

Nonlinear Flight Dynamics and Stability Analysis of Hybrid Airship with Suspended Payload using Bifurcation Techniques with Controller Design

A Thesis submitted to the

UPES

For the Award of

Doctor of Philosophy

in

Aerospace Engineering

By

Abhishek Kumar

JULY 2025

Supervisor

Dr. Om Prakash



Department of Aerospace Engineering

School of Advanced Engineering

UPES

Dehradun- 248007: Uttarakhand, INDIA

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DECLARATION

I declare that the thesis entitled “*Nonlinear Flight Dynamics and Stability Analysis of Hybrid Airship with Suspended Payload using Bifurcation Techniques with Controller Design*” has been prepared by me under the guidance of Dr. Om Prakash, Professor of the Department of Aerospace Engineering, Mechanical Cluster, School of Advanced Engineering, UPES, Dehradun. No part of this thesis has formed the basis for the award of any degree or fellowship previously.



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I certify that Mr. Abhishek Kumar, SAP ID- 500051832 has prepared his thesis entitled “*Nonlinear Flight Dynamics and Stability Analysis of Hybrid Airship with Suspended Payload using Bifurcation Techniques with Controller Design*”, for the award of Ph.D. degree in Aerospace Engineering of UPES, under my guidance. He has carried out the work at the Department of Aerospace Engineering, School of Advanced Engineering, UPES, Dehradun.

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ABSTRACT

The objective of the research entitled “*Nonlinear Flight Dynamics and Stability Analysis of Hybrid Airship with Suspended Payload using Bifurcation Techniques with Controller Design*” is to derive a mathematical model for longitudinal dynamics analysis as single body dynamics approach and two body dynamics approach of hybrid airship to identify the design parameters after analysis of the model. In this regard 3 DOF model as single body from 6DOF model and 4DOF model for two body of hybrid airship from 9DOF model has been developed here. The system taken for the research is a small sized winged hybrid airship with control maneuvers like elevator and rigging angle. Bifurcation theory is based on continuation algorithms. Trim and stability analysis is done on the nonlinear model in level flight for longitudinal dynamics. Simulation is carried out in MATLAB, Scilab and AUTO bifurcation tools. Control system design tool and Simulink is used in designing controllers. The general equation of motion of an airship is very similar to that of a conventional aircraft, with the significant difference being the addition of the added mass and inertia effects and buoyancy force. Hybrid airship is designed for low height and for stationary atmosphere. It is developed with the same concept of aircraft dynamics and dynamics of parafoil payload delivery system. Apparent mass effect including added mass and inertia effect, Buoyancy effect and wing incidence angle with rigging mechanism is taken care in mathematical modelling of hybrid airship. The maximum allowable operational weight for the hybrid airship is 12.47 kg. The calculations are restricted for the flight domain of 20 m/s cruise speed and an altitude of 457 m. For 3DOF nonlinear model of hybrid airship, a linearized model is developed in state space form as well as in transfer function approach for designing a linearized controller (PID) for pitch control and trajectory tracking. NDI controller is designed with the help of two body dynamic model for heading tracking and collision avoidance. Through bifurcation theory a proper trim value with eigen values in nonlinear equations can be identified which will be used in developing a hybrid airship fully autonomous in future. Model can be designed more accurately if aerodynamic flight data beyond stall condition is available.

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LIST OF ABBREVIATIONS

AC = Aerodynamic Center

B - Buoyancy force

CAD - Computer Aided Design

CB - Center of buoyancy

CG - Center of Gravity

CV - Center of volume

CP- Center of pressure

DOF = Degree of Freedom

MAC - Mean aerodynamic chord

STOL - Short take-off and landing

VTOL - Vertical take-off and landing

LIST OF SYMBOLS

a	Wing or wing-body lift curve slope: Acceleration. Local speed of sound
a'	Inertial or absolute acceleration
a_0	Speed of sound at sea level. Tailplane zero incidence lift coefficient
a_1	Tailplane lift curve slope
a_2	Elevator lift curve slope
a_3	Elevator tab lift curve slope
a_h	Local lift curve slope at coordinate h
a_y	Local lift curve slope at spanwise coordinate y
ac	Aerodynamic centre
A	State Matrix
b	Wing span
b_1	Elevator hinge moment derivative with respect to α_T
b_2	Elevator hinge moment derivative with respect to η
b_3	Elevator hinge moment derivative with respect to β_η
B	Input matrix
c	Chord: Viscous damping coefficient. Command Input
$\bar{c}$	Standard mean chord (<i>smc</i>)
cg	Centre of gravity
cp	Centre of pressure
C	Output matrix
C_D	Drag coefficient
C_l	Rolling moment coefficient
C_L	Lift coefficient
C_{L_w}	Wing or wing-body lift coefficient
C_{L_T}	Tailplane lift coefficient
C_m	Pitching moment coefficient
C_{m_α}	Slope of C_m - α plot

C_n	Yawing moment coefficient
C_x	Axial force coefficient
C_y	Lateral force coefficient
C_z	Normal force coefficient
C_τ	Thrust coefficient
D	Drag
$\mathbf{D}$	Direction cosine matrix: Direct matrix
D_c	Drag due to camber
D_α	Drag due to incidence
e	The exponential function
e	Oswald efficiency factor
F	Aerodynamic force: Feed forward path transfer function
F_c	Aerodynamic force due to camber
F_α	Aerodynamic force due to incidence
F_η	Elevator control force
g	Acceleration due to gravity
G	Controlled system transfer function
h	Height: Centre of gravity position on reference chord: Spanwise coordinate along wing sweep line
h_0	Aerodynamic centre position on reference chord
h_m	Controls fixed manoeuvre point position on reference chord
h_n	Controls fixed neutral point position on reference chord
H	Elevator hinge moment: Feedback path transfer function
H_F	Fin span measured perpendicular to the roll axis
H_m	Controls fixed manoeuvre margin
i_x	Dimensionless moment of inertia in roll
i_y	Dimensionless moment of inertia in pitch
i_z	Dimensionless moment of inertia in yaw
i_{xz}	Dimensionless product of inertia about ox and oz axes
I_x	Moment of inertial in roll
I_y	Moment of inertial in pitch
I_z	Moment of inertial in yaw

I	Identity matrix
I_{xy}	Product of inertia about ox and oy axes
I_{xz}	Product of inertia about ox and oz axes
I_{yz}	Product of inertia about oy and oz axes
j	The complex variable ($\sqrt{-1}$)
k	General constant: Spring stiffness coefficient
k_q	Pitch rate transfer function gain constant
k_u	Axial velocity transfer function gain constant
k_w	Normal velocity transfer function gain constant
k_θ	Pitch attitude transfer function gain constant
k_τ	Turbo-jet engine gain constant
K	Feedback gain: Constant in drag polar
K	Feedback gain matrix
K_n	Controls fixed static stability margin
l_f	Fin arm measured between wing and fin quarter chord points
l_t	Tail arm measured between wing and tailplane quarter chord points
l_F	Fin arm measured between cg and fin quarter chord points
l_T	Tail arm measured between cg and tailplane quarter chord points
L	Lift: Rolling moment
L_c	Lift due to camber
L_w	Wing or wing-body lift
L_F	Fin lift
L_T	Tailplane lift
L_α	Lift due to incidence
m	Mass
M	Local Mach number
M_0	Free stream Mach number
M	Pitching Moment
M	“Mass” matrix
M_0	Wing-body pitching moment about wing aerodynamic centre
M_T	Tailplane pitching moment about tailplane aerodynamic centre

n	Total normal load factor
n_α	Normal load factor per unit angle of attack
N	Yawing moment
o	Origin of axis
p	Roll rate perturbation
q	Pitch rate perturbation
Q	Dynamic pressure
r	Yaw rate perturbation
R	Radius of turn
s	Wing semi-span: Laplace operator
S	Wing reference area
t	Time: Maximum aerofoil section thickness
T	Time constant
T_r	Roll mode time constant
T_s	Spiral mode time constant
T_u	Numerator zero in axial velocity transfer function
T_w	Numerator zero in normal velocity transfer function
T_θ	Numerator zero in pitch rate and attitude transfer function
T_τ	Turbo-jet engine time constant
T_2	Time to double amplitude
u	Axial velocity perturbation
$\mathbf{u}$	Input vector
U	Total axial velocity
U_e	Axial component of steady equilibrium velocity
U_E	Axial velocity component referred to datum-path earth axes
v	Lateral velocity perturbation
$\mathbf{v}$	Eigenvector
V	Perturbed total velocity: Total lateral velocity
V_e	Lateral component of steady equilibrium velocity
V_E	Lateral velocity component referred to datum-path earth axes
V_0	Steady equilibrium velocity
$\mathbf{V}$	Eigenvector matrix

w	Normal velocity perturbation
W	Total normal velocity
W_e	Normal component of steady equilibrium velocity
W_E	Normal velocity component referred to datum-path earth axes
x	Longitudinal coordinate in axis system
x_τ	Axial coordinate of engine thrust line
$\mathbf{x}$	State Vector
X	Axial force component
y	Lateral coordinate in axis system
y_B	Lateral body “drag” coefficient
y_r	Lateral coordinate of engine thrust line
$\mathbf{y}$	Output vector
Y	Lateral force component
z	Normal coordinate in axis system: System zero
z_τ	Normal coordinate of engine thrust line
$\mathbf{z}$	Transformed state vector
Z	Normal force component
α	Angle of attack or incidence perturbation
α_e	Equilibrium incidence
α_T	Local tailplane incidence
α_{w0}	Zero lift incidence of wing
α_{wr}	Wing rigging angle
β	Sideslip angle perturbation
β_e	Equilibrium sideslip angle
β_η	Elevator trim tab angle
γ	Flight path angle perturbation: Imaginary part of a complex number
γ_e	Equilibrium flight path angle
Γ	Wing dihedral angle
δ	Control angle: Increment: Unit impulse function
δ_ξ	Roll control stick angle
δ_η	Pitch control stick angle

δ_ζ	Rudder pedal control angle
δ_m	Mass increment
Δ	Characteristic polynomial: Transfer function denominator
ε	Throttle level angle: Downwash angle at tailplane: Closed loop system error
ε_0	Zero lift downwash angle at tail
ζ	Rudder angle perturbation: Damping ratio
ζ_d	Dutch roll damping ratio
ζ_p	Phugoid damping ratio
ζ_s	Short period pitching oscillation damping ratio
η	Elevator angle perturbation
η_e	Elevator trim angle
η_T	Tailplane setting angle
θ	Pitch angle perturbation: A general angle
θ_e	Equilibrium pitch angle
κ	Thrust line inclination to aircraft ox axis
λ	Eigenvalue
Λ	Wing sweep angle
Λ	Eigenvalue matrix
μ_1	Longitudinal relative density factor
μ_2	Lateral relative density factor
ξ	Aileron angle perturbation
ρ	Air Density
σ	Aerodynamic time parameter: Real part of a complex number
τ	Engine thrust perturbation: Time parameter
τ_e	Trim thrust
ϕ	Roll angle perturbation: Phase angle: A general angle
Φ	State transition matrix
ψ	Yaw angle perturbation
ω	Undamped natural frequency
ω_b	Bandwidth frequency
ω_d	Dutch roll undamped natural frequency

ω_n Damped natural frequency
 ω_p Phugoid undamped natural frequency
 ω_s Short period pitching oscillation undamped natural frequency
 p, q, r = roll, pitch, and yaw rates
 R = link length
 S_b = payload (body) cross-section area
 S_h = hull reference area
 T_b = transformation matrix from inertial reference frame to payload reference frame
 T_h = transformation matrix from inertial reference frame to hull reference frame
 u, v, w = velocity components along reference frame
 V = total velocity
 X, Y, Z = body-fixed reference frame
 x, y, z = position components
 α = angle of attack
 γ = glide angle
 μ = rigging angle
 δ = side rigging(tilt) angle
 ρ = air density
 ϕ, ψ, θ = Euler roll, yaw, and pitch angles, respectively
Subscript
 b = referred to payload body
 c = referred to joint C
 cb = measured from joint C to payload CG
 ch = measured from joint C to hull CG
 F = referred to apparent mass
 h = referred to hull
 ha = measured from hull CG to quarter chord point
 B = Buoyancy Force
 b = wing span
 CD = drag coefficient
 CL = lift coefficient

C_Y = side force coefficient

$C_{lp}; C_{lr}$ = rolling moment damping coefficients

$C_{m\alpha}/4$ = pitching moment coefficient at quarter chord

C_{mq} = pitching moment damping coefficient

$C_{nr}; C_{np}$ = yawing moment damping coefficients

c = wing chord length

F = force

g = acceleration due to gravity

I = inertia matrix

$I_A; I_B; I_C$ = apparent inertia terms

M = mass matrix

l = length of control line pulled

m = mass

CHAPTER 1

INTRODUCTION

The research deals with the flight dynamics and control domain, with the primary focus on the autonomous hybrid lighter than air vehicle (airship). The research title is:

“Nonlinear Flight Dynamics and Stability Analysis of Hybrid Airship with Suspended Payload using Bifurcation Techniques with Controller Design”.

The principal research achievements for airship throughout the world can be summarized as follows:

The Aurora Airship is predominantly employed for the purpose of carrying out environmental monitoring missions at the “Autonomous Institute of CTI Campinas”, located in Brazil. Furthermore, during the period from 2003 to 2008, an airship dynamic model was incorporated within the system to augment its operational capabilities. The notion of optimal visual advice pertains to the utilization of visual cues or aids in order to offer proficient and efficacious support to persons. The examination and documentation of the incorporation of internet-based technologies in airship control have been extensively explored and recorded in several scholarly references [1], [2], [3] and [4]. The individual attended the University of Pennsylvania, situated in the United States, during the period from 1989 to 2003. The navigation mechanism of the “GRASP Blimp” incorporates computer vision technology. The task of motion planning holds significant importance in the field of robotics and autonomous systems [5, 6, 7, 8]. The study was carried out at “LAAS/CNRS” in France over the timeframe of 2002-2003. Terrain mapping refers to the systematic procedure of generating comprehensive depictions or visualizations that accurately depict the physical attributes and qualities of a certain geographical area or locality. The citations provided include references [9], [10], [11], and [12]. From 2003 to

2005, researchers at Kyoto University in Japan conducted a study focused on the control of an indoor blimp using image-based tracking. The subject matter under consideration pertains to Inverse Optimal Tracking Control, as cited in references [13], [14], and [15]. The “AURORA Airship”, which was operational at the “Technical university of Lisbon in Portugal” between 2005 and 2008, was outfitted with an advanced control system. The method of regulating movement by hovering has been thoroughly examined and recorded in prior scholarly investigations [16, 17]. The research undertaken at the “University of Évry-Val d'Essonne”, situated in France, aimed to characterize non-trim trajectories in the “LSC Airship” over the year of 2005-2006. Previous research has examined the determination of trim trajectories [18], [19]. The subject of inquiry at “Hokkaido University” in 2006 pertains to the motion control of a balloon robot, as elucidated in references [20] and [30]. During the 2006-2007 academic year, the “University of Tokyo” engaged in a scholarly discourse on the application of model predictive control in the domain of indoor blimps. The user has presented a compilation of three references, identified as [22], [23], and [24]. The focus of conversation at the “Jet Propulsion Laboratory”, a division of the “National Aeronautics and Space Administration” situated in the United States, revolves around the enhancement of trajectory planning for a theoretical airborne apparatus referred to as the "Planet Blimp". The primary objective of this study is to ascertain the optimal trajectory for the aerobot in question, which was operational over the period from 2003 to 2008. The investigation of lunar exploration employing humanoid robots has garnered attention in scholarly discourse, as indicated by citations [25] and [26].

1.1 STATEMENT OF THE PROPOSAL

1.1.1 PROBLEM STATEMENT: Unmanned airships possess notable attributes such as strong lifting capacities, minimal environmental impact, and extended operational durability. Nevertheless, these airships encounter difficulties pertaining to maneuverability during landing and the reliability of control systems. The achievement of autonomous flight in an aerial vehicle necessitates the utilization of a mathematical model that possesses a high level of accuracy. A more accurate non-linear model of hybrid airship is required for

tracking trajectories more accurately with collision avoidance features. For longitudinal dynamics analysis of the model and for collision avoidance and trajectory tracking nonlinear controller along with PID controller should be designed. An airship can act as an eye in the sky for better surveillance in remote boarder area or in your locality. This results in considerable cost savings when compared to conventional aerial systems such as airplanes or helicopters. Analysis of a system requires accurate mathematical model. Here a nonlinear mathematical model for longitudinal dynamic analysis of hybrid airship with suspended payload is required to perform pitch control. Airship modelling is done in two approaches 1) single body dynamics and 2) two body dynamics. Two body dynamic analysis is required because the part of airship as hull filled with helium gas and gondola considered as payload. All masses are attached with hull. Analysis of hybrid airship is in line with the combination of aircraft and parafoil. Drones are widely used these days. An autonomous airship refers to an unmanned aerial vehicle that operates without human intervention, characterized by its ability to manipulate six varied control inputs. Lighter-than-air flying systems has highly favorable features. Specifically, the proposed aerial vehicle has several advantages, including increased payload capacity, extended flight duration due to its utilization of buoyancy for hovering, exceptional aerial stability, and the absence of significant motor-induced oscillations that may potentially interfere with the performance of sensors and cameras. In contrast to an aeroplane, the wing exhibits the highest degree of variability, but the fuselage demonstrates the greatest level of flexibility within the aircraft. Airships are typically classified into three distinct types based on their hull design: rigid airships, non-rigid airships, and semi-rigid airships. Rigid airships are equipped with a motorized fuselage that houses numerous inert gas cells or balloons, which serve the purpose of generating lift. Both semi-rigid and non-rigid airships employ an air-filled envelope. In contrast to non-rigid airships, semi-rigid airships possess a rigid keel positioned beneath the hull, which serves to evenly distribute substantial loads throughout the entirety of the hull. Currently, non-rigid aircraft have become the prevailing norm in the aviation industry.



Fig.1.1 The Skyship-500 airship [56]

By adjusting the capacity of the balloon, a notable difference is established between the ambient atmospheric air and the less dense gas enclosed within. This balloon serves as a protective airbag within the vehicle's fuselage, as depicted in Fig. 1.1. The dynamics of airships are inherently nonlinear due to their large, flexible structures, and the interaction between aerodynamic, buoyant, and inertial forces. As a result, bifurcation analysis has become a crucial tool for understanding the complex stability behavior of airships under varying operational conditions.

1.1.2 MOTIVATION: Airships are employed for the purpose of facilitating substantial airlift operations and conducting surveillance missions. These aircraft, being characterised by their low-speed flying capabilities, exhibit significant potential in the context of search applications and the collecting of geographic system information. In the stratospheric region, airships exhibit superior performance compared to other types of aircraft, including satellites. Consequently, in recent years, scholars have endeavoured to advance the development of robots inside this stratum. There must also be a management strategy that ensures collision avoidance. The status of the research development highlighting lift control strategies of different types of hybrid airship (conventional rigid airships, conventional variable buoyancy airships, conventional semi rigid airships and conventional non rigid airships) are shown

below in Fig. 1.2a to 1.2d.

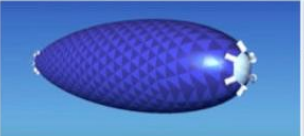
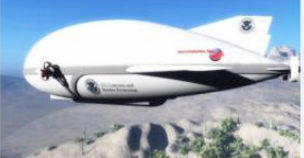
Airship	Country	Airship type	Lift control	Graphic	Status
Euro Airship Corsair	France	Rigid, variable buoyancy, fixed volume	Air pressurization / release + vector thrust + ballonets		Construction-ready design prepared.
Euro Airship DGPatt	France	Rigid, variable buoyancy, fixed volume	Air pressurization / release + vector thrust + ballonets		Construction-ready design prepared.
Skylite Aeronautics GeoShip	USA	Variable buoyancy, fixed volume, rigid, solar powered	Lift gas pressurization / release + aero lift + ballonets		Conceptual design circa 2011.
AeroVehicles Inc. (AVI) Aerocat R-12 & R-40	Argentina	Variable buoyancy, fixed volume	Lift gas pressurization / release + vector thrust + aero lift + ballonets		MiniCat prototyped under Development.

Fig. 1.2a: Conventional rigid airship (source: <https://lynceans.org/tag/aeroscraft/>)

Airship	Country	Airship type	Lift control	Graphic	Status
Global Airships Atlas	USA	Variable buoyancy, fixed volume	Ballonets + vector thrust + proprietary system		Conceptual design phase.
LTA Aerostructures LTA-10 & LTA-70	Canada	Variable buoyancy, fixed volume, lenticular	Lift gas pressurization / release + vector thrust + aero lift + ballonets		Conceptual design phase 2014-2015. Company failed circa 2018. Assets acquired by Global Airships.
Imaginative Alert, Invitation & Kugaaruk	Canada	Variable buoyancy, fixed volume	Air pressurization / release + aero lift + ballonets		Conceptual designs circa 2014 - 2018, intended for Arctic use.
EADS Tropospheric airship	France	Variable buoyancy, variable volume, twin-hull	Alter volume of lift gas envelope + vector thrust + aero lift + ballonets		Conceptual design, model displayed at Paris Air Show 2013.

Fig. 1.2b: Conventional variable buoyancy airship (source: <https://lynceans.org/tag/aeroscraft/>)

Airship	Country	Airship type	Lift control	Graphic	Status
Dirisolar DS-12, DS-900 & DS-1500	France	Variable buoyancy, rigid, solar powered	Vector thrust + aero lift + ballonets + helium volume control		Conceptual design phase.
SolarAirShip High-Speed Solar Airship (HSSA)	USA	Semi-rigid, solar powered	Vector thrust + ballonets		Concept, circa 2010.
Atlas (former Augur RosAeroSystems) HAA Berkut	Russia	Semi-rigid, solar powered, high-altitude, unmanned	Vector thrust + ballonets		Conceptual design phase.
Egan Airships PLIMP Model D drone	USA	Semi-buoyant plane / blimp hybrid (PLIMP)	Vector thrust + aero lift + ballonets		Prototype flew in 2018. FAA certification in progress.

Fig. 1.2c: Conventional Semi rigid Airship (source: <https://lynceans.org/tag/aeroscraft/>)

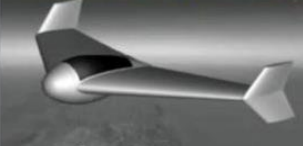
Airship	Country	Airship type	Lift control	Graphic	Status
Egan Airships Model J	USA	Semi-buoyant plane / blimp hybrid (PLIMP)	Vector thrust + aero lift + ballonets		Announced in November 2018. FAA certification in progress.
Festo b-IONIC Airfish	Germany	Non-rigid	Aero lift		First aircraft to demonstrate use of solid state (ionic) propulsion in 2005
Phoenix	UK	Non-rigid, solar powered	Ballonets + helium control + center of gravity control		Sub-scale model demonstrated variable buoyancy propulsion in 2019
New Mexico State University Advanced High-Altitude Aerobody (AHAB)	USA	Non-rigid, solar powered	Ballonets + helium control + center of gravity control		Sub-scale model demonstrated variable buoyancy propulsion in early 2000s

Fig. 1.2d: Conventional Non rigid Airship (source: <https://lynceans.org/tag/aeroscraft/>)

1.1.3 OBJECTIVE OF THE RESEARCH PROPOSAL:

Major objectives for this research are:

1. Bifurcation analysis for longitudinal dynamics, stability, and trim analysis of a non-linear mathematical model for single body dynamics (using wind tunnel and CFD data) of hybrid airship with suspended payload.
2. Modelling and simulation for longitudinal dynamics, stability, and trim analysis of a non-linear mathematical model for two-body dynamics of hybrid airship with suspended payload.
3. Designing a PID controller using single dynamic model for maintaining longitudinal control for closed loop analysis.
4. Designing a non-linear dynamic inversion (NDI) controller using two body dynamic model to achieve directional control for trajectory tracking and collision avoidance.

1.2 RESEARCH METHODOLOGY:

It will be necessary to develop tools and procedures to make various research phases—such as modelling, analysis, control design, and validation—easier. Create control relevant modes ranging in complexity and dependability to be used in controller analysis and design to offer vehicle simulation in order to assist with the design and development of advanced control methods. To enable the best possible use of the simulation, design the framework such that the models may integrate in a common and modular way. The uncertainty in monitoring expectations is mainly due to the inability to accurately estimate the aerodynamic and control coefficients of hybrid airships. However, the controller designed for the airship will ensure system stability and provide good control performance. The goal of final control is the stability, direction control and trajectory tracking.

The research has been conducted in the following major phases:

Phase I: The literature is an unending process of exploring, which provides context to the research work by establishing the control objective of the research, exploring the non-linear flight mechanics and dynamics of airship, defining the theoretical argument under discussion, and refining the aims,

objectives and motives for conducting the study.

Phase II: Using the laws of physics describing the aerodynamics and nonlinear dynamics of the hybrid airship, a mathematical model of the hybrid airship is created to achieve the desired goals. Determining a sufficiently simplified model requires deciding which physical variables and relationships are unimportant and which are important to the accuracy of the model. In general, to get an overview of the solution, it is advisable to build a simplified model first, and then a more detailed mathematical model can be created and used for a more comprehensive analysis. However, in the case of a simplified model, it is necessary to decide which physical variables and relationships are unimportant and which are important for the accuracy of the model.

Phase III: Modelling, simulation, and bifurcation analysis.

A modelling environment for control engineers to use early in the design process to understand the complex interactions between aerodynamics, parameter estimation, control system, and structural dynamics in each configuration and improve trajectory tracking and control strategies is the goal. One-body and two-body dynamics are modelled. Flight control engineers can use such models to understand how these interactions affect system dynamics and make trade-offs to improve vehicle handling. Airship stability involves analyzing the vehicle's response to small disturbances in various flight regimes. Traditional linear stability analysis can only provide insights into stability near equilibrium points. However, airships often operate in regimes where nonlinear effects dominate, making bifurcation analysis essential.

Phase IV: Development of methodology.

The methodology proposed as thesis workflow for this study is shown below in Fig. 1.3. The flow of the research started with literature survey in the primary focus area of modelling of hybrid airship, bifurcation theory, controller design and “LTA” aerodynamic modelling. With literature survey gap analysis was done on the identified research gap. Based on that, the research problems are formulated with proposed solutions in the form of research objectives.

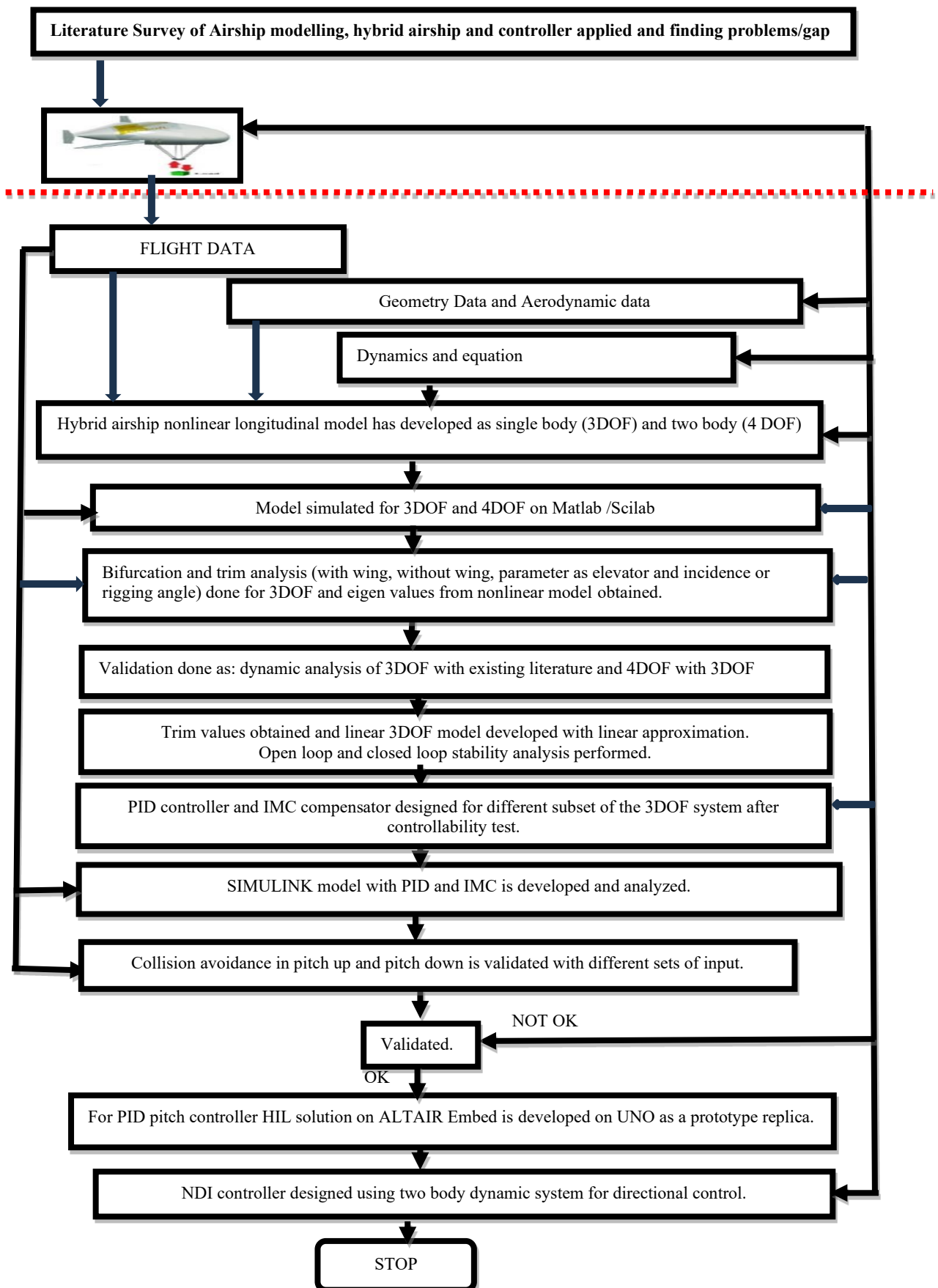


Fig. 1.3 Thesis Workflow

Phase V: Validation.

This phase has carried the validity analysis and modifications has made in the simulation model as per design requirements. Debugging and system analysis will be carried out throughout the different stages of the model. 3DOF model is verified with [27] and [28]. 4DOF is verified with 3DOF. Bifurcation result of 3DOF verified with result of [29].

Phase VI: Results, Conclusions.

Results will be analyzed with the help of the simulation environment, and results obtained during the research work have complied with the thesis. Conclusions are based on the simulation results which are achieved during the research work. Analysis of texts and documents will be referred from the available resources form the worldwide research institutions focusing on hybrid airship research. Research papers from journals and conferences have been consulted.

1.3 TENTATIVE STRUCTURE OF THE THESIS:

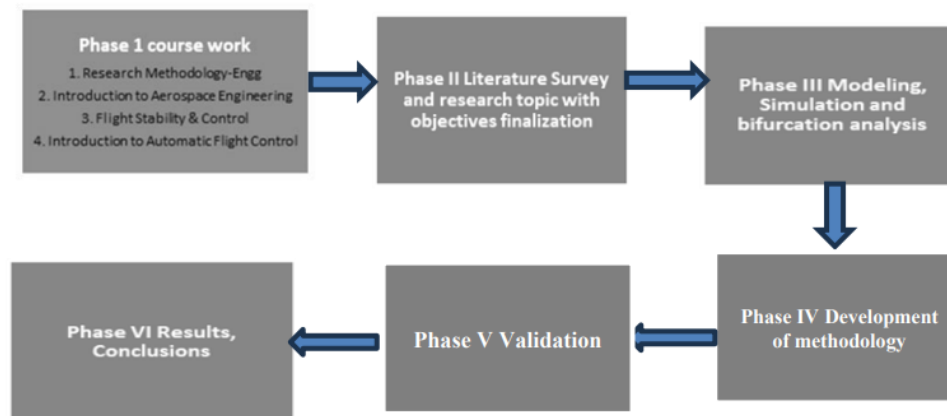


Fig. 1.4 Flow chart of tentative structure of research work

To describe section 1.3 as research methodology, Fig. 1.4 describes the tentative structure of the research work in the form of flow chart.

1.4 ORGANISATION OF THESIS:

At the start of each chapter there is an introduction part in which it is described, which topic and subtopic are covered in that chapter.

The chapter 1 ‘Introduction’ of the thesis covers statement of the proposal having brief introduction about the research objective work followed by the area of the work with present day scenario. Motivation to do the research includes shortcomings in the previous work, research developments of airship, problem statement, uniqueness of the methodology that has been adopted and significance of the possible result. Motivation, objective of the research proposal, research methodology, chapter outline of the thesis, target specification and research work schedule are also described in this chapter.

The chapter 2 ‘literature review’ includes hybrid airship, controllers for airship, development of airship design and application, literature review on airship modelling, analysis and control and summarized outcome of the literature review, Literature review is elaborated as present state/recent development in the work area, brief background theory and literature survey. In this chapter theory related to hybrid airship and controller design for airship is included.

The chapter 3 ‘Mathematical Modelling, includes methodology, body and wind frame transformation, longitudinal dynamic model equation, single body dynamics for 6 DOF and 3DOF as longitudinal dynamics of hybrid airship, two body dynamics include 9DOF and 4 DOF as longitudinal dynamics for hybrid airship with suspended payload. Assumption made for mathematical modelling of airship dynamics is included in this chapter.

Chapter 4 ‘controller design for airship includes NDI controller, MPC controller, PID controller and SMC controller. The controller is designed for heading tracking and trajectory tracking for airship.

Chapter 5 ‘trim and stability analysis using bifurcation techniques includes introduction to bifurcation analysis and types of bifurcation analysis.

The obtained results with explanations are given in chapter 6 as a simulation and Result analysis.

In chapter 7, summary of work with objective and work methodology, general conclusion, future scope of the work, and publications are included.

1.5 SUMMARY:

The chapter deals with the introduction of the system, topics of the research, research objectives and the outline of the thesis. The system taken for the research is small sized winged hybrid airship with control maneuvers like elevator and rigging angle. The nonlinear model for longitudinal dynamic analysis is designed for single body and two body dynamic approaches. Linearized models are also derived for designing a pitch controller. Bifurcation theory is based on continuation algorithms. Trim and stability analysis is done on the nonlinear model in level flight for longitudinal dynamics. Simulation is carried out in MATLAB, Scilab and AUTO bifurcation tools. Control system design tool and Simulink are used in designing controllers. Tentative structure of the thesis is also described.

CHAPTER 2

LITERATURE REVIEW

Literature review is done for hybrid airship, controllers for hybrid airship, airship modelling includes single body and two body dynamic approaches. Due to the novelty in development of two body models for hybrid airship, enough literature is not available directly. To develop two body dynamic models for hybrid airship, idea is taken in existing literature survey of two body parafoil systems. Modelling of hybrid airship is done with modification in the standard dynamic model of aircraft and parafoil system.

2.1 HYBRID AIRSHIP:

The six-degrees-of-freedom equations of motion and “HLASIM” simulation were provided by [30] to test aircraft characteristics in flying field high pressure and air turbulence. The aerodynamic lift due to hull filled with helium gas makes this hybrid airship different from aeroplanes. Estimating aerodynamic forces, moments, and inertia parameters is the hardest part of airship flight dynamics modelling. Determined equations of motion or linear equations can model flight dynamics. Airships have lower weight and center of gravity than planes. The drag moment destabilizes the plane, but the wing force rotates the gravitational moment to stabilize it. Different scenarios require different controls because aircraft models change quickly. Aerodynamic forces affect stability. Thus, stable turning radius can be calculated without inertial factors. Aeroplane flight models are needed for trajectory optimization and stability and external assessments. These studies explore the AS500 airship [31-34]. In aeroplane equations of motion, Reference [3] regarded the balloon as a fixed surface and variable mass. Reference [81] optimized stratospheric airship lifting by converting huge balloons to point-size models. To understand how balloons

affect Aeroplane power, consider balloon inflation and deflating and terms that enhance balloon volume relative to the fuselage. The airbag volume varies greatly with altitude. Airbag mass change influences the aircraft air matrix and should be modelled. If aircraft acceleration is low, airbags have little force. The balloon's rock is harder to model than the body. We study airbag slosh modelling utilising a spring-mass-buffer model and airship response forces. The YEZ-2A blimp prototype was chosen for the tethered airship design because Gomes studied its behaviour and wind tunnel data determined its aerodynamic performance in [35]. Aeroplane's long performance, high cost-to-weight ratio, and low fuel consumption have drawn researcher's attention [36–39]. Airship's revival demands realistic dynamic models and simulation to evaluate flight behaviour and design control systems. Extensive research efforts have been dedicated to a range of programmes pertaining to unmanned aerial vehicles, including the “AURORA” [40], [41], [42] and [43] initiative, which involved piloting from “LAAS/CNRS” [44] and [45] in Brazil throughout the late 1990s, Lotte in Germany Airship [2], Korea KARI [46],[47], Portugal DIVA [48] and IIT Mumbai's autonomous open-air dirigible project [49]. One downside is that all these airships are sensitive to atmospheric conditions, especially storms. Modelling and control of heavy aircraft loading and unloading using a combined power distribution system (CDPM) [50]. Most CDPM studies use the inertial model [51], [52], [53], [54]. Then fix the cable as a straight section. Hybrid airships, low profile airships with aerofoil section, long twin hull airships, combined airships and wingspans, delta airships, airship quadcopters [55], [56], [57], [58] available in configurations. The hybrid airship system consists of an airship body (having both apparent and air mass) and a heavy load attached to the bottom of the fuselage. The ropes hanging from the hull are connected to the payload riser via tie-down points, allowing independent rotational movement of the hull and the load. Therefore, unlike the 6-DOF flight dynamics model of the aircraft, the system needs a 9-DOF flight dynamics model. The 9 DOF model derivative is similar to parafoil payload system that is used in [59], [60], [61], [62] but does not model for the airship pitch and yaw motion and load with springs and dampers originating from the line. By writing the payload displacement and rotation equations and the hull translation and rotation

equations, the nine degree-of-freedom equations of the airship-payload combination in the form of cascading matrices [27] has been developed.

2.2 AIRSHIP CONTROLLER:

A parafoil/launch system may safely deliver a payload to a target area or numerous targets from a single launch [63]. A full dynamic 9 DOF two-body model can represent these motions and the pendulum stability effect. Using a 9-DOF model, this work designs a general tracking control rule for a parafoil/payload system with multibody dynamics and nonlinear aerodynamic data using nonlinear dynamic inversion (NDI). In [64], a four-rotor heavy airship hover flying mode feedback linearization control approach was developed. In [65], a vision-based airship control system and station-keeping PID controller are presented. Under different operational conditions, this method may not work. A loop shape control technique for high-altitude aircraft and attitude-keeping performance analysis were given in [8]. This study neglected non-linear effects and longitudinal-lateral coupling due to a linear model. Adaptive control for stability theory and online estimations of stratospheric airship inertial parameters was proposed in [66]. The estimated values were not near to their true values. An adaptive nonlinear control algorithm was used to track a stratospheric airship with parametric inertial errors in [67] to explore its stability. An adaptive technique for a stratospheric airship was given in [68], and the linear parameterization property and online gradient parameter update rule estimate the first comparative dynamic inversion control method despite the uncertain assumption of inertial parameters. The control rule and nonlinear equations of inertial motion and adaptive control are based on passive airship control; therefore, the motion algorithm was designed accordingly. In [69], sliding mode controlled the airship. Tracking minimized distance error and rate relative to the reference track. Feedback control was used to stabilize an airship resistant to unmodified dynamics in [70]. Nonlinear controllers were created for the nonlinear model because: Fuzzy logic controllers can be used with standard control methods, are easy to design, and tolerate model imperfections. Airship stabilization and control during wind disturbances are essential in any application. Many applications employ PID

control. The AURORA airship project develops sensor, control, and navigation technology for autonomous or semi-autonomous airships [4]. PID, PD altitude, and PD heading controllers govern longitudinal speed, altitude, and heading. Another solution for airship heading control uses a PID controller with H_2/H_∞ gain design [7]. Inversion control of the synthetic neural network model is used to control airships [71]. It combines feedback linearization with linear control. Neural network theory includes feedback linearization correction and modelling error. A dynamic inversion control law was designed to track the airship's trajectory and force the closed-loop system's outputs to follow the position commands [72]. The control law shows how to swiftly rectify trajectory errors. Also, Lyapunov stability-based state feedback rules solved the airship control problem [73]. A step-back technique creates a closed-loop trajectory tracking controller for an underactuated airship [74]. The authors say reversing gives designs flexibility and robustness to parametric uncertainties in aerodynamic modelling and airflow disturbances. It fits vehicle dynamics' cascading nature. Other airship control methods include sliding mode control and fuzzy logic design. This work uses linear quadratic (LQ) methods to create a gain scheduling non-optimal feedback control rule. Lanes-specific navigation algorithms will tell the controller to fly the airship across planar intermediate spots. Its simplicity and ease of implementation indicate LQ-based controller design's suitability for multivariable systems [75]. The work presented is similar to the work of [76] and [77] implementing his LQ controller to control the guided flight of airships, but the precise gain scheduling planning and navigation methods are not discussed here in detail. In [77], 11 trim points are required to execute the gain scheduling method, and the guidance method produces a commanded yaw rate based on the course of the airship and the commanded course to the next increased waypoint. The purpose of this study is to conduct a rapid investigation on the topic of random free obstacle avoidance path planning, with the aim of mapping the trajectory in the presence of impediments. The purpose of this controller is to accurately track the reference trajectory [78]. It is feasible to develop control strategies that can effectively stabilize the complete state space of an unmanned aerial system

(UAS), without the necessity of partitioning the system into distinct positional and translational subsystems. In order to evaluate the effectiveness of the control laws, a set of simulations is conducted. The results of this study indicate that the suggested methodology effectively directs the unmanned aerial system (UAS) along the desired trajectory while maintaining a consistent altitude [79]. In reference [80], it is observed that the characteristics of lighter-than-air flight exhibit notable distinctions when compared to the dynamics of conventional heavier-than-air flight. The research indicates that the airship had considerable impacts from the wind. The AS500 airship has the capability to be controlled using a rudimentary LQ controller. This controller receives navigation instructions that are derived from either a track-specific guiding law or a relative navigation law. These instructions are designed to guide the airship over several waypoints. It is important to note that these navigation commands are applicable only in windless conditions and when there are time-varying winds.

2.3 DEVELOPMENT OF AIRSHIP DESIGN AND APPLICATION

 Piasecki Helistat hybrid rotary airship	 US Navy YEZ-2A airship mockup	 1988 Pepsi airship tour
 Cargo Lifter heavy lift airship concept	 DARPA ISIS stratospheric concept	 SAIC SKYBUS 80K



LEMV hybrid airship



Lockheed Martin LMH-1 hybrid airship concept



Ohio Airships demonstrator rotational flight test



Millennium Airships Sky freighter with its four Thrust Wings



Dirisolar concept installation of solar cells



Aerovehicles AerocatR-40



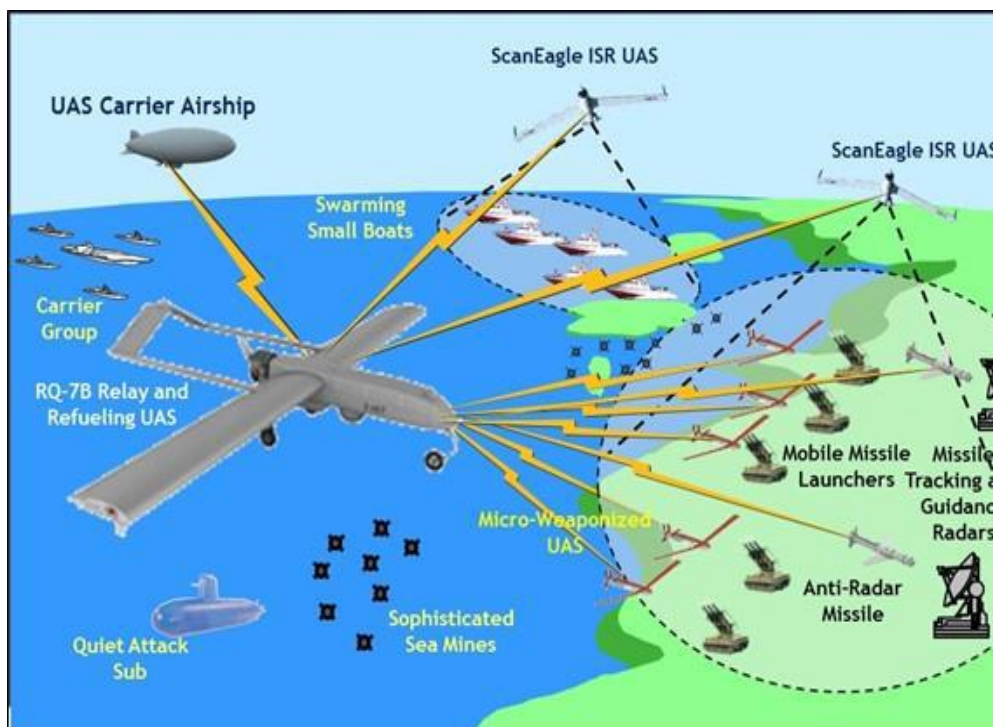
Airship GP Super-hybrid



Lockheed Martin hybrid with sling load



Notional UAS carrier airship with deployed UAS squadron, (Source: <https://ntrs.nasa.gov/api/citations/20180001627/downloads/20180001627.pdf>)



OTH(Over the horizon) control of various UAS from airship via relay UAS (source : <https://ntrs.nasa.gov/api/citations/20180001627/downloads/20180001627.pdf>)

Fig.2.1 collection of images as development in airship Design and applications
(Source: <https://lynceans.org/tag/aeroscraft/>.)

Figure 2.1 shows the development of Airship design and applications like over the horizon control of various “UAS” from airship, “UAS” Carrier airship with

deployed “UAS” squadron and as a payload delivery system.

2.4 DETAILED LITERATURE REVIEW:

"Modelling and Simulation of Airship Dynamics" was released in 2007 [2]. The model incorporates flight mechanics, aerostatics, and aerodynamics. This work presents a thorough aerodynamics estimation approach that accounts for additional weight, viscous effects, fin forces, and axial drag. Flight test results and frequency domain control responses from control surface deflections are significantly correlated. The airship's stability is then analyzed at various speeds. All motion is neutral at zero velocity.

"Aerodynamic Estimation Techniques for Aerostats and Airships" was published in 2013 [6]. A mathematical model was created to explain an axisymmetric finned body's steady-state behavior. After that, the model was used to analyze wind tunnel data from five aerostats and airships. This analysis sought to determine fuselage-wing interaction and crossflow drag coefficients. The local airflow analysis over the fuselage and fin cross-sections is integrated into the existing model to account for unstable motion. The present dynamic model is developed for nonlinear simulations and rotational stability derivative and equation calculations. When convective acceleration is included, the rotational stability derivatives from static wind tunnel data match the swing arm traction box data.

The paper "Modelling and Near-Space Station keeping Control of a Large High-Altitude Airship" (2007) [8] explores modelling and controlling a large airship for near-space station maintenance. In strong winds, spacecraft must use propulsion to maintain station keeping, incurring significant limits. This study investigates vehicle influence and directional dynamics, wind conditions, and basic control and guidance approaches. The vehicle's frontal or impact position, lateral position, and cornering rotation will be examined in the following study. The "pendulum mode," or roll degree of freedom, will be ignored.

The article [29] titled as “Airship Trim and Stability Analysis Using Bifurcation Techniques” published in 2016. In this paper, bifurcation analysis of an airship model with full six degrees of freedom is performed under both unconstrained

and constrained conditions. The airship's equations of motion are first modified into a suitable form and standard and bound bifurcation analysis is performed with respect to the airship's control and design parameters. In this analysis, a coupled nonlinear airship model with six degrees of freedom (DOF) is used, assuming that the airship is rigid. The equations of motion consist of force equations written in the wind axis and moment equations written in the body axis assuming fixed centres of gravity (CG) and buoyancy (CB). The location of the CG was chosen as the origin for deriving the equations of motion. The added mass and inertia terms are also considered in the equation, and since they are a function of body shape, they are assumed to be constant, satisfying the rigid body assumption. The kinematic equations associated with the force equations are also preserved. The known equations of motion of airplanes are therefore extended to the airship by buoyancy, added weight and inertia. A complete set of equations of motion used for global stability analysis.

In [30], "Flight Dynamics Simulation of a Heavy Lift Airship" (1982) examined the performance, stability and control characteristics of the four-rotor heavy airship concept, which can be easily removed, were determined using the test model of this configuration. Nonlinear equations of motion were used to generate hybrid computer simulations of the systems and to provide linear models of configuration dynamics. Here it is assumed that the equation generated by the payload is non-linear and depends on the vehicle and payload motion variables. Keep a distance equal to the length of the cable from the hanger.

The article [39] titled "Dynamic Modeling, Control and Station-Keeping Guidance of a Large High-Altitude 'Near-Space' Airship" was published in 2006. Next, use the 6-degree-of-freedom body equation to describe the dynamics of the aircraft in flight, including the effects of added weight and inertia. Numerical simulations were performed to verify the displacement of the cruise ship without minor disturbances. Finally, the report showed that the length and rear of the aircraft were unstable, and within a few seconds the length and back were weak.

The article titled "Modelling and control of an aerial robocrane using a wire

driven system” was published in 2019 and referred in [50]. This research focuses on the flight path of a light aircraft while holding a container. However, a terminal driven by a tethered robot is more difficult to control than a stationary robot on the ground. In particular, the aircraft will be exposed to various external influences caused by the winds, which will cause many problems such as interference with the operation of the operation. The contribution of this article is the development of a powerful controller that makes up the terminal. In addition, before the dynamic control system is developed, the dynamic model of the dual system including the airship and the wired parallel robot needs to be clarified. The CDPM formula is derived using the Newton-Euler method.

In "Adaptive Trajectory Tracking of Stratospheric Airship Based on Input-output Stability Theory" [66], input-output stability theory is applied to stratospheric airship trajectory monitoring in year 2011. This work proposes using entrance and exit stability to adaptively regulate flight trajectory tracking in a fully operated stratospheric airship. The suggested method uses online inertial parameter estimate and trajectory tracking law to autonomously track a continuously time-varying desired trajectory from any beginning condition. This is especially important when inertial parameters are uncertain.

Reference [67] states, in 2015, "Nonlinear Adaptive Trajectory Tracking Control for a Stratospheric Airship with Parametric Uncertainty" was published. This paper presents a control method for nonlinear adaptive trajectory tracking. The fundamental idea is to create a new filter design for airship dynamics regressor terms. Several second-order mechanical systems use the proposed strategy. Future study could provide controller design methods and robust analytic methods for airships with parametric errors, unknown aerodynamic coefficients, and external disturbances.

The article titled "Adaptive and Robust Control of a Stratospheric Airship" was published in 2019 [68]. This work presents the formulation of a dynamic model for a fully operated 6-degree-of-freedom (DOF) stratospheric airship, utilizing generalized coordinates. The application of adaptive and resilient control techniques, specifically those based on passivity, is employed for the purpose of controlling the airship. The utilization of the Lyapunov stability theory serves

to present a succinct illustration of the stability of the closed-loop control system.

Reference [69] states, the 2007 paper "Sliding Mode Control for the Path Following of an Unmanned Airship" was published. This paper details the design of two MIMO Sliding Mode Controllers. These controllers focus on longitudinal and lateral modes for the AURORA airship's guiding and navigation system. Path tracking reduces angle and distance tracking errors relative to reference trajectory.

"Airship Hover Stabilization Using a Backstepping Control Approach" was published in 2006 [70]. This research study takes a new approach to airship hover stability. The model is used to build feedback formulation for aircraft hover control. The control design combines saturations and creates global asymptotic stability under saturation situations to manage reduced actuation limits. To handle the large lateral vacuum, the control target is changed.

The article [73] titled as "Stabilization and Trajectory Tracking of Autonomous Airship's Planar Motion" published in 2008. In a recent study, Valdmanis achieved the localization of an airship in a constrained state space—three translational DOFs and a yaw rotation—and a known environment using an advanced Monte Carlo Localization (MCL) algorithm and a LIDAR system. The main purpose of this balloon is the ability to operate independently, maintain a safe distance and avoid obstacles in the indoor hall.

The article [76] titled as "Gain Scheduled LQR Control for an Autonomous Airship" published in 2011. In this paper, we explored this potential of an artificial neural network for characterizing the nonlinear dynamics of an unmanned airship. A sufficiently trained neural network model captured and satisfactorily demonstrated the longitudinal dynamics of the airship. We have also demonstrated the usefulness of the proposed technique for the Lotte airship, where the performance of the proposed model is validated and analysed for Lotte airship flight test data. In this paper, a DRNN-based methodology was presented to identify the longitudinal dynamics of an airship using data from flight experiments.

The article titled “Control of Airship Motion in Presence of Wind” was published in 2011 and referred in [77]. A closed loop can be used to dampen the movement of the load relative to the vehicle, but it does not provide adequate control of the load during groundwork. It has been found that tighter control is required to control the vehicle's position than when the truck is lifted. The strength of the vehicle's payload suspension elements is important to predict the response of the payload to impact. The length of the suspension cable is an important parameter in determining the degree of dynamic linkage between the vehicle and the payload and creating a closed-loop control system to improve the vehicle's performance. The system's response to atmospheric turbulence increases as the magnitude of the turbulence approaches the size of the vehicle 1 payload configuration geometry. Other studies conducted by Goodyear will explore the impact of the linkage with other lateral and directional dynamics of the system in response to crosswinds. A similar wind distortion model would include turbulence components at all rigid body degrees of freedom of the vehicle. The detailed design of a vehicle's closed-loop control system can be a complex task.

The article [78] titled as “Autonomous obstacle avoidance manoeuvring of thrust-vector airship with neural network control” published in 2019. Starting from the mathematical modelling of a thrust vector airship, this paper deals with the integrated motion planning and control unit development for vehicle autonomy, taking into account various uncertainties. A fast exploration of random tree obstacle avoidance path planning is performed to map the trajectory in the presence of obstacles. Then, a neural network-based sliding mode controller is subsequently designed to learn the unknown equivalent control under the sliding mode control to follow the reference trajectory. In this work, the development of a mathematical model for a 6-degree full thrust vector airship is demonstrated along with motion planning and intelligent control. In many practical engineering problems, the directional definiteness of the control gain matrix cannot be determined. In order to correct these problems, the author is currently working on an integral learning control algorithm for optimal tracking of nonlinear continuous-time systems.

The article [79] titled as “Nonlinear Control and Trajectory Tracking Nonlinear Control and Trajectory Tracking” published in 2018. In this paper, the problem of tracking the trajectory of autonomous lighter-than-air vehicles in the presence of wind gusts is studied. The airship is represented as a single point, which is its centre of volume, and only kinematic equations are considered for trajectory tracking design. The control algorithm uses Lyapunov theory and Sontag's universal stabilization feedback. The proposed control laws are robust, structurally, with respect to uncertainties in the model and unknown external parameters. The problem was formulated using optimal control theory to generate a 3D flight path, with travel time as a cost function to be minimized. Future work involves generalizing this method by introducing system dynamics that will allow us to cover the hovering flight regime and take into account the characteristics and limitations of the airship.

The article titled "Equations of Motion of a Vehicle in a Moving Fluid" was published in 2000 [82]. This model provides an equivalent set of instructions describing the movement of underwater vehicles, airships, parafoils and aircraft in moving water.

Reference [83] discusses "A review of airship structural research and development" from 2009. This paper thoroughly examines airship design, structural analysis, and simulation methods. The discourse includes a brief consideration of airship body configuration optimization. Analysis of structural components is the focus of this review.

"Influence of Wind Speed on Airship Dynamics" was published in 2002 [84]. A novel formulation of the equation of motion for an aeroplane is provided in this work to investigate how wind affects aircraft power. The Lagrangian approach, which integrates three types of kinetic energy, was used to formulate the equation of motion. Internal ship energy, relative velocity-induced environmental energy, and wind power-powered propulsion energy are included.

The article titled "The Shenandoah flies again: a computer simulation" was published in 1981 [85]. The purpose of this study is to measure the kinematic

parameters and the nature of the load in the plane to control the intake and pressure in the air. This is done through nonlinear flight dynamics simulations that use crossflow aerodynamics theory to model the behavior of the fuselage and fins at “higher angle of attack. The US Navy ZR-1 airship "Shenandoah" is used for simulation as an example; The special cases examined are: (1) right turn at maximum altitude in smooth air; and (2) average flight with 1 cosine lateral wind at maximum speed.

The article titled "Modeling and Simulation of Flexible Airships" was published in 2009 [86]. The combination of flexibility and aerodynamics is achieved by deforming the local velocity of the airship. The resulting dynamic model is represented by a set of differential equations describing the physical parameters, elastic deformation, and the interrelationship between them. A dynamic simulation was developed in the MATLAB environment to examine the dynamic and frequency response of the Skyship-500 airship.

The article [87] titled "An investigation into the flight dynamics of airships with application to the YEZ-2A" was published in 2015. This document creates a non-linear model of the aircraft sound channel and uses the traditional PID control method to control aircraft sound. The simulation results show that the aircraft can follow the desired pitch attitude in 1000 seconds and remain stable. A six-degree-of-freedom model of the YEZ-2A airship is used.

The article [88] titled “Modelling of a heavy-lift airship carrying a payload by a cable-driven parallel manipulator” was published in 2019. It suggests a kinematic model and a design model that were developed by considering the aircraft and the robot as an integrated system. A heavy aircraft is a strong fuselage made up of many interconnected fuselages. The motion of the plane is usually represented by a set of kinematic and dynamic equations that describe its evolution in 6-DOF space. The numerical simulation concept was developed using MATLAB/Simu-link. The flow dynamics model is derived from Kirchhoff's equations.

The article [89] titled “Autonomous Flight Control System for an Airship” published in 2010. The main purpose of the paper is to investigate and design

longitudinal and transverse directional control allocation systems to optimize the operation and performance of an unconventional control system consisting of fixed and thrust vector propellers. The airship is considered a rigid body without aeroelastic effects and symmetric about the XZ plane so that both CV and CG lie in the plane of symmetry. The airship's mathematical model is based on a 6 DOF nonlinear model that follows the standard dynamic modelling of a conventional aircraft. However, due to the large gas volume and aerostatic lift, there are significant additions to the known aircraft equations of motion, such as the lift force B and the apparent mass and inertia terms. The 6 DOF equations of motion can be developed by implementing Newton's second law of motion for each degree of freedom.

The article [90] titled as “Collision Avoidance and Stabilization for Autonomous Vehicles in Emergency Scenarios” published in 2017. This paper presents a single controller capable of mediating between the sometimes, conflicting goals of collision avoidance, vehicle stabilization, and route tracking. The driver immediately reacts to new emergency scenarios and can use all the vehicle's performance capabilities to avoid an accident. Prioritizing collision avoidance above any other goal, including stabilization, leads to a different paradigm from existing approaches.

The article [91] titled as “Simulation of Proportional Guidance Law with an Airship” published in 2015. A method for simulating the missile guidance process with an unmanned airship controlled by a homing seeker for the law of proportional guidance is designed and verified by the flight test. According to the control characteristics of the airship, we designed the corresponding control law of the airship based on its prepared PID controllers and successfully realized the proportional homing flight with the airship piloted by the line-of-sight angular velocity signal produced by the TV seeker loaded on the airship. . It stands to reason that if this technique is used to simulate a jammed missile object in an electronic warfare test and evaluation, it would be possible to test the jamming effect of electronic warfare weapons on the tracking status of the seeker and further on the flight control system. , about the flight status of its carrier, so he could visually demonstrate and verify the effect of electronic

interference on missiles.

The article [92] titled as “Aerodynamics of a Hybrid Airship” published in 2012. The obtained results show that the addition of a wing to a conventional airship increases the lift at a positive angle of attack by three times the lift compared to an airship without wings, while the drag increases in the range of 19% to 58%. Longitudinal and directional stability have been found to be statically stable, but both conventional airships and hybrid or winged airships have poor pitch stability. An airship without wings has slightly higher longitudinal stability than an airship with wings. A winged airship has better directional stability than a wingless airship. An airship without wings has only static stability when turning in the range of -5° to 5° yaw angles. On the contrary, the originally tested winged airship has no rolling stability at all. Computational fluid dynamics (CFD) simulations show that modifications to the location of the wing and its dihedral have a strong positive effect on roll stability. Raising the wings to the centre of gravity and introducing an angle of 5° stabilizes the rolling motion of the winged airship.

The article [93] titled as “Nonlinear Simulation Framework Based on Bifurcation Analysis for Controlled Airship Manoeuvring” published in 2019. A simulation framework for constructing a combined, nonlinear, six-degrees-of-freedom stratospheric airship model by combining continuous flow with a nonlinear floating control system. The continuous bifurcation method was used to provide information on the horizontal slide turn performance, which has important implications for the design of uncontrollable systems for autonomous manoeuvring of airships. The target is not a sliding control frame used in a closed loop for steering and control of planned aircraft manoeuvres. The relevant simulation results show the effectiveness of the proposed method. In this study, a stratospheric aircraft with an altitude of 21 KM and a payload of 1500 KG is considered. The -30 -degree aileron deflection results in a bank angle of 3.54 degrees, which is too small for effective turn manoeuvring. Non-linear SMCs designed for the steering function generate the Euler values required to steer the aircraft to the desired path, while non-linear SMCs designed for the control function generate commands to reach the Euler values required by the

regulation. In this paper, a bifurcation-based continuation method is combined with a nonlinear sliding-mode-based control process to create a method for manoeuvre design and control.

The article [94] titled as “Adaptive sliding mode trajectory tracking control of robotic airships with parametric uncertainty and wind disturbance” published in 2017. The Modified Floating Mode Orbit Tracking Controller is designed for piloting aircraft with uncertain and unknown wind fluctuations. The main idea includes a change in the gain scheduling controller to avoid offline competition. In particular, control pattern difference is used to solve unknown physical and unknown wind effects, and boundary layer processing is used to avoid unwanted management surge event. Computer experiments were conducted to demonstrate the efficiency and effectiveness of the control system. Future work will focus on the problem of airship trajectory tracking with a saturated finite-time adaptive sliding mode control.

2.5 SUMMARY:

As a result of the literature review, the following problems have been identified, and solutions have been proposed:

✓ Problems in landing maneuverability and control reliability were experienced in unmanned ships.

- Hybrid airships include wings, and tethered suspension loads as a solution. The 9DOF model is selected for two body dynamics.

- The parafoil/freight transport system can be used to deliver cargo safely across a designated area or to deliver cargo to multiple destinations in a single launch referred to [63]. A full 9 DOF two-body model can be used to capture these differences as well as pendulum stability effects.

- Airship's mode changes rapidly, so different situations require different controllers.

Stability behavior is also affected by aerodynamic forces. Therefore, stable turning radius can be estimated without using inertial parameters. In addition to

stability and external analysis, aircraft flight modeling is an essential part of the trajectory optimization solution.

✓ Unlike an airplane, the most variable part is the wing, and the most flexible part of the airplane is the fuselage. Both semi-rigid and non-rigid airships use air-filled envelopes as the hull, and hull quality is controlled by pressure levels higher than ambient air pressure.

- To understand the effect of the balloon on the power of the aircraft, two interrelated problems must be considered: the variation of the balloon during inflation and deflating, and the movement of the balloon affects the fuselage. For flying at different altitudes, the volume of air in the airbag can vary greatly. Therefore, the effect of airbag mass change on the entire air matrix of the airship should be considered in the modeling process.

- The visible dimension matrix is preserved in the hybrid airship model. Articles [82,84,95] improve our understanding of flow connectivity and added aerodynamic elements.

✓ A control strategy is required that guarantees stability and strength against uncertainty, accident avoidance and monitoring, and can produce timely solutions.

- Scroll mode is used to control the plane in [69]. The monitoring strategy reduces the amount of error and the error rate relative to the method used. In [70], a feedback control technique was introduced to stabilize the aircraft against unstable changes.

- The rise time rule and control rule must be followed to command 11 trim points in [77], a legal value down the plane and the next ground. Speed and search random tree-based problem avoidance exercise plan to guide walking in the presence of obstacles.

✓ The uncertainty in the trajectory is mainly due to the inability to accurately predict the hybrid airship aerodynamics and control coefficients.

- Nonlinear dynamic inversion (NDI) techniques use 9-DOF models to propose

dynamic control rules for parafoil/load bearing systems with multiple engines and non-aerodynamic data.

- PID follows the conventional method for single body dynamics of hybrid airship to predict trajectories and to analyze closed loop stability and controllability.

After going through the literature surveys of modelling of airship, aircraft, parafoil payload delivery systems and airship controllers along with project development, a hypothetical model has been proposed in a pictorial way shown in Fig. 2.2, named as “winged hybrid airship flight vehicle with suspended payload”, located at 1st row 1st column, having developed with combining the modelling features of winged aircraft, airship with aircraft feature known as Plimp and parafoil payload delivery flight system.

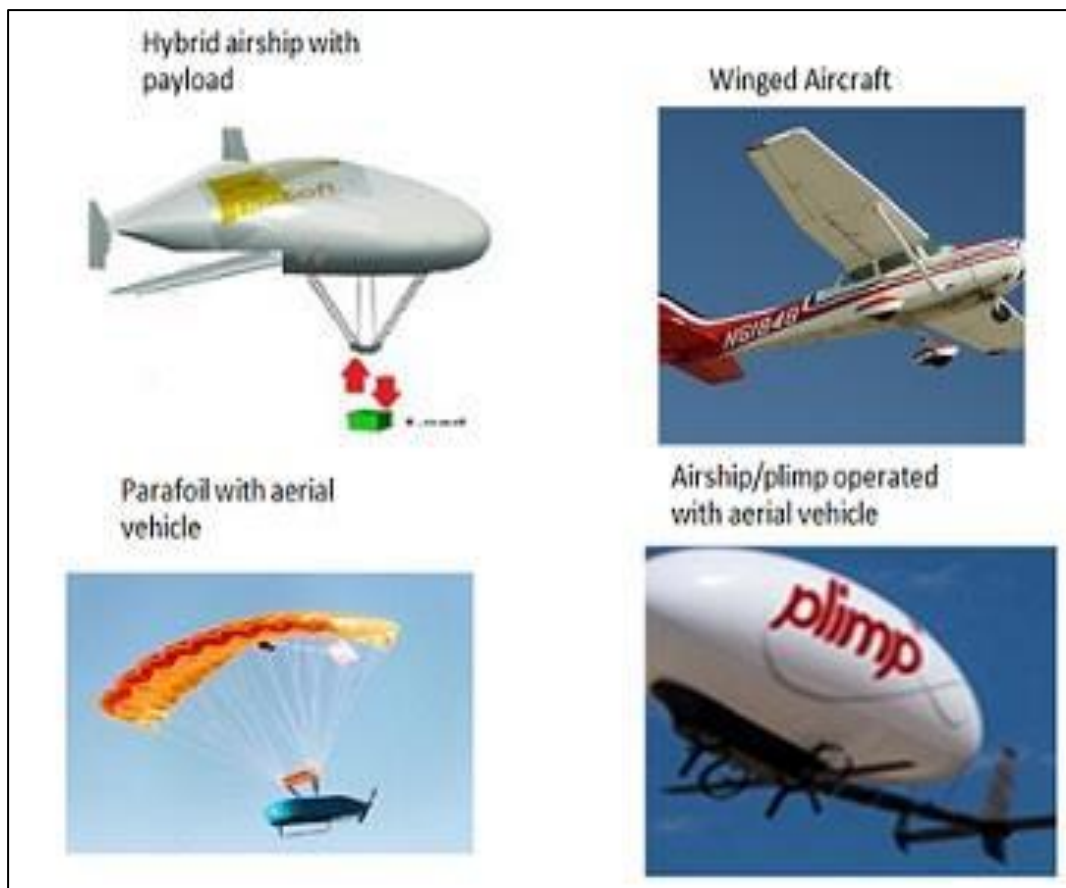


Fig.2.2 Pictorial Idea for winged hybrid airship flight vehicle with suspended payload

CHAPTER 3

MATHEMATICAL MODELLING

Mathematical modelling derivation for hybrid airship is done as single body and two body dynamics to study and analyze longitudinal dynamics behavior of the hybrid airship.

3.1 METHODOLOGY:

To meet the outcome of the research objectives based on the proposed logical hypothetical model shown in Fig. 2.2, firstly a nonlinear mathematical model has developed for longitudinal dynamic model of hybrid airship as 3DOF (single body dynamic approach) and 4DOF (Two body dynamic approach). 3DOF longitudinal model is developed from 6DOF nonlinear mathematical model of hybrid airship that is proposed in the research work [28]. 4DOF longitudinal model of hybrid airship with suspended payload as two body dynamic approach has been developed and proposed with the help of 9DOF model of parafoil payload system, taken from [59] and [60]. 9DOF model of hybrid airship work [27] is also referred to develop 4DOF model of hybrid airship.

Hybrid airships are a type of aircraft that combines buoyant lift, aerodynamic lift, and sometimes thrust to achieve flight. In the literatures airship is basically considered as a hull filled with helium gas. The proposed hybrid airship consists of hull, gondola, Clark Y wings, horizontal and vertical fins connected as cross structure, suspended payload with rigging mechanism and elevator control surface on horizontal tail fins. Based on these add-on configurations the hybrid airship model for single body is shown in Fig. 3.1 and for two body dynamics, hybrid airship model is shown in Fig. 3.2. The anticipated result of a hybrid airship incorporating both a wing and a suspended payload is a collective

outcome influenced by the dynamics of the parafoil payload delivery system as well as the dynamics of the aircraft.

Hybrid airship is designed for low height and for stationary atmosphere. It is developed with the same concept of hull dynamics, aircraft dynamics and dynamics of parafoil payload delivery system. Hybrid airship is basically developed as a payload delivery system. The payload can be either attached to the surface bottom of the airship or suspended to joint knots of raiser lines that is connected to all side of airship. The downward lines originating from the hull are affixed to the payload riser via a connection point, which facilitates autonomous rotating movement of both the hull and the payload.

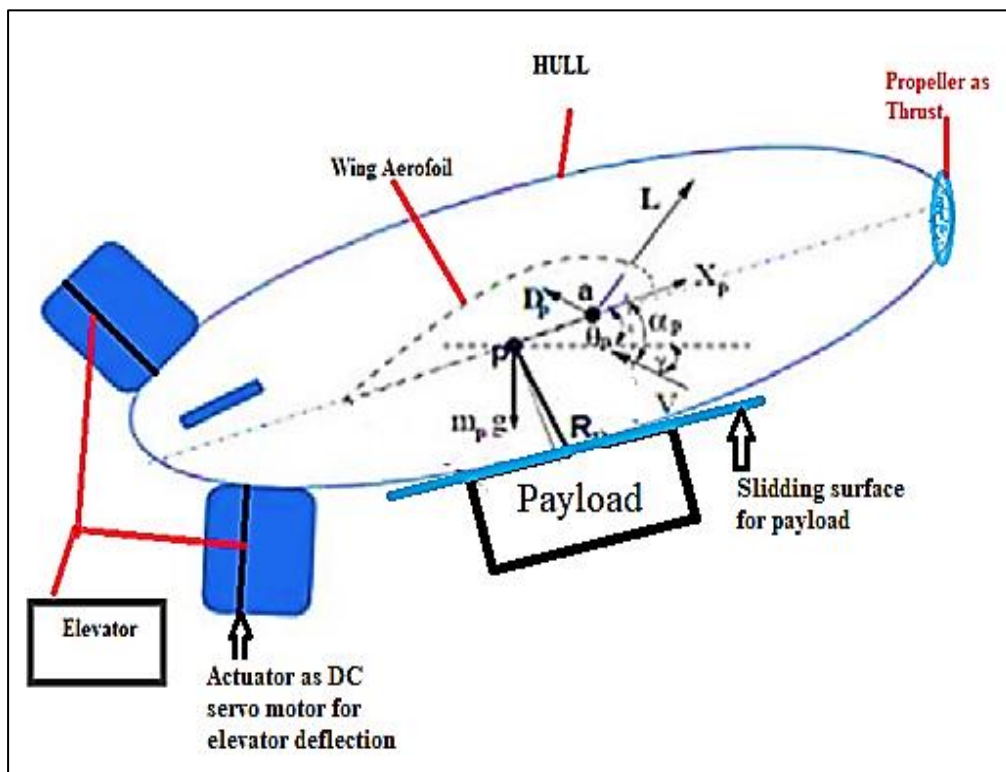


Fig. 3.1 Hybrid Airship as single body dynamics

In hybrid airship model as single body dynamic approach shown in Fig. 3.1, the payload is attached to the surface of hull and can be moved on a sliding surface forward and backward. This concept of sliding surface is similar to providing wing incidence angle / rigging angle effect of a parafoil system and here used to provide the pitching effect in hybrid airship. To deflect the control maneuvers

elevator, a servo dc motor can be used as an actuator. In hybrid airship model as two body dynamic approach shown in Fig. 3.2, the payload is suspended in the nature through a point ‘c’ (joint of ropes). The net effect of the dynamics of hybrid airship and the dynamics of payload is combined for proposing a two-body longitudinal dynamic model of hybrid airship with suspended payload. A servo and pulley mechanism is there at point ‘c’ for varying the length of the joint ropes on either side to provide rigging mechanism in pitching control of the system. The rigging effect is obtained by varying the rigging angle (μ).

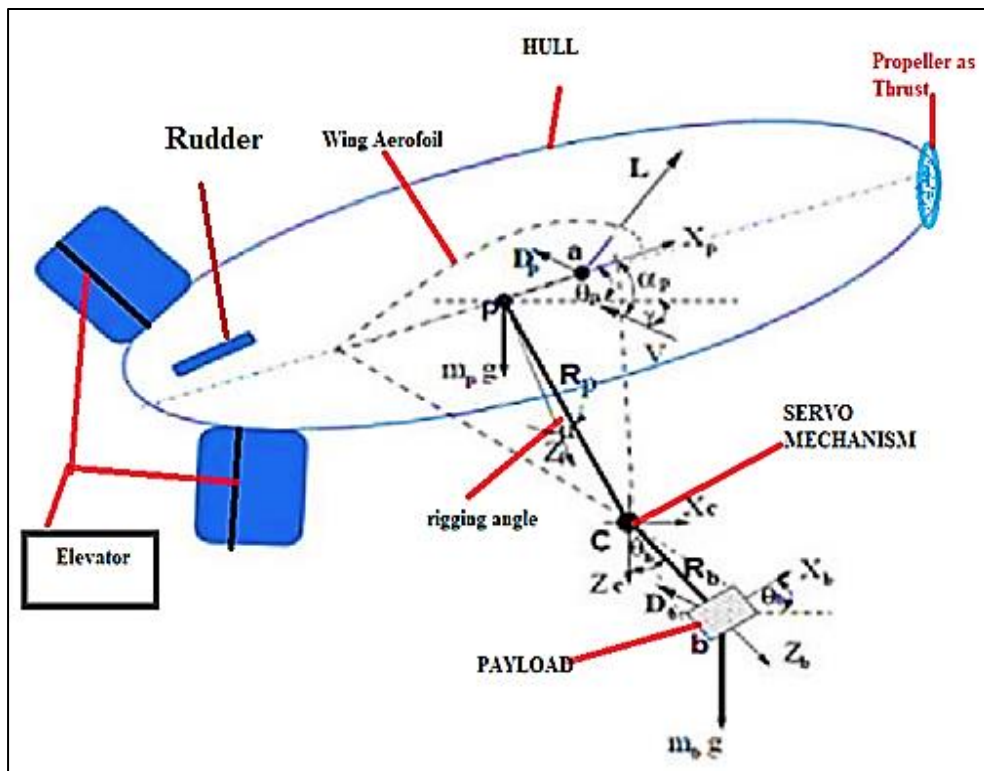


Fig. 3.2 Hybrid Airship as two body dynamics

In aircraft all masses are within the envelop of aircraft. So single body complete dynamic modelling and analysis is required. In airship payloads are always attached/suspends with gondola. So along with single body, two body longitudinal dynamic modelling and analysis is required in hybrid airship.

Three angles can be used to depict an object's orientation in three dimensions using Euler angles. For the rotation of a hybrid airship using Euler angles, it might consider using three angles (often represented as roll, pitch, and yaw) to describe its orientation. The sequence in which these rotations are applied

matters because Euler angles are sensitive to the order of rotations. Let's consider an example where we want to rotate the hybrid airship shown in Fig. 3.3 using Euler angles:

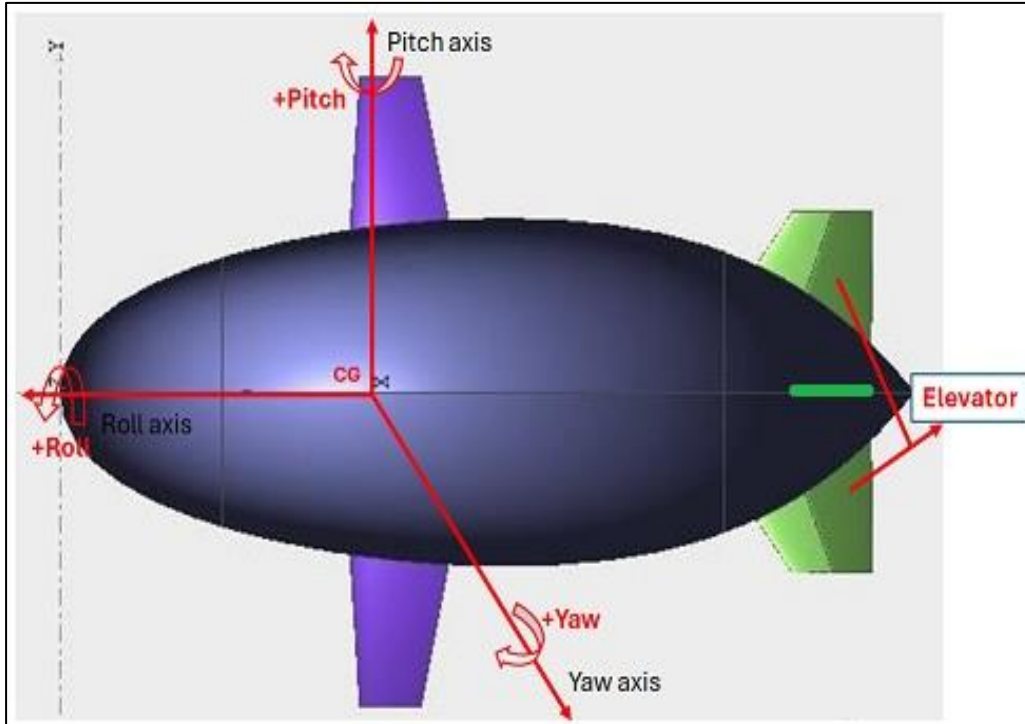


Fig. 3.3 A hybrid airship's body axes system with longitudinal control surface.

Roll (Rotation around the longitudinal axis): This is the rotation along the airship's longitudinal axis. It's like the movement of a plane's wings during a roll.

Pitch (Rotation around the lateral axis): This rotation is around an axis perpendicular to the wings, causing the nose of the airship to move up or down, like the movement of a seesaw.

Yaw (rotation across the vertical axis): This rotation lets in the airship to show left or right, just like a compass needle does.

To perform these rotations, we can use mathematical matrices or quaternion representations to manipulate the orientation of the hybrid airship. Each Euler angle represents a specific rotation matrix that can be combined to achieve the desired orientation.

The vehicle geometry data shown in table 3.1 is used to develop the model

shown in Fig. 3.1 and in Fig. 3.2. The similar data is also used in [28], where the vehicle is considered as a “HULL” without payload concept. The hybrid airship 3D model has been designed based on the vehicle geometry as a input to a 3D CAD software tool called CATIA referred from the work [28] and is also developed here on 3D CAD software tool called XFLR5 as shown in Fig. 3.4.

Table 3.1 Vehicle geometry data:

Airship Geometry Parameter of a HULL	Value
Mass of airship, m	10 kg
Overall Airship length, l	4.2 m
Maximum Airship Diameter, D	1.6 m
Hull Volume, V	5.6 m ³
Reference Area of a Hull, S	3.16 m ²
Reference length of a Hull, c	1.78 m
Ellipsoid shaped semi-minor axis, b	0.8 m
a_1 (first semi major axis of ellipsoid)	1.55 m
a_2 (second semi major axis of ellipsoid)	2.75 m
Airship Wingspan, b_w	3.06 m
Airship Wing area, S_w	1.72 m ²
Airship tail span, b_t	3.06 m
Airship tail area, S_t	0.916 m ²
Rck	0.8 m
Wing MAC $\bar{c}$	0.47 m
Tail MAC $\bar{c}$	0.51 m
Payload Geometry	
Mass of a payload, m_b	4kg
Mass of hull, m_h	6kg
Payload surface area, S_b	0.1x0.2 m ²
R_{cb} ; distance between joint c to suspended payload box	0.25 m
C_D^B	1.05

The moments and products of inertia for the airship configuration corresponding to the inputted mass are obtained in table 3.2 by moving the sub model to get

the desired CG location. For the mathematical model development of hybrid airship and for the simulation work, the used aerodynamic data, airship geometry and geometry data, wind tunnel and CFD data is shown in Appendix A3. Aerodynamic Coefficient data with approximate equations as a design

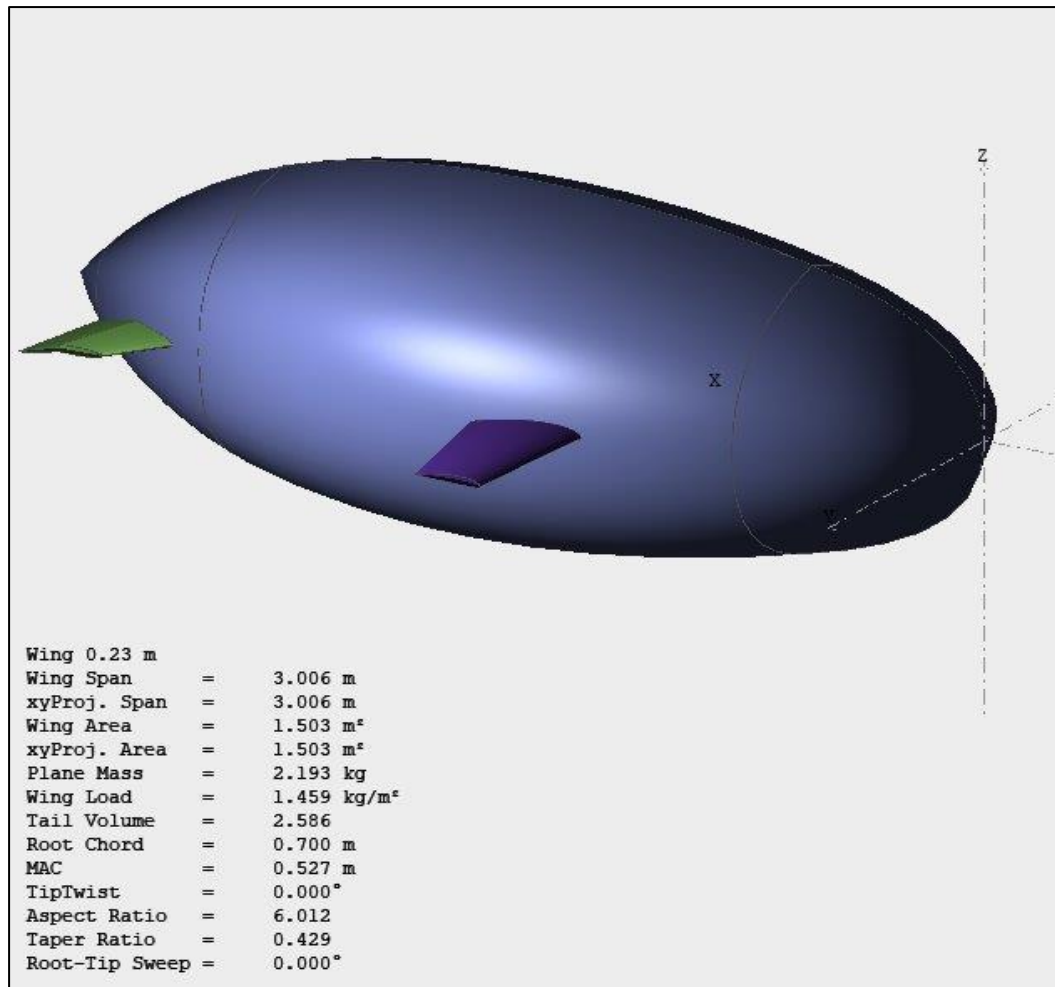


Fig. 3.4 SIDE View of 3D model of hybrid Airship on XFLR5 software

Table 3.2 Inertia matrix obtained from 3D CAD software:

Airship mass, m	10 kg
I_{xx}	4.94 kgm ²
I_{yy}	14.916 kgm ²
I_{zz}	15.289 kgm ²
I_{xz}	0.03389 kgm ²
I_{xy}	0 kgm ²
I_{yz}	0 kgm ²

parameter for development of mathematical model of hybrid airship is shown in Table 3.3. Aerodynamic modelling is based on the work of [96] and the work of [97]. For a complete analysis, the aerodynamic database that includes all the expression of the aerodynamic coefficients must be constructed for the vehicle. Since the hybrid airship is newly developed, such database has not been created. The main challenge here is that there is no source in the open literature that tackles the problem of estimating the aerodynamic coefficients of a winged hybrid airship, while there are several approximation methods for conventional

Table 3.3. Aerodynamic coefficient data (source: [96])

Aerodynamic Coefficient	Approximate equations	Estimated Values for hybrid airship
$C_{D\alpha}$	$2C_{L_1}C_{L\alpha}K$ Where $K = \frac{1}{\pi e AR}$ Is induced drag factor.	0.0011
$C_{L\alpha}$	CFD	0.0462
$C_{m\alpha}$	CFD	-0.0066
C_{Dq}	$-2\eta_{ht}C_{D\alpha_{ht}}V_{ht}$	negligible
C_{Lq}	$-2\eta_{ht}C_{L\alpha_{ht}}V_{ht}$ Where $V_{ht} = \frac{l_h S_{ht}}{\bar{c} S}$, $\eta_{ht} = 1$	-0.0104 $C_{L\alpha_{ht}} = 0.0048$ -ve for canard configuration.
C_{mq}	$-2\eta_{ht}C_{L\alpha_{ht}}V_{ht}\frac{l_{ht}}{\bar{c}}$	-0.0359
$C_{D\delta_e}$	$-\eta_{ht}C_{D\alpha_{ht}}V_{ht}\zeta\frac{S_{ht}}{S}$	$C_{D\alpha_{ht}}$ not available. 0.120
$C_{L\delta_e}$	$-\eta_{ht}C_{L\alpha_{ht}}V_{ht}\zeta\frac{S_{ht}}{S}$	1.24
$C_{m\delta_e}$	$-\eta_{ht}C_{L\alpha_{ht}}V_{ht}\zeta$ Where $\zeta = 0.62$	-6.8

aircraft [98], [99], [89]. With the assumption that the general equation for an aircraft is adequate for the winged airship application as well, the remaining aerodynamic coefficients are estimated using estimation methods and equations provided by [75] and [100]. The estimates are only sufficient for this preliminary modelling and analysis of the winged hybrid airship. To ensure the accuracy of these aerodynamic coefficients, CFD and wind tunnel test must be performed on a model of the hybrid airship.

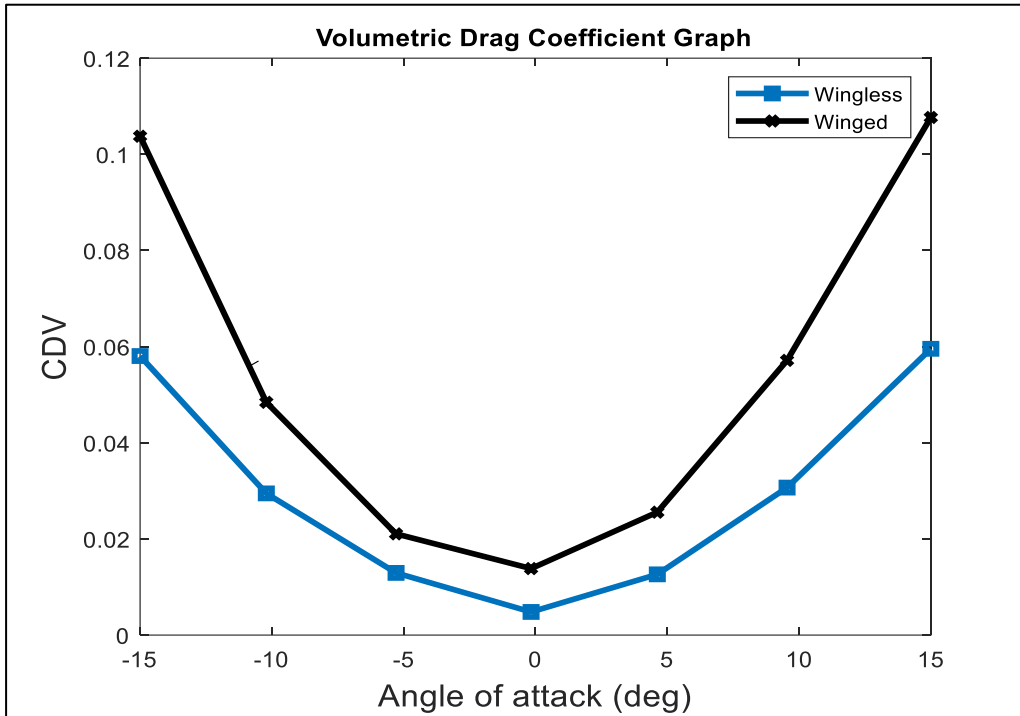


Fig. 3.5a. CFD Aerodynamic data C_{DV} vs α (source: [96])

Fig. 3.5a to Fig. 3.5c is the aerodynamic data (CFD) taken from [96] used in simulation of the hybrid airship model. Simulation work on MATLAB and Scilab along with bifurcation analysis on AUTO 2000 was done for 3DOF and 4DOF models and validated with 6DOF results of reference [28] and 9DOF results respectively. After analysis, trim and equilibrium points, stability, and instability regions, and eigen values are obtained.

Longitudinal dynamic modes are obtained based on the eigen values obtained from bifurcation analysis with parameter as elevator deflection of nonlinear model. With all these information a NDI controller has been designed for two body dynamics for making the direction control of the system. Linearised

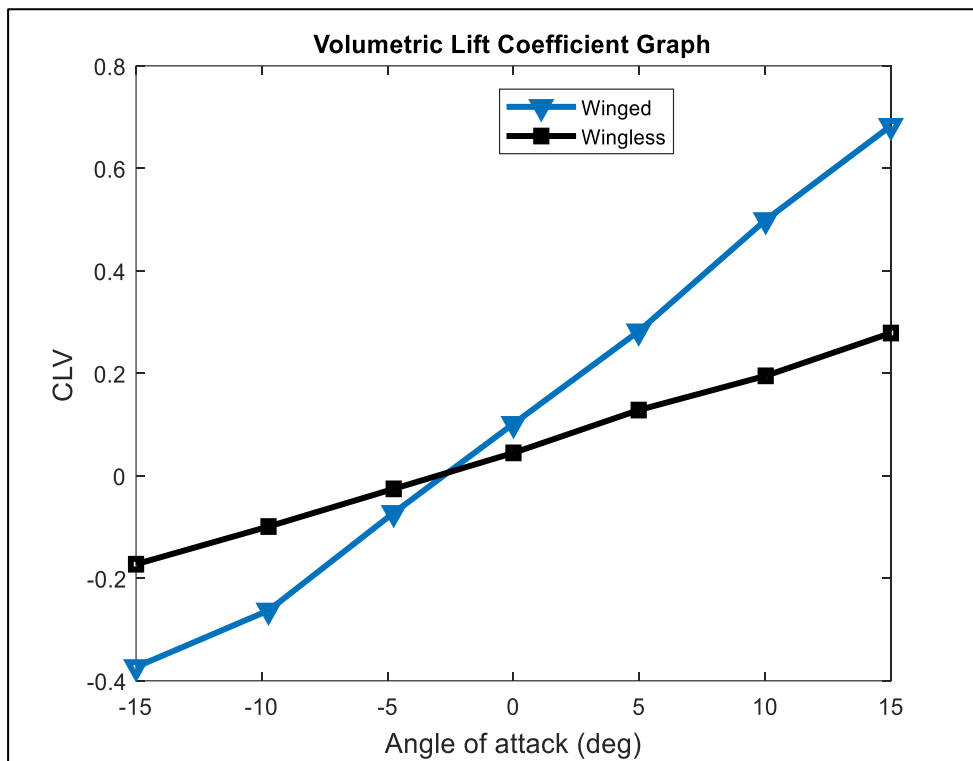


Fig. 3.5b. CFD Aerodynamic data CL_V vs α (source: [96])

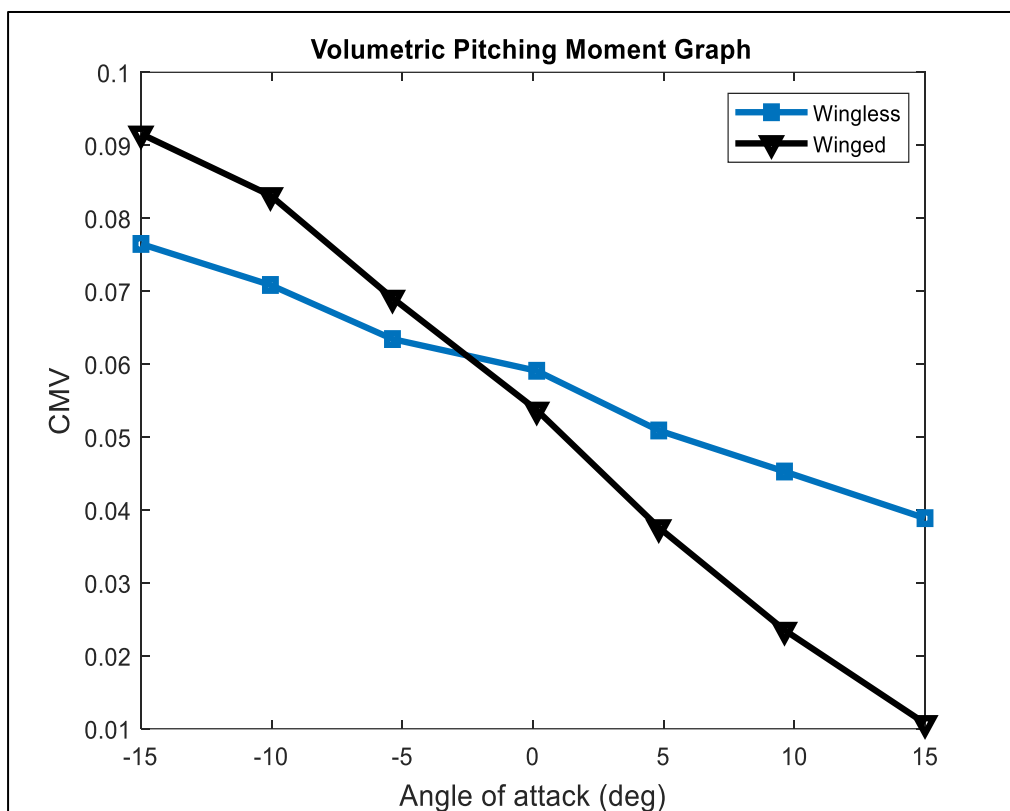


Fig. 3.5c. CFD Aerodynamic data CM_V vs α (source: [96])

model has also been developed for 3 DOF model to design a linearized PID controller for trajectory tracking and collision avoidance. For PID design, a HIL solution has also been developed on UNO board with Altair Embed Software for pitch control. Lastly the open loop and closed loop analysis is performed in time domain and frequency domain both. These all findings and controller design will make the hybrid airship model as autonomous in future.

The mathematical modelling of the hybrid airship is basically based on the idea of mathematical derivation of Aircraft which is shown in Appendix A1. The rotational axis and transformation to another reference frame with Euler angle derivation is also discussed in Appendix A1. Mathematical modelling of hybrid airship is done as single body dynamic model and two body dynamic model. For longitudinal dynamics of hybrid airship, single body dynamic model as 3DOF and two body dynamic model as 4DOF are developed and proposed.

