Name: **Enrolment No:**	

UPES

End Semester Examination, December 2025

Course: Advanced Engineering Mathematics-I **Semester: I**
Program: B.Tech. (SoCS) **Time : 03 hrs.**
Course Code: MATH1059 **Max. Marks: 100**

Instructions: Attempt all the questions. There are internal choices in the question number 9 and 11. Calculator is allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Discuss the maxima and minima of $x^2 + y^2 + 6x + 12$.	**4**	**CO1**
Q 2	Determine the value of $\Gamma\left(-\frac{5}{2}\right)$.	**4**	**CO2**
Q 3	Change the order of integration for $$\int_{-a}^{a}\int_{0}^{\frac{y^2}{a}} f(x,y)dx\,dy.$$	**4**	**CO2**
Q 4	Compute $div(grad\ \phi)$, for the scalar valued function $\phi(x,y,z)$.	**4**	**CO3**
Q 5	Find all the possible values of a and b, for which given differential equation is exact $$(y + x^3)dx + (ax + by^3)dy = 0.$$	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Use Mean value theorem to show that for any $x \in \left(0, \frac{\pi}{2}\right)$, the ratio $\frac{\sin x}{x}$ satisfies the inequality $$\frac{2}{\pi} < \frac{\sin x}{x} < 1.$$	**10**	**CO1**
Q 7	Evaluate the double integral using polar coordinates (r,θ) $$\int_{0}^{1}\int_{x}^{\sqrt{2x-x^2}} (x^2 + y^2)dxdy.$$	**10**	**CO2**
Q 8	A competitive interaction is described by the Lotka-Volterra competition model	**10**	**CO5**

	$$\frac{dx}{dt} = 0.004x(50 - x - 0.75y)$$ $$\frac{dy}{dt} = 0.001y(100 - y - 3x)$$ Find all the critical points of the system.		
Q 9	Determine the general solution of the differential equation $$\frac{dy}{dx} = \frac{xy^2 \sin(xy) + y\cos(xy)}{x\cos(xy) - x^2 y \sin(xy)}.$$ **OR** A particle of mass m moves in a horizontal straight line with acceleration $\frac{mk}{x^3}$ directed towards the origin at a distance x from the origin with equation of motion given by $mv\frac{dv}{dx} = -\frac{mk}{x^3}$. If initially the particle was at rest at a distance a from the origin, show that it will be at a distance $\frac{a}{2}$ from the origin at $t = \frac{a^2}{2}\sqrt{\frac{3}{k}}$. $\left(\text{Hint: } v = \frac{dx}{dt}\right)$.	**10**	**CO4**
SECTION-C (2Qx20M=40 Marks)			
Q 10	a) Find the constant α such that the force field $$\vec{F} = (e^x z - \alpha xy)\hat{\imath} + (1 - \alpha x^2)\hat{\jmath} + (e^x + \alpha z)\hat{k}$$ is a conservative field. Also, find the scalar potential $\phi(x, y, z)$ such that $\nabla.\vec{F} = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$. b) Using Green's theorem find the area enclosed by the circle described as: $$\vec{r}(t) = (\text{a}\cos t)\hat{\imath} + (\text{a}\sin t)\hat{\jmath},\ \ 0 \le t \le 2\pi.$$	**10+10**	**CO3**
Q 11	Using the method of variation of parameter, solve the differential equation given by $(D^2 - 3D + 2)y = \frac{e^x}{1+e^x}$, where $D \equiv \frac{d}{dx}$. **OR** An RLC circuit consists of a resistance (R) of 0.06 Ω, an inductance (L) of 0.01 H, and a capacitance (C) of $\frac{50}{89}F$. The circuit is powered by a voltage source $E(t) = \cos(10t)$ V. Initially, the current (I) and charge (q) on the capacitor are zero. The problem is mathematically modeled as: $$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t)$$ with $I = q = 0$ at $t = 0$. Determine the charge in the circuit as a function of time.	**20**	**CO4**