldsymbol{u}_1\boldsymbol{v}_1, \boldsymbol{u}_2\boldsymbol{v}_2)$ and define scalar multiplication by scalar a to be $\boldsymbol{au} = (\boldsymbol{au}_1, \boldsymbol{au}_2+1)$. Determine whether or not this set under these operations is a vector space. **or** Find the standard matrix for the stated composition of linear operators on R^2 (a) A rotation of 90^0, followed by a reflection about line y=x. (b) An orthogonal projection on the y-axis, followed by a contraction with factor k= 0.5. (c) A reflection about the x-axis, followed by a dilation with factor k=3.	[8]	CO3
Q7.	What is the weight of a minimum spanning tree of the following graph? Apply the Kruskal's algorithm.	[8]	CO5



or

Using Dijkstra's Algorithm, find out the shortest distance from the source vertex 'S' to the rest of the vertices in the given graph. Also, write the order in which all the vertices of the graph are visited. **CO5**



Q8. Solve the following recurrence relation **[8]** **CO2**

$a_{n+2} - 2\, a_{n+1} + a_n = 2^n + 5$ $\qquad$ $a_0 = 2,\ a_1 = 1$

Q9. Let f, g and h be functions from N to N defined by **[8]** **CO1**

$f(x) = x$, $g(x) = x^2 + 1$ and $h(x) = 2x + 1$ for every $x \in$ N.

a) Determine whether each function is one-to-one or onto.

b) Find $h \circ (g \circ f)$ and $(h \circ g) \circ f$, and verify the Associative law for composition of functions.

Q10. Find the chromatic number for the following graphs. **[8]** **CO5**

a) K_3
b) C_5
c) C_3
d) W_3
e) W_4
f) $K_{3,3}$
g) Q_2
h) Tree

SECTION-C

Q11. Determine whether $T_1 o T_2 = T_2 o T_1$

(a) T_1: $R^2 \rightarrow R^2$ is the orthogonal projection on the x-axis, and T_2:$R^2 \rightarrow R^2$ is the orthogonal projection on the y-axis.
(b) T_1: $R^2 \rightarrow R^2$ is the rotation through an angle α_1, and T_2:$R^2 \rightarrow R^2$ is the rotation through an angle α_2. **[20]** **CO3**
(c) T_1: $R^2 \rightarrow R^2$ is the orthogonal projection on the x-axis, and T_2:$R^2 \rightarrow R^2$ is the rotation through an angle α.

Q12. Let G be a connected planar graph with p vertices, q edges, and r regions. Then prove that **CO5**

$p - q + r = 2$.

or

Write short note on the following: **[20]** **CO5**

(a) Kuratowski's non-planar graphs
(b) Chromatic Polynomial
(c) Chromatic Polynomial using Decomposition Theorem
(d) Cut set