3.2 HYBRID AIRSHIP SINGLE BODY DYNAMIC MODEL:

Understanding the concept of the axis system is crucial for appropriately deriving equations for force and moment. Fig. 3.6 illustrates two distinct types of axis systems. The symbol O_body denotes the location of the origin of the body reference frame, which remains fixed to the airship. On the other hand, $O_inertial$ indicates the location of the origin of the inertial reference frame, which is equivalent to the reference frame fixed to the Earth. The decision to establish the body axis system at the center of volume rather than the center of gravity is made. This feature allows for the adjustment of the aircraft's center of gravity during flight without the need to redefine the aerodynamic coefficients in the dynamic equation. Let $r_G = [a_x \ a_y \ a_z]^T$ be the vector representing the coordinate location of the center of gravity relative to the origin. The weight of the airship is exerted at its center of gravity. The vertical displacement of the center of gravity from the center of volume is a common occurrence, primarily attributed to the inclusion of the gondola and engine components. In the preliminary analysis, it is assumed that the center of gravity coincides with the center of volume. When the center of gravity is situated at a distinct point from the body origin, the equation of motion considers rotational velocity at the

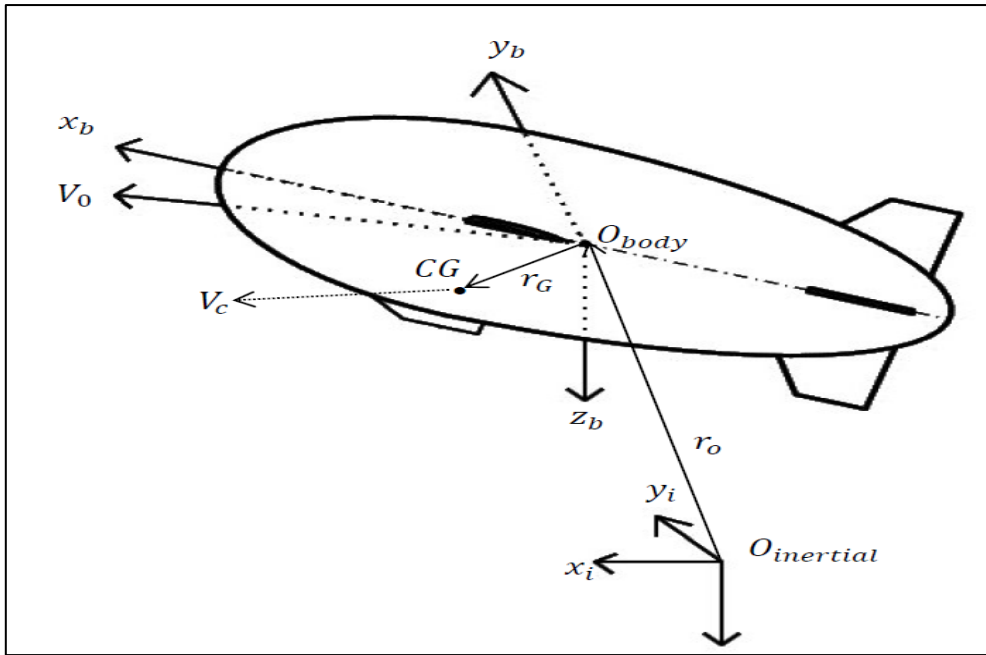


Fig. 3.6 Axis system for hybrid airship

center of gravity in relation to the body frame. The sign convention for the x , y , and z axes in both the body and inertial reference frames adheres to the standard aircraft axis system. In the body frame, the x -axis extends outward from the airship's nose along the reference line, the y -axis extends outward from the airship's right wing, and the z -axis extends downward through the airship's bottom, following the right-hand rule. In the inertial frame, the x -axis points towards the North, the y -axis points towards the East, and the z -axis points in order to facilitate calculations, the gravitational term is represented in the earth axis system, as the weight vector aligns with the positive z direction in the earth frame. Conversely, it is advantageous to represent the aerodynamic and thrust forces in the body axis due to the fact that the drag and lift forces are exerted along the x and z axes of the body frame. The application of Newton's second law is limited to the inertial reference frame, whereas the comprehensive equation of motion for the hybrid airship is formulated within the body frame. Hence, the phrases that are articulated within alternative frames undergo a conversion process into the suitable frame by means of a transformation matrix. Fig. 3.7 gives a pictorial idea for developing nonlinear flight dynamics models and simulation for airships. The general equation of motion of an airship is very similar to that of a conventional aircraft, with the significant difference being the addition of the

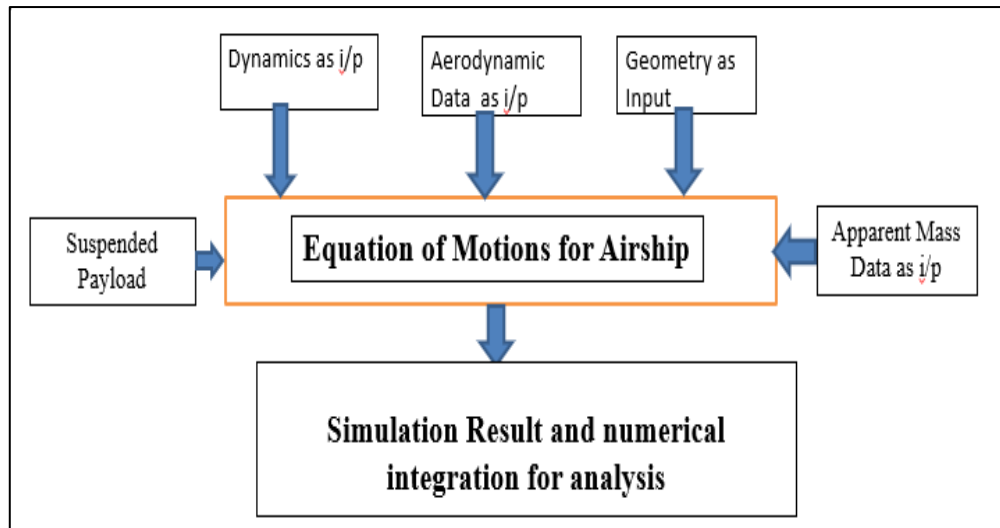


Fig. 3.7 Flow Chart for mathematical modelling

added mass and inertia effects and buoyancy force. In developing the equation of motion of the winged hybrid airship, the familiar aircraft dynamic modelling methods apply to the airship.

The following assumptions form the hybrid airship dynamic model:

1. Aeroelasticity and fluid motion inside the hull are neglected because the airship is rigid, and the hull is solid.
2. The hybrid airship maintains a consistent mass and volume. In practical terms, the mass of an airship undergoes changes over time as a result of the reduction in fuel mass and the expansion of the lifting gas as the altitude increases. Given that the airship is designed for low-altitude flight, specifically below 500 metres, it is reasonable to disregard any major changes in fuel mass for a short duration during the aerodynamic analysis. Additionally, the expansion of the lifting gas can be considered negligible.
3. The buoyancy and volume centres are considered to be the same.
4. Airship symmetry about oxz plane.
5. The motion of airship is portrayed as a perturbation about an initial trim condition.
6. Stationary atmosphere is assumed. This assumption is important because airships in general are much more sensitive to wind.
7. The airship is assumed to move in a steady low speed rectilinear flight.

8. At low Mach number, the airship's motion does not affect air density locally, and couplings between dynamics and thermal phenomena are ignored.
9. The airship exhibits the property of volume displacement. This suggests that the presence of additional fluid leads to a notable change in inertia that is directly proportional to the volume of air displaced by the hull.
10. The inertial reference frame is the earth axis.

In order to derive the equations of motion for a six-degree-of-freedom (6-DOF) model of an airship, it is postulated that the Center of Gravity (CG) is situated below the Center of Volume (CV), and that the Center of Buoyancy (CB) coincides with the CV. The study does not consider any fluctuations in the volume and mass of the airship. There exists an extensive collection of literature pertaining to airship modelling, which has been clearly summarised in reference [101]. The dynamics of airships can be effectively described by a set of six interconnected nonlinear equations. The mathematical model of an airship encompasses the equation of motion with six degrees of freedom, which counts for three translational and three rotating motions along the x, y, and z axes of the body frame. The equation of motion is derived from Newton's second law, which states that the rate of change of linear momentum over time is equal to the sum of the forces applied to the centre of gravity of the airship. The overall movement of a hybrid airship can be characterised by the application of Newton's second law of motion, both in terms of translational and rotational aspects. The equations are expressed in vector form as follows:

$$F = \frac{d}{dt}(mV) \quad (3.1)$$

$$\zeta = M = \frac{d}{dt}(I\omega) \quad (3.2)$$

In this context, the symbol F represents the applied force, mV denotes the linear momentum, m represents the mass, V signifies the translational velocity, M represents the applied moment about the center of gravity, Iω represents the angular momentum about the center of gravity, ω represents the angular velocity, and I represents the inertia matrix. The equation of motion for a hybrid airship is formulated in the body axis coordinate system. Eqn. (3.1) and (3.2) provide vector equations that describe the translational motion of the

center of gravity (c.g.) and the rotational motion around the center of gravity, respectively. The vector equation presented denotes three scalar equations that correspond to the vector components. Consequently, this results in a cumulative total of six scalar equations, which account for the six degrees of freedom pertaining to the motion of the aeroplane. Eqn. (3.1) and (3.2) hold true inside an inertial reference frame. However, it is advantageous to represent the respective quantities in relation to their components within the body axes. These axes typically undergo translation and rotation with respect to the inertial axes. There are several causes for this phenomenon. Firstly, it is common practice to conduct measurements using the body-axes system. Additionally, the inertial matrix I remains constant when expressed in body axes, whereas it would vary over time if expressed in inertial axes. The body-axis components of the quantities described in Eqn. (3.1) and (3.2) are as follows.

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \quad V = V_0 = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad M = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \quad \omega = \omega_0 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3.3)$$

$$I = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \quad (3.4)$$

$$\begin{aligned} I_x &\equiv \int_{\text{volume}} x^2 dm & I_y &\equiv \int_{\text{volume}} y^2 dm & I_z &\equiv \int_{\text{volume}} z^2 dm \\ I_{xy} &\equiv \int_{\text{volume}} xy dm = I_{yx} & I_{yz} &\equiv \int_{\text{volume}} yz dm = I_{zy} \\ I_{xz} &\equiv \int_{\text{volume}} xz dm = I_{zx} \end{aligned} \quad (3.5)$$

The sign standards for the components of force (F), velocity (V), moment (M), and angular velocity (ω) are in accordance with the sign conventions for the body axes and the right-hand rule. The constituents of the angular velocity vector ω consist of the roll rate p (with a positive value indicating a downward movement of the right wing), the pitch rate q (with a positive value indicating an upward movement of the nose), and the yaw rate r (with a positive value indicating a rightward movement of the nose).

The variables x , y , and z represent the body axis coordinates of the mass elements dm within the integrals stated in Eqn. (3.5). These mass elements

collectively form the hybrid airship mentioned in Etkin's work [102]. Based on the definitions provided in Eqn. (3.5), it is evident that in the case of a rigid body exhibiting symmetry with respect to the Oxz plane in body axes, the inertia matrix I possesses symmetry, hence resulting in $I_{xy} = I_{yx} = I_{yz} = I_{zy} = 0$. The inertia matrix is then simplified to.

$$I = \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \quad (3.6)$$

In the context of rotating axis systems, such as body axes, the derivative operator applied to vectors can be decomposed into two components. The first component captures the rate of change of vector components stated in the rotating system, while the second component accounts for the rotation of the axis system.

$$\frac{d}{dt}(\cdot) = \frac{\delta}{\delta t}(\cdot) + \omega \times (\cdot) \quad (3.7)$$

Combining Eqn (3.1), (3.2) and (3.7) we get aircraft equation as

$$F = m\dot{V} + \omega \times mV \quad (3.8)$$

$$\zeta = M = I\dot{\omega} + \omega \times I\omega \quad (3.9)$$

3.2.1 6DOF (single body) Dynamic Model:

When the origin of the body frame does not align with the center of gravity, it becomes necessary to account for the rotation of velocity at the center of gravity in relation to the body frame. The velocity of the center of gravity of a hybrid airship, relative to the origin of the body frame, can be expressed as:

$$V_{G/0} = \omega_0 \times r_G \quad (3.10)$$

Hence, the velocity of the center of gravity of a hybrid airship, as represented in the rotating body frame, can be determined as follows:

$$V_c = V_0 + \omega_0 \times r_G \quad (3.11)$$

In an alternative approach, the acceleration of the airship can be determined by taking the derivative of the velocity of the center of gravity, denoted as V_c in Eqn. (3.11), with respect to time. The equation of motion for translation is given by:

$$F = m \left(\dot{V}_0 + \omega_0 \times V_0 - r_G \times \dot{\omega}_0 + \omega_0 \times (\omega_0 \times r_G) \right) \quad (3.12)$$

Force F is defined in the body-fixed frame.

Similar rotation in the inertial frame is carried for the rotational equation of motion.

$$\zeta = T = M = I_0 \dot{\omega}_0 + \omega_0 \times I_0 \omega_0 + m r_G \times (\dot{V}_0 + \omega_0 \times V_0) \quad (3.13)$$

It should be noted that the inertia matrix is specified with respect to the origin of the body frame. The airship is subject to several external forces and moments, including aerodynamic forces and moments, gravitational forces, buoyancy forces, and propulsion forces and moments. The forces of gravity and buoyancy are considered to be static forces, and as such, they can be analyzed together for the sake of simplicity.

$$F = F_a + F_s + F_T \quad (3.14)$$

$$M = M_a + M_s + M_T \quad (3.15)$$

The dynamics of an airship bear resemblance to those of an aircraft, but with the inclusion of additional factors such as mass, inertia effects, and buoyancy. The concept of added mass and inertia can be seen as the alteration in momentum that arises from the displacement of air during movement.

The Complexity in mathematical derivation of Hybrid airship is because of the centre of mass or centre of gravity and centre of volume is positioned at two distinct locations as shown in the Fig. 3.8. Moment equation and force equation is obtained easily on centre of mass or centre of gravity. In Hybrid airship, the centre of volume is almost located at the centre position of HULL that is filled with Helium or Hydrogen gas. Centre of mass or centre of gravity in hybrid airship is located near gondola where payload is attached, its mass is larger compared to HULL mass. Airship is lighter than air vehicle, and hybrid airship is HTA vehicle with attached HULL filled with helium gas. so, we must take care of apparent mass of the system in the mathematical derivation which is not required in the final equation of motion of the aircraft as shown in Eqn. (3.57). For calculating the lift in airship design, Archimedes principle required volume for hydrostatic lift obtained as in Eqn. (3.16)

$$L_{hst} = vol(\rho_a - \rho_g) \quad (3.16)$$

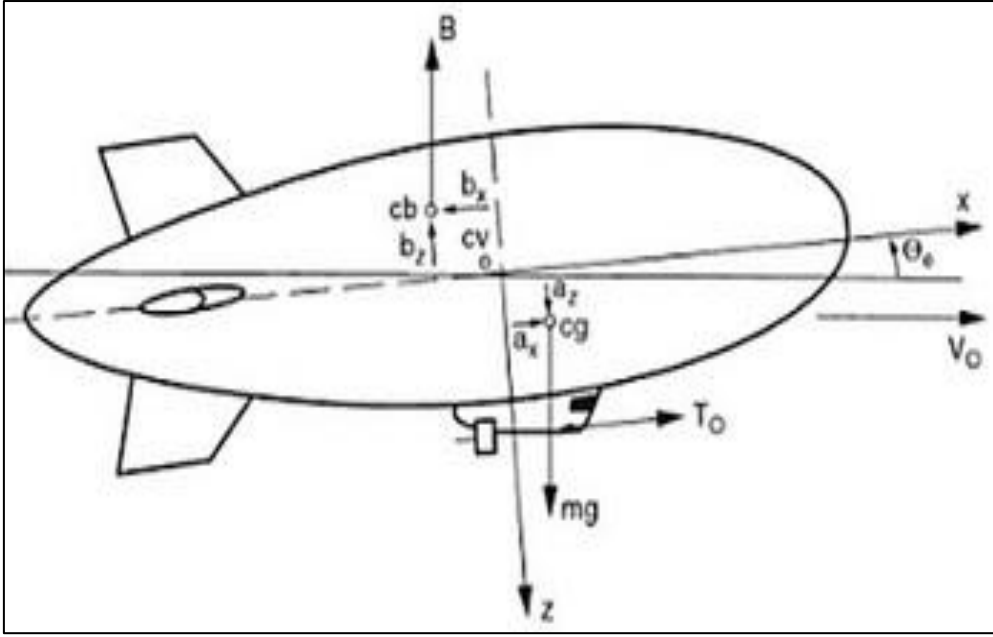


Fig. 3.8 Coordinate location of CG and CV [36]

Where ρ_a and ρ_g are densities of air and lifting gas.

$$w_{net} = (m_{GTW} \times g - L_{hst}) \quad (3.17)$$

Net weight w_{net} expressed in Eqn. (3.17) is gross take-off weight minus the weight balanced by the hydrostatic lift.

Hybrid airship 6 DOF Non-linear equation of motion obtained with Eqn. (3.12) and (3.13):

$$\begin{bmatrix} mI_3 + M' & -mr_G^X \\ mr_G^X & I_0 + I'_0 \end{bmatrix} \begin{bmatrix} \dot{V}_0 \\ \dot{\omega}_0 \end{bmatrix} + \begin{bmatrix} m(\omega_0 \times V_0 + \omega_0 \times (\omega_0 \times r_G)) \\ \omega_0 \times (I_0 \omega_0) + mr_G \times (\omega_0 \times V_0) \end{bmatrix} = \begin{bmatrix} F \\ \zeta \end{bmatrix} \quad (3.18)$$

Where,

$$r_G^X = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \quad (3.19)$$

$$(r_G^X)^T = -(r_G^X) \quad (3.20)$$

Aircraft 6 DOF Non-linear equation of motion obtained from Eqn. (3.8) and

Eqn. (3.9):

$$\begin{bmatrix} mI_3 & 0 \\ 0 & I_0 \end{bmatrix} \begin{bmatrix} \dot{V}_0 \\ \dot{\omega}_0 \end{bmatrix} + \begin{bmatrix} m(\omega_0 \times V_0) \\ \omega_0 \times (I_0 \omega_0) \end{bmatrix} = \begin{bmatrix} F \\ \zeta \end{bmatrix} \quad (3.21)$$

Airship and hybrid airship is lighter than air vehicle, so we have to take of apparent mass of the system in the mathematical derivation which is not required in the final equation of motion of the aircraft as shown in Eqn. (3.18) and (3.21).

The simplified version of the equation for propulsion system of hybrid airship is given in Eqn. (3.22). The typical propulsion system geometry used in hybrid airship is shown in APPENDIX A3, Fig. A3.5.

$$M_P = M_T = T d_z \quad (3.22)$$

Where M_T is the moment due to propulsion system or thrust, $X_T=T$ is the thrust in axial direction, $Z_T=0$ is the thrust in normal direction. T is total thrust generated by the engine and d_z is the distance of propulsion system from CV.

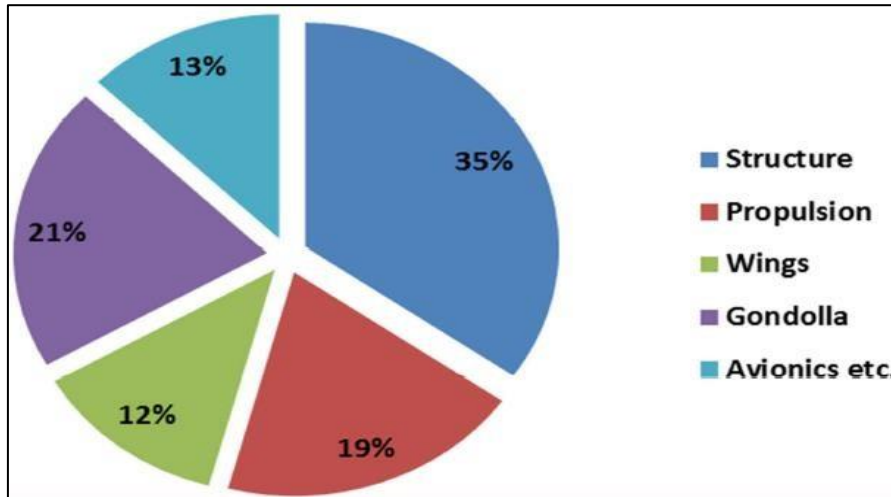


Fig. 3.9 Mass distribution in hybrid airship [36]

Contribution of mass components in hybrid airship is shown in Fig. 3.9 and the approx. information taken from [36]. Calculation of the apparent mass in hybrid airship is shown in Fig. 3.10. The hybrid airship 6 DOF non-linear equation Eqn. (3.21) is expanded, and the full nonlinear forces and moments equation is given below in Eqn. (3.23) to (3.28):

Axial force:

$$m_x \dot{U} + (ma_x - X_{\dot{q}}) \dot{q} + m_z qW - m_y rV - ma_x(q^2 + r^2) - ma_z pr = X_a + X_S + X_T \quad (3.23)$$

Side force:

$$m_y \dot{V} + (ma_z - Y_{\dot{p}}) \dot{p} + (ma_x - Y_{\dot{r}}) \dot{r} + m_x rU - m_z pW + ma_x pq + ma_z qr = Y_a + Y_S + Y_T \quad (3.24)$$

Normal force:

$$m_z \dot{W} - (ma_x + Z_{\dot{q}}) \dot{q} + m_y pV - m_x qU + ma_x pr + ma_z(p^2 + q^2) = Z_a + Z_S + Z_T \quad (3.25)$$

Rolling moment:

$$J_x \dot{p} - (J_y - J_z)qr - J_{xz}(\dot{r} + pq) - (ma_z + L_{\dot{v}}) \dot{V} - ma_z(rU - pW) = L_a + L_S + L_T \quad (3.26)$$

Pitching moment:

$$J_y \dot{q} - (J_x - J_z)pr - J_{xz}(r^2 - p^2) + (ma_z + M_{\dot{u}}) \dot{U} - (ma_x + M_{\dot{w}}) \dot{W} - ma_x(pV - qU) + ma_z(qW - rV) = M_a + M_S + M_T \quad (3.27)$$

Yawing moment:

$$J_z \dot{r} - (J_x - J_y)pq - J_{xz}(\dot{p} + pr) + (ma_x + N_{\dot{v}}) \dot{V} + ma_x(rU - pW) = N_a + N_S + N_T \quad (3.28)$$

Kinematic equation:

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi) \quad (3.29)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (3.30)$$

$$\dot{\psi} = \frac{(q \sin \phi + r \cos \phi)}{\cos \theta} \quad (3.31)$$

Airship Added Mass

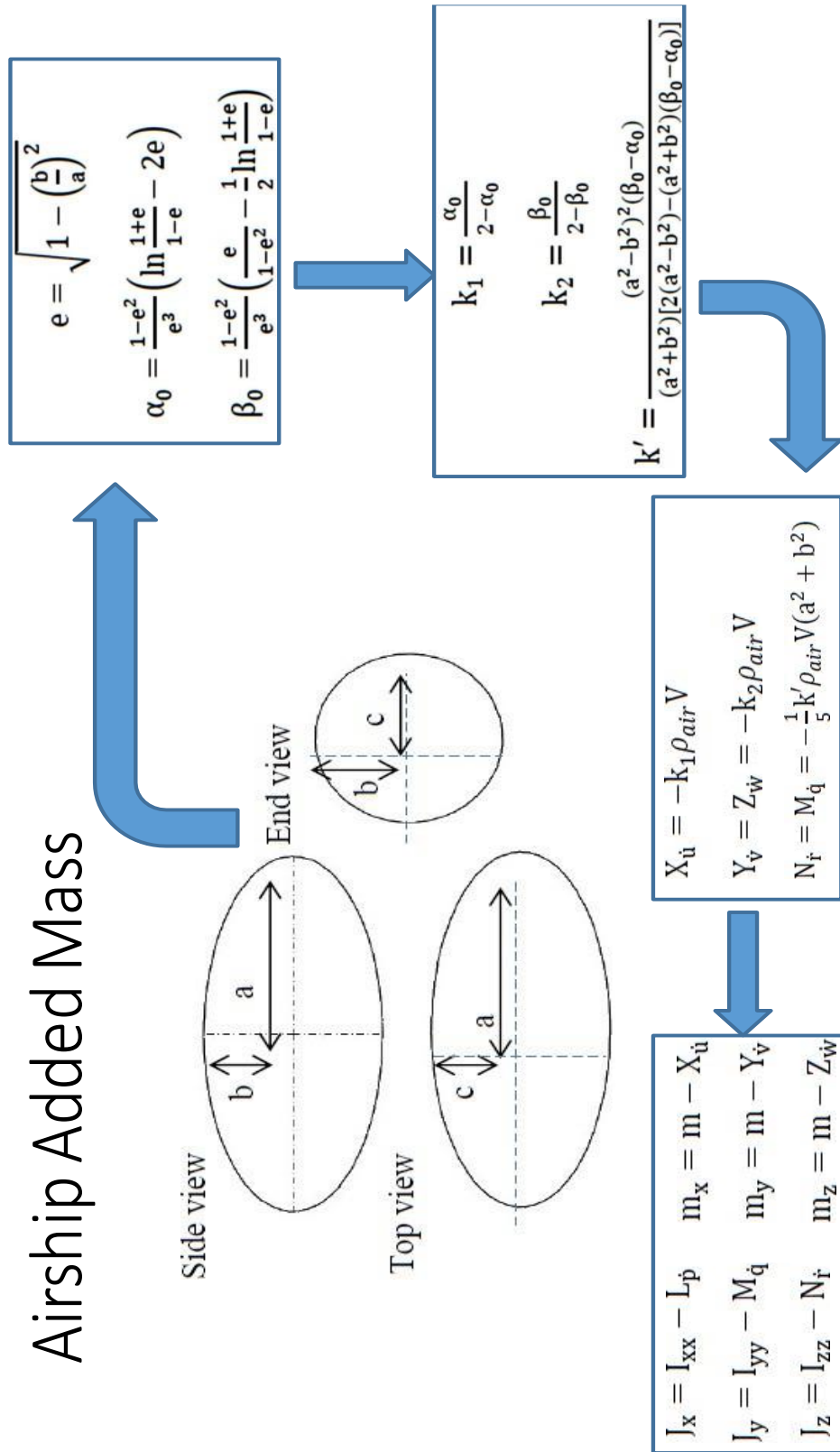


Fig. 3.10 Calculation of apparent mass in hybrid airship

3.2.2 3DOF (single body) Longitudinal Dynamic Model in Body Fixed Frame:

For simplifying the complexity in the 6DOF non-linear dynamic equations of motion for hybrid airship as shown in Eqn. (3.21), the dynamics can be analysed in two parts as longitudinal dynamics consists of 3DOF equation of motion and lateral dynamics consists of another 3DOF equation of motion. This can be done because, the shape of the hybrid airship is also symmetrical along x-axis just as an aircraft. The longitudinal dynamics consists of axial force equation ($\dot{U}$), normal force equation ($\dot{W}$), pitching moment equation ($\dot{q}$) and kinematic equation ($\dot{\theta}$). All the 6 DOF non-linear equation of motion is given in body axis frame of reference. The longitudinal 3 DOF non-linear equation of motion for single body dynamics of hybrid airship by considering the associated terms used for longitudinal dynamics and dropping out the remaining terms in the given 6 DOF nonlinear equation of motion for airship is proposed here.

Non-Linear Longitudinal Dynamic Equations for Hybrid Airship:

Axial force:

$$m_x \dot{U} + (ma_x - X_{\dot{q}})\dot{q} + m_z qW - ma_x q^2 = X_a + X_S + X_T \quad (3.32)$$

Normal force:

$$m_z \dot{W} - (ma_x + Z_{\dot{q}})\dot{q} - m_x qU + ma_z q^2 = Z_a + Z_S + Z_T \quad (3.33)$$

Pitching moment:

$$J_y \dot{q} + (ma_z + M_{\dot{u}})\dot{U} - (ma_x + M_{\dot{w}})\dot{W} + ma_x(qU) + ma_z(qW) = M_a + M_S + M_T \quad (3.34)$$

Kinematic equation:

$$\dot{\theta} = q \quad (3.35)$$

In linearizing the nonlinear equation of motion from Eqn. (3.32) to (3.35), small perturbation theory is applied, where squares of variables and coupling derivatives are omitted and small angle assumption is used.

An example of a complete expression of the aerodynamic force in x-axis is given in Eqn. (3.36).

$$X_a = X_1 + X_u \cdot u + X_v \cdot v + X_w \cdot w + X_p \cdot p + X_q \cdot q + X_r \cdot r \quad (3.36)$$

Where X_1 is the trim equilibrium aerodynamic force which is defined as:

$$X_1 = \frac{1}{2} \rho_{air} V^2 S (C_L \sin \alpha - C_D \cos \alpha) \quad (3.37)$$

The rest of the components in the aerodynamic expression are the dynamic perturbations terms, which will be zero if the aircraft is in equilibrium. Some of the terms in the aerodynamic force and moment representation can be neglected if its contribution to the motion is very small [36].

Linearized 3DOF Longitudinal Dynamic Equations for Hybrid Airship:

Axial force:

$$m_x \dot{U} + (ma_x - X_{\dot{q}}) \dot{q} + m_z q W_1 = X_a + X_S + X_T \quad (3.38)$$

Normal force:

$$m_z \dot{W} - (ma_x + Z_{\dot{q}}) \dot{q} - m_x q U_1 = Z_a + Z_S + Z_T \quad (3.39)$$

Pitching moment:

$$J_y \dot{q} + (ma_z + M_{\dot{u}}) \dot{U} - (ma_x + M_{\dot{w}}) \dot{W} + ma_x (qU) + ma_z (qW) = M_a + M_S + M_T \quad (3.40)$$

Kinematic equation:

$$\dot{\theta} = q \quad (3.41)$$

The expression of the aerodynamic force in x-axis shown in Eqn. (3.36) is expressed below for longitudinal dynamics of the Airship.

$$X_a = X_1 + X_u \cdot u + X_w \cdot w + X_q \cdot q + X_{\delta_e} \cdot \delta_e \quad (3.42)$$

The expression for the stability derivatives is given in equations (3.43a) to (3.43l).

Longitudinal stability derivatives:

u derivatives

$$X_u = \frac{-2C_{D1}q_1S}{U_1} \quad (3.43a)$$

$$Z_u = \frac{-2C_{L1}q_1S}{U_1} \quad (3.43b)$$

$$M_u = \frac{C_{m1}q_1S\bar{c}}{U_1} \quad (3.43c)$$

w derivatives

$$X_w = \frac{-(C_{D\alpha} - C_{L1})q_1S}{U_1} \quad (3.43d)$$

$$Z_w = \frac{-(C_{L\alpha} + C_{D1})q_1S}{U_1} \quad (3.43e)$$

$$M_w = \frac{c_{m_\alpha} q_1 S \bar{c}}{U_1} \quad (3.43f)$$

q derivatives

$$X_q = \frac{-C_{Dq} q_1 S \bar{c}}{2U_1} \quad (3.43g)$$

$$Z_q = \frac{-C_{Lq} q_1 S \bar{c}}{2U_1} \quad (3.43h)$$

$$M_q = \frac{c_{m_q} q_1 S \bar{c}^2}{2U_1} \quad (3.43i)$$

δ_e derivatives

$$X_{\delta_e} = \frac{-C_{D\delta_e} q_1 S \bar{c}}{2U_1} \quad (3.43j)$$

$$Z_{\delta_e} = \frac{-C_{L\delta_e} q_1 S \bar{c}}{2U_1} \quad (3.43k)$$

$$M_{\delta_e} = \frac{c_{m_{\delta_e}} q_1 S \bar{c}^2}{2U_1} \quad (3.43l)$$

The aerodynamic coefficients (such as, C_{m_α} , C_{m_q} and $c_{m_{\delta_e}}$) are obtained either from wind tunnel test, numerical solutions, or semi-empirical approach [97], [28], [96], [92] & [102].

Buoyancy or static forces and moments:

The buoyancy forces and moments for hybrid airship are given in equation (3.44a) to (3.44c).

$$X_s = -(mg - B) \sin \theta \quad (3.44a)$$

$$Z_s = (mg - B) \cos \phi \cos \theta \quad (3.44b)$$

$$M_s = -(mga_z + Bb_z) \sin \theta - (mga_x + Bb_x) \cos \phi \cos \theta \quad (3.44c)$$

The Expanded Longitudinal Dynamic (3DOF) Equations for Hybrid Airship in body fixed frames:

By using the equation (3.22), (3.42), and (3.44a) to (3.44c) in equation (3.38) to (3.40), we get the expanded equation as equation (3.45) to (3.47).

Axial force:

$$m_x \ddot{U} + (ma_x - X_{\dot{q}}) \dot{q} = X_1 + X_u \cdot u + X_w \cdot w + X_{\delta_e} \cdot \delta_e + (X_q - m_z W_1) \cdot q - (mg - B) (\sin \theta_1 + \theta \cos \theta_1) + T \quad (3.45)$$

Normal force:

$$m_z \dot{W} - (ma_x + Z_{\dot{q}}) \dot{q} = Z_1 + Z_u \cdot u + Z_w \cdot w + Z_{\delta_e} \cdot \delta_e + (Z_q + m_x U_1) \cdot q + (mg - B) (\cos \theta_1 - \theta \sin \theta_1) \quad (3.46)$$

Pitching moment:

$$J_y \dot{q} + (ma_z + M_{\dot{u}}) \dot{U} - (ma_x + M_{\dot{w}}) \dot{W} = M_1 + M_u \cdot u + M_w \cdot w + M_{\delta_e} \cdot \delta_e + (M_q - ma_x U_1 - ma_z W_1) \cdot q - \theta [(mga_z + Bb_z) \cos \theta_1 - (mga_x + Bb_x) \sin \theta_1] - (mga_z + Bb_z) \sin \theta_1 - (mga_x + Bb_x) \cos \theta_1 + T d_z \quad (3.47)$$

All perturbation variables are zero at longitudinal trim state. The trim equations are given from equation (3.48) to (3.50).

$$\frac{1}{2} \rho_{air} V^2 S (C_L \sin \alpha - C_D \cos \alpha) - (mg - B) \sin \theta_1 + T = 0 \quad (3.48)$$

$$-\frac{1}{2} \rho_{air} V^2 S (C_D \sin \alpha + C_L \cos \alpha) + (mg - B) \cos \theta_1 = 0 \quad (3.49)$$

$$\frac{1}{2} \rho_{air} V^2 S \bar{c} C_m - (mga_z + Bb_z) \sin \theta_1 - (mga_x + Bb_x) \cos \theta_1 + T d_z = 0 \quad (3.50)$$

3.2.3 State Space Representation of Linearized Longitudinal Dynamic Equations for Hybrid Airship:

Equation (3.45) to (3.47) and (3.35) are expressed in state space form as:

$$m \dot{x} = ax + bu \quad (3.51)$$

Where the state variables expressed as:

$$x^T = [u \ w \ q \ \theta] \quad (3.52)$$

$$u^T = [\delta_e \ T] \quad (3.53)$$

$$\text{Where } m = \begin{bmatrix} m_x & 0 & (ma_x - X_{\dot{q}}) & 0 \\ 0 & m_z & -(ma_x + Z_{\dot{q}}) & 0 \\ (ma_z + M_{\dot{u}}) - (ma_x + M_{\dot{w}}) & J_y & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a

$$= \begin{bmatrix} X_u & X_w & (X_q - m_z W_1) & -(mg - B) \cos \theta_1 \\ Z_u & Z_w & (Z_q + m_x U_1) & -(mg - B) \sin \theta_1 \\ M_u & M_w & (M_q - ma_x U_1 - ma_z W_1) - [(mga_z + Bb_z) \cos \theta_1 - (mga_x + Bb_x) \sin \theta_1] & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} [X_{\delta_e} + C_{L_{\delta_e}} \sin \alpha - C_{D_{\delta_e}} \cos \alpha] & 1 \\ [Z_{\delta_e} - C_{D_{\delta_e}} \sin \alpha - C_{L_{\delta_e}} \cos \alpha] & 0 \\ [M_{\delta_e} + C_{m_{\delta_e}}] & d_z \\ 0 & 0 \end{bmatrix}$$

The general form of the state equation is

$$\dot{x} = Ax + Bu \quad (3.54)$$

Where $A = m^{-1}a$

$B = m^{-1}b$

The equations for aerodynamic forces and moments are provided below in equation (3.55 to 3.60) are referred from [36] and [102]. Let δ_{ELVL} and δ_{ELVR} be the deflections of the left and right trailing edge flaps of the elevator.

$$X = \frac{1}{2} \rho V_0^2 [C_{X_1} \cos^2 \alpha \cos^2 \beta + C_{X_2} \sin 2\alpha \sin(\frac{\alpha}{2})] \quad (3.55)$$

$$Z = \frac{1}{2} \rho V_0^2 \left[C_{Z_1} \cos\left(\frac{\alpha}{2}\right) \sin 2\alpha + C_{Z_2} \sin(2\alpha) + C_{Z_3} \sin(\alpha) \sin(|\alpha|) + C_{Z_4} (\delta_{ELVL} + \delta_{ELVR}) \right] \quad (3.56)$$

$$M = \frac{1}{2} \rho V_0^2 \left[C_{M_1} \cos\left(\frac{\alpha}{2}\right) \sin 2\alpha + C_{M_2} \sin(2\alpha) + C_{M_3} \sin(\alpha) \sin(|\alpha|) + C_{M_4} (\delta_{ELVL} + \delta_{ELVR}) \right] \quad (3.57)$$

$$C_L = \left\{ C_{L_0} + C_{L_\alpha} \cdot \alpha + C_{L_q} \cdot \frac{q\bar{c}}{2V_1} + C_{L_{\delta_e}} \cdot \delta_e \right\} \quad (3.58)$$

$$C_D = \left\{ C_{D_0} + C_{D_\alpha} \cdot \alpha + C_{D_q} \cdot \frac{q\bar{c}}{2V_1} + C_{D_{\delta_e}} \cdot \delta_e \right\} \quad (3.59)$$

$$C_m = \left\{ C_{m_0} + C_{m_\alpha} \cdot \alpha + C_{m_q} \cdot \frac{q\bar{c}}{2V_1} + C_{m_{\delta_e}} \cdot \delta_e \right\} \quad (3.60)$$

3.2.4 The Longitudinal Dynamic (3DOF) Equations for Hybrid Airship in wind fixed frame:

The simulation is done in wind axis coordinate system, so the terms used for simulation is $\dot{V}$ (resultant of U and W velocity in wind axis), $\dot{\gamma}$ (*gamma*), $\dot{q}$ and $\dot{\theta}$ for 3 DOF longitudinal dynamics of the system. 3DOF hybrid airship Longitudinal translational and rotational dynamics eqns. (3.33-3.36) of the model can be transformed and modelled into the eqns. (3.62- 3.65) in Wind - fixed frame axes. The derivation of wind axis representation of Aircraft shown in Appendix A1 is used to develop the model of hybrid airship in wind -fixed frame and are given below.

$$\dot{V} = (T \cos(\alpha) - \bar{q}S C_D - (mg - B) \sin(\gamma)) / (m - X_{ud}) \quad (3.61)$$

$$\dot{\gamma} = (T \sin(\alpha) + \bar{q}S C_L + z_q - (mg - B) \cos(\gamma)) / \{(m - z_{wd}) \cdot V\} \quad (3.62)$$

$$\dot{q} = M_z / (I_{yy} - M_{qd}) \quad (3.63)$$

$$\dot{\theta} = q \quad (3.64)$$

$$\dot{R} = V \cos(\gamma) \quad (3.65)$$

$$\dot{h} = V \sin(\gamma) \quad (3.66)$$

Considering $\alpha = \theta - \gamma$, So

$$\alpha = q - [(T \sin(\alpha) + \bar{q}S C_L + z_q - (mg - B) \cos(\gamma)) / \{(m - z_{wd}) \cdot V\}] \quad (3.67)$$

$$M_z = -M_s + (Mq - ma_x uu - ma_z ww) \cdot q + \bar{q}S \bar{c} C_{m_{mp}} + T * 0 \cdot 1 \quad (3.68)$$

3.2.5 Wing Incidence Angle in Single Body Dynamics of Hybrid Airship.

The system is postulated to undergo a smooth gliding motion at a constant velocity V , characterized by an angle of attack α and a flight path angle γ , as depicted in Fig. 3.11. The incidence angle, denoted as Γ , is defined as the angle formed between the mean chord of the airfoil and an imaginary line that is perpendicular to the line OC. The variable θ represents the pitch angle of the system.

The overall angular momentum with respect to the system's center of mass can be expressed as,

$$\sum M = M_0 + r_2 \{D \cos(\alpha + \Gamma) - L \sin(\alpha + \Gamma) + W_2 \sin \theta\} - W_1 \sin \theta \quad (3.69)$$

Dividing Eqn. by $\frac{1}{2} \rho V^2 S c$, it is possible to state it in a dimensionless form.

$$C_m = C_{m_0} + \frac{r^2}{2} \{C_D \cos(\alpha + \Gamma) - C_L \sin(\alpha + \Gamma)\} + \frac{(w_2 r_2 - w_1 r_1)}{1/2 \rho V^2 S c} \sin \theta \quad (3.70)$$

The equation in its ultimate form is shown in Eqn. (3.71), employing the steady state glide relation $L = W \cos \gamma$ and $D = W \sin \gamma$.

$$C_m = C_{m_0} + \left(\frac{dm_1}{m_1 + m_2} \right) \left(\frac{1}{c} + \frac{\rho}{2} \frac{C_{B_0}}{W/S} \right) [C_D \cos(\alpha + \Gamma) - C_L \sin(\alpha + \Gamma)] \quad (3.71)$$

The work shown in [103]. The similar wing incidence angle idea is very useful in hybrid airship. In designing a mathematical model of single body dynamics of hybrid airship the same approach is incorporated as rigging mechanism with hybrid airship for enhancing the flight performance.

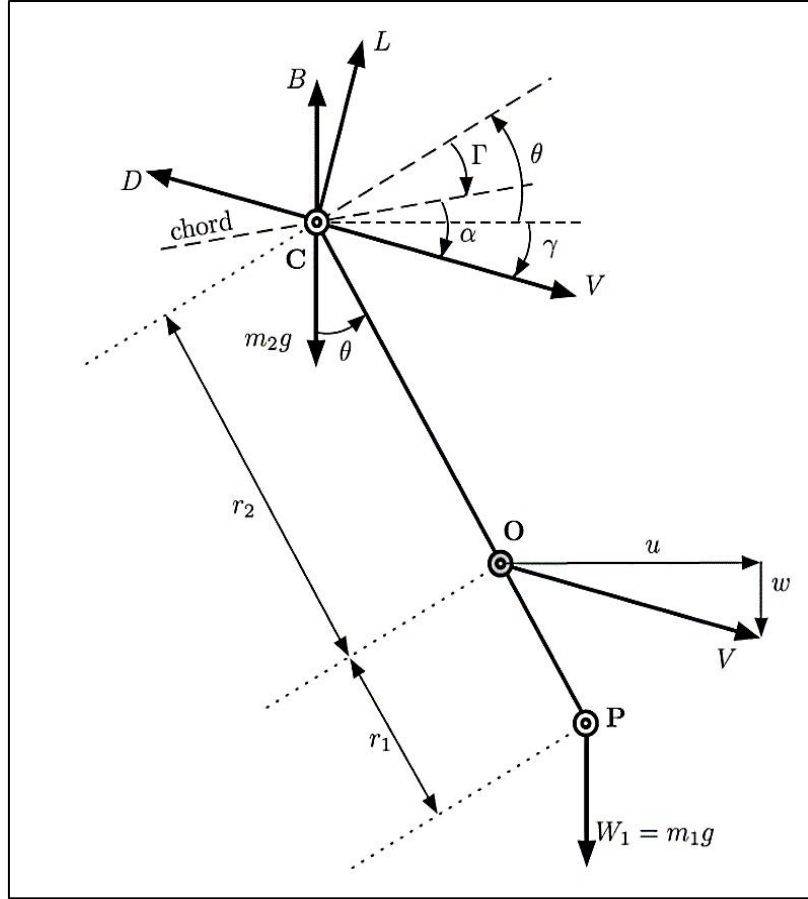


Fig. 3.11 parafoil-payload system in longitudinal flight [103]

Wing incidence angle Γ is considered as rigging angle μ . In simulation of single body dynamics of hybrid airship, angle of attack α is replaced with $(\alpha + \Gamma)$ or $(\alpha + \mu)$.

3.3 HYBRID AIRSHIP TWO BODY DYNAMIC MODEL:

Two body dynamic model of hybrid airship is developed with control manoeuvres as elevator (δ_e) and rigging angle (μ). A payload attached to the hybrid airship is suspended in nature. The two dynamic body concepts in hybrid airship modelling are taken from the mathematical model of parafoil payload delivery system. Hybrid airship is primarily a combination of HULL and GONDOLA considered as payload. The working of hybrid airship with suspended payload is somehow like the working of parafoil/parachute delivery system with few addons and changes in associated terms used in equation of

motion. Two body dynamic model developed here as 9DOF model and 4DOF longitudinal model.

3.3.1 9DOF Dynamic model with Suspended Payload:

This study focuses on the application of a cable-driven parallel manipulator (CDPM) for the modelling and control of the loading and unloading system of a heavy-lift airship [50]. Most studies on Cable-Driven Parallel Manipulators (CDPM) employ a non-elastic massless cable model [51], [52], [53], [54]. The cables are thereafter regarded as linear segments. Hybrid airships are available in various configurations, encompassing a range of designs such as low aspect ratio wing shape airships with aerofoil cross-sections, slender shape double fuselage structure airships, airships combined with wings, delta-shaped airships, and airship quadcopters. References [55], [56], [57], and [58] are provided in support of it. The Hybrid airship system comprises of the airship hull mass, which encompasses both the apparent mass and the incorporated air mass, together with the rigid payload mass that is affixed beneath the hull. The downward lines originating from the hull are affixed to the payload riser via a connection point, which facilitates autonomous rotating movement of both the hull and the payload. In contrast to the 6-DOF flight dynamic model employed by aeroplanes, the system necessitates the utilisation of a 9-DOF flight dynamics model. The development of the 9-degrees-of-freedom (9-DOF) model follows a similar approach as that employed in references [59], [60], [61] & [62], with the exception that it does not incorporate the spring and damper modelling of the relative yawing motion between the airship and the cargo caused by the lines. By compiling the equations governing the translational and rotational motion of the payload and airship, as depicted in Fig. 3.12, we can get the 9 degrees of freedom (DOF) equations of motion for the combined airship-payload system [27]. These equations can be represented in a concatenated matrix format. In deriving a two-body dynamic model of hybrid airship, the hybrid airship is made with the help of two sub-model as

- 1) Suspended payload box sub-model: for this suffix b is used as shown in Fig. 3.13.
- 2) Hybrid Airship sub-model: This is shown in Fig. 3.14. For derivation suffix P or suffix h is used for indicating hybrid airship. Suffix C is for connecting points of suspended payload with hybrid airship.

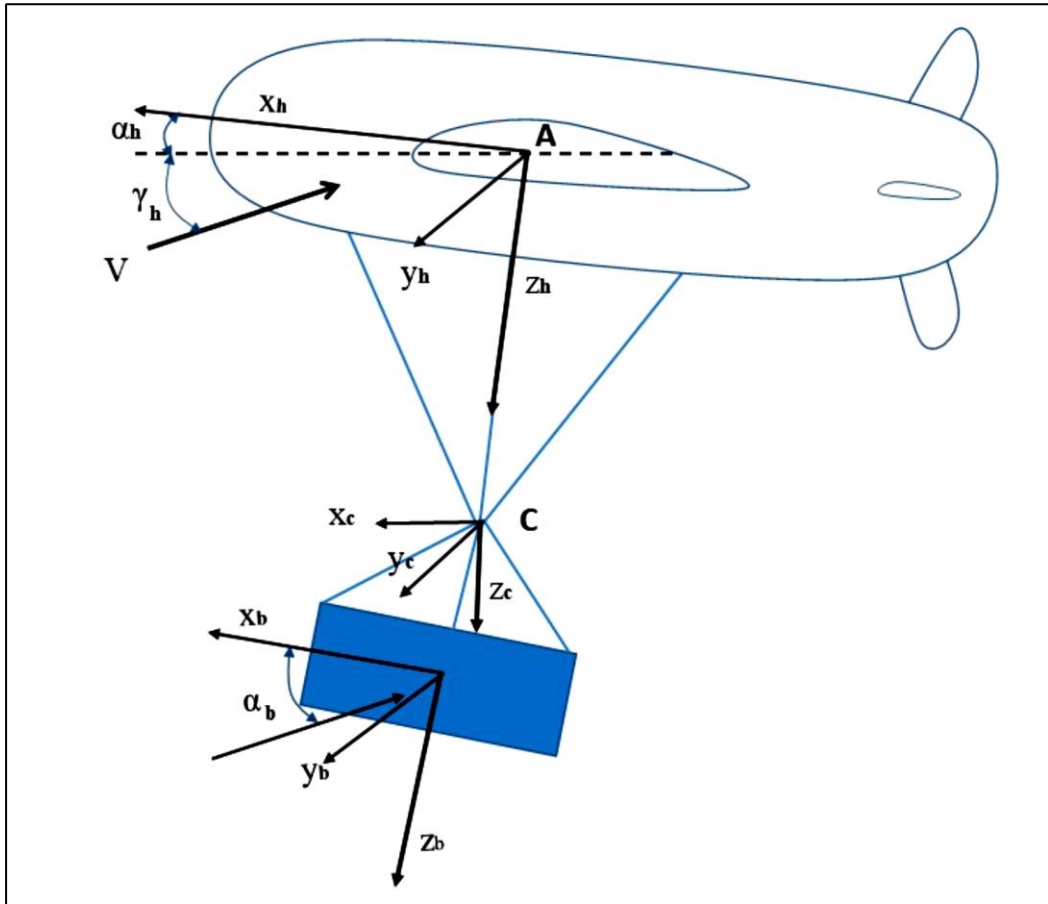


Fig. 3.12 Airship-payload system reference frames 9 DOF

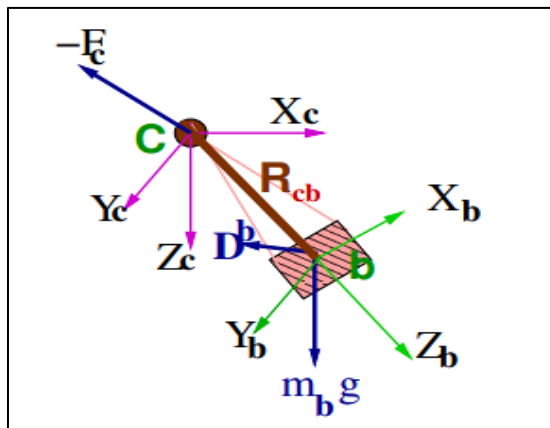


Fig. 3.13 Forces on payload sub model [60]

Detailed derivation of two body dynamic model of hybrid airship is given in Appendix A2. Only important equation is discussed in this section to develop two body dynamic model.

9 DOF two body dynamic model is shown in equations (3.72) and (3.73) in matrix vector form.

$$\begin{bmatrix} -M_b R_{Cb} \times & 0 & M_b T_b & T_b \\ 0 & -(M_h + M_F) R_{CP} \times & (M_h + M_F) T_h & -T_h \\ I_b & 0 & 0 & -R_{Cb} \times T_b \\ 0 & I_h + I_M & 0 & R_{CP} \times T_h \end{bmatrix} \begin{bmatrix} \dot{\Omega}_b \\ \dot{\Omega}_h \\ \dot{V}_c \\ F_c \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \quad (3.72)$$

Where

$$B_1 = F_b^A + F_b^S - \Omega_b \times M_b \Omega_b \times R_{Cb} \quad (3.73a)$$

$$B_2 = F_h^A + F_h^S + F_h^T - \Omega_h \times (M_h + M_F) \Omega_h \times R_{CP} + M_F \Omega_h \times T_h V_c - \Omega_h \times M_F T_h V_c \quad (3.73b)$$

$$B_3 = -\Omega_b \times I_b \Omega_b \quad (3.73c)$$

$$B_4 = M_h^A + M_h^S + M_h^T - \Omega_h \times (I_h + I_M) \Omega_h \quad (3.73d)$$

Eqn. (3.72) provides a set of 12 equations, comprising 9 equations of motion and 3 equations related to joint forces. Hence, the set of nine equations derived from Eqn. (3.72) and Eqn. (3.73a - 3.73d), in conjunction with the kinematic equations, yield a total of twenty-one equations of motion for the integrated Airship-payload system, encompassing eighteen state variables. The three supplementary equations presented in Eqn. (3.72) provide information regarding the amount of the internal forces acting at joint C. The mathematical model of the hybrid airship is constructed by utilising fundamental principles of physics to elucidate the aerodynamics and non-linear dynamics associated with the hybrid airship, hence facilitating the achievement of the desired target.

The expanded nonlinear equations in scalar form for 9DOF two body dynamic models of hybrid airship are shown below for deriving 4DOF longitudinal two body dynamics.

Sub model Payload box: Force equations- Side force ($\dot{q}_b$):

$$\begin{aligned}
 & -m_b z_{cb} \dot{p}_b + m_b x_{cb} \dot{r}_b + m_b S_{\phi_b} S_{\theta_b} C_{\psi_b} \dot{u}_c - m_b C_{\phi_b} S_{\psi_b} \dot{u}_c + \\
 & m_b S_{\phi_b} S_{\theta_b} S_{\psi_b} \dot{v}_c + m_b C_{\phi_b} C_{\psi_b} \dot{v}_c + m_b S_{\phi_b} C_{\theta_b} \dot{w}_c + S_{\phi_b} S_{\theta_b} C_{\psi_b} F_{Cx} - \\
 & C_{\phi_b} S_{\psi_b} F_{Cx} + S_{\phi_b} S_{\theta_b} S_{\psi_b} F_{Cy} + C_{\phi_b} C_{\psi_b} F_{Cy} + S_{\phi_b} C_{\theta_b} F_{Cz} = 0 + (m_b g - \\
 & B) S_{\phi_b} C_{\theta_b} + p_b q_b m_b x_{cb} - r_b^2 m_b y_{cb} - p_b^2 m_b y_{cb} + q_b r_b m_b z_{cb} \quad (3.74)
 \end{aligned}$$

Sub model hybrid Airship: Force equations- Side force ($\dot{q}_h$):

$$\begin{aligned}
 & z_{cp}(m_h - Y_{\dot{v}}) \dot{p}_h - x_{cp}(m_h - Y_{\dot{v}}) \dot{r}_h + (m_h - Y_{\dot{v}})(S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) \dot{u}_c + \\
 & (m_h - Y_{\dot{v}})(S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h}) \dot{v}_c + (m_h - Y_{\dot{v}}) S_{\phi_h} C_{\theta_h} \dot{w}_c - \\
 & (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) F_{Cx} - (S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h}) F_{Cy} - S_{\phi_h} C_{\theta_h} F_{Cz} = \\
 & \bar{q}_p S_p C_y + (m_h g - B) S_{\phi_h} C_{\theta_h} - p_h (m_h - Z_{\dot{w}}) x_{cp} r_h^2 + (m_h - X_{\dot{u}}) y_{cp} + \\
 & p_h^2 (m_h - Z_{\dot{w}}) y_{cp} - q_h r_h (m_h - X_{\dot{u}}) z_{cp} + [-r_h Y_{\dot{v}} C_{\theta_h} C_{\psi_h} + \\
 & p_h Y_{\dot{v}} (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})] u_c + [-r_h Y_{\dot{v}} C_{\theta_h} S_{\psi_h} + p_h Y_{\dot{v}} (C_{\phi_h} S_{\theta_h} S_{\psi_h} - \\
 & S_{\phi_h} C_{\psi_h})] v_c + [r_h Y_{\dot{v}} S_{\theta_h} + p_h Y_{\dot{v}} C_{\phi_h} C_{\theta_h}] w_c + [-r_h X_{\dot{u}} C_{\theta_h} C_{\psi_h} + \\
 & p_h Z_{\dot{w}} (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})] u_c + [-r_h X_{\dot{u}} C_{\theta_h} S_{\psi_h} + p_h Z_{\dot{w}} (C_{\phi_h} S_{\theta_h} S_{\psi_h} - \\
 & S_{\phi_h} C_{\psi_h})] v_c + [r_h X_{\dot{u}} S_{\theta_h} + p_h Z_{\dot{w}} C_{\phi_h} C_{\theta_h}] w_c \quad (3.75)
 \end{aligned}$$

Sub model Payload box Moment equations- Axial ($\dot{u}_c$):

$$\begin{aligned}
 & I_{xb} \dot{p}_b - I_{xyb} \dot{q}_b - I_{xzb} \dot{r}_b + [(S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) z_{cb} - (S_{\phi_b} S_{\theta_b} C_{\psi_b} - \\
 & C_{\phi_b} S_{\psi_b}) y_{cb}] \dot{u}_c - [-(S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b}) z_{cb} + (C_{\phi_b} S_{\theta_b} S_{\psi_b} - \\
 & S_{\phi_b} C_{\psi_b}) y_{cb}] \dot{v}_c - [-S_{\phi_b} C_{\theta_b} z_{cb} + C_{\phi_b} C_{\theta_b} y_{cb}] \dot{w}_c = -[I_{yxb} r_b - I_{zxb} q_b] p_b - \\
 & I_{ybr_b} - I_{zyb} q_b q_b - [I_{yzb} r_b + I_{zbb} q_b] r_b \quad (3.76)
 \end{aligned}$$

Normal ($\dot{w}_c$):

$$\begin{aligned}
 & -I_{zxb} \dot{p}_b - I_{zyb} \dot{q}_b + I_{zbb} \dot{r}_b + [y_{cb} C_{\theta_b} C_{\psi_b} - (S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) x_{cb}] \dot{u}_c - \\
 & [-y_{cb} C_{\theta_b} S_{\psi_b} + x_{cb} (S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b})] \dot{v}_c - [y_{cb} S_{\theta_b} + \\
 & x_{cb} S_{\phi_b} C_{\theta_b}] \dot{w}_c = -[I_{xbr_b} - I_{yxb} p_b] p_b - [I_{xyb} q_b + I_{ybr_b}] q_b - [I_{xzb} q_b + \\
 & I_{yzb} p_b] r_b \quad (3.77)
 \end{aligned}$$

Sub model hybrid airship Moment equations: Axial (F_{Cx})

$$\begin{aligned} & (I_{xx}-L\dot{p})\dot{p}_h - (I_{zx}-N\dot{p})\dot{r}_h + [-(S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})z_{CP} + (S_{\phi_h} S_{\theta_h} C_{\psi_h} - \\ & C_{\phi_h} S_{\psi_h})y_{CP}]F_{Cx} + [-(S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h})z_{CP} + (C_{\phi_h} S_{\theta_h} S_{\psi_h} - \\ & S_{\phi_h} C_{\psi_h})y_{CP}]F_{Cy} + [-S_{\phi_h} C_{\theta_h} z_{CP} + C_{\phi_h} C_{\theta_h} y_{CP}]F_{Cz} = \bar{q}_h S_h b C_l + \\ & (mga_z + Bb_z) + (I_{xz}-L\dot{r}) q_h p_h + (I_{yy}-M\dot{q}) r_h q_h - (I_{zz} - N\dot{r}) q_h r_h \end{aligned} \quad (3.78)$$

Normal (F_{Cz}):

$$\begin{aligned} & -(I_{xz}-L\dot{r})\dot{p}_h + (I_{zz} - N\dot{r})\dot{r}_h + [-y_{CP}C_{\theta_h}C_{\psi_h} + (S_{\phi_h} S_{\theta_h} C_{\psi_h} - \\ & C_{\phi_h} S_{\psi_h})x_{CP}]F_{Cx} + [-y_{CP}C_{\theta_h}S_{\psi_h} + x_{CP}(S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h})]F_{Cy} + \\ & [y_{CP}S_{\theta_h} + x_{CP} S_{\phi_h} C_{\theta_h}]F_{Cz} = \bar{q}_h S_h b C_n + (mga_x + Bb_x) \sin \phi_h \cos \theta_h + \\ & (I_{xx}-L\dot{p}) q_h p_h + (I_{yy}-M\dot{q}) p_h q_h - (I_{zx} - N\dot{p}) q_h r_h \end{aligned} \quad (3.79)$$

In accordance with the citation provided [59], [60] and [27], the aerodynamic forces and moments exerted on the mass centre of the parafoil canopy are represented through the utilisation of aerodynamic force and moment coefficients.

$$C_L = C_L(\alpha_p, \delta_s) + c_{L\delta a} \delta_a \quad (3.80)$$

$$C_D^P = C_D^P(\alpha_p, \delta_s) + C_{D\delta a} \delta_a + c_{L\delta e} \delta_e \quad (3.81)$$

$$C_Y = C_Y \beta + C_{Y_r} r_p \frac{b}{2V_P} + c_{Y\delta a} \delta_a \quad (3.82)$$

In terms of hybrid airship-fixed