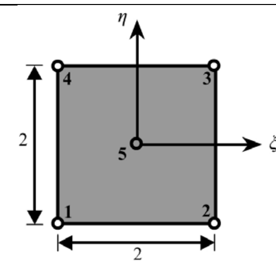
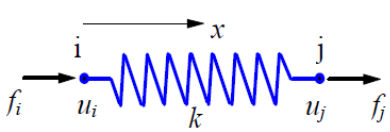
Name: **Enrolment No:**	UPES

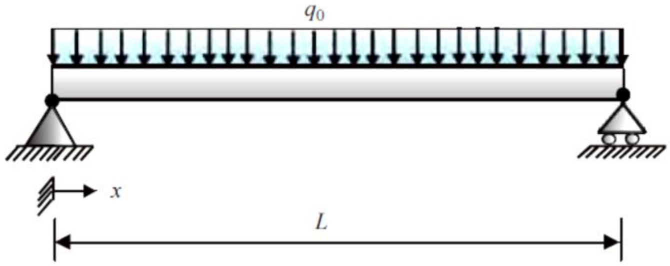
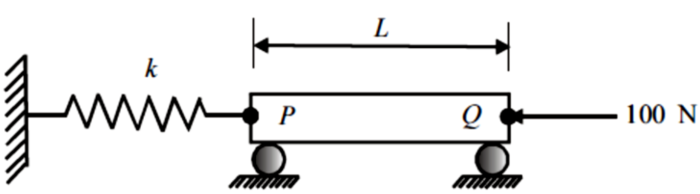
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, April 2018

Course: Finite Element Analysis **Semester: VIII**
Program: B. Tech Aerospace Engineering
Time: 03 hrs. **Max. Marks: 100**

Instructions: Make use of *sketches/plots* to elaborate your answer. Brief and to the point answers are expected.
The Question paper has three sections: Section A, B and C. Section B and C have internal choices.

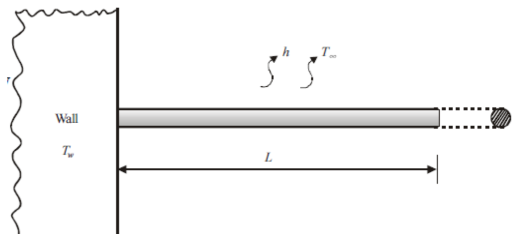
SECTION A [20 Marks]

S. No.		Marks	CO
Q 1.	Determine the shape functions for the five-node rectangular element shown in the fig.  	**[04]**	**CO1**
Q 2.	What do you mean by weak form of the differential equation? State the advantages of the weak form over the weighted residual method.	**[04]**	**CO3**
Q 3.	Consider a single spring element with the given notations,  Two nodes: i j Nodal displacements: u_i u_j Nodal forces: f_i f_j Spring constant (stiffness) k Using the spring-displacement relationship, derive the expression, **ku** = **f** where, k = (element) stiffness matrix, u = (element nodal) displacement vector f = (element nodal) force vector	**[04]**	**CO2**
Q 4.	What is the difference between "sub-structuring" and "sub-modeling"?	**[04]**	**CO2**
Q 5.	State the type of finite element(s) that are best to use when performing the structural analysis for each of the following situations. (i) A calculator housing under load from being sat on (ii) The floor of a house loaded with furniture. The floor has wooden joists (beams) and plywood flooring. (iii) A coffee cup loaded with coffee, where we are interested in the stresses where the handle joins the cup.	**[04]**	**CO4**

SECTION B [40 Marks]			
Q 6.	Consider a simply supported beam under uniformly distributed load as shown in figure below. The governing differential equation and the boundary conditions are given by, $$EI\frac{d^4v}{dx^4} - q_0 = 0; \qquad v(0) = 0, \frac{d^2v}{dx^2}(0) = 0, v(L) = 0, \frac{d^2v}{dx^2}(L) = 0$$  Find the approximate solution using the point collocation technique at $x = L/2$. Assume a one parameter trial solution: $v(x) \approx \hat{v}(x) = c_1 sin(\pi x/L)$	**[10]**	**CO3**
Q 7.	Describe briefly the Method of Weighted Residuals (MWR). Furthermore, explain the application of MWR in the Method of Collocation by Sub-Regions.	**[10]**	**CO4**
Q 8.	Solve the following equation using a two-parameter trial solution by (a) the point collocation at x = 1/4 and x = 1/2; (b) the Rayleigh-Ritz method. $$\frac{dy}{dx} + y = 0; \qquad y(0) = 1$$	**[10]**	**CO2**
Q 9.	Consider the spring mounted bar as shown in the figure. Solve for the displacements of points *P* and *Q* using bar elements (assume *AE* = constant) 	**[10]**	**CO4**

SECTION-C [40 Marks]

Q 10. Consider a 1 *mm* diameter, 50 *mm* long aluminum pin fin as shown in the figure below that is used to enhance the heat transfer from a surface wall maintained at 300°C. The governing differential equation and the boundary conditions are given by,



$$k\frac{d^2T}{dx^2} = \frac{Ph}{A_c}(T - T_\infty); \qquad T(0) = T_w = 300^oC, \qquad \frac{dT}{dx}_{(L)} = 0$$

Let k = 200W/m/°C for aluminum, h = 20 W/m^{2o}C, T_∞ = 30°C. Estimate the temperature distribution in the fin at 10 equal points using the Galerkin residual method using an appropriate polynomial trial function.

[20] **CO3**

Q 11. Derive the Euler-Lagrange equation for a functional given by,

$$I(u) = \int_a^b F\left(u, \frac{du}{dx}, x\right) dx$$

Thus, obtain the corresponding Euler-Lagrange for the functional given below,

$$I = \frac{1}{2}\int_0^L \left[\propto \left(\frac{dy}{dx}\right)^2 - \beta y^2 + ryx^2\right] dx - y(L)$$

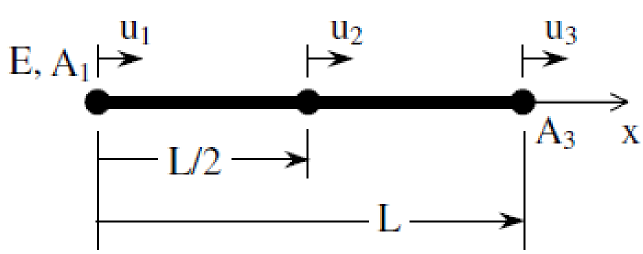
or

A 3 node rod element has a quadratic shape function matrix:

$$\mathbf{N} = \left\langle 1 - \frac{3x}{L} + \frac{2x^2}{L^2} \quad \frac{4x}{L} - \frac{4x^2}{L^2} \quad -\frac{x}{L} + \frac{2x^2}{L^2} \right\rangle$$

For L = 1m, E = 200x10^9 Pa, U_1 = 0, U_2 = 5x10^{-6}m, and U_3 = 5x10^{-6}m, find:

a. The displacement U at x = 0.25 m.
b. The strain as a function of x.
c. The strain at x = 0.25 m.
d. The stress at x = 0.25 m.



[20] **CO4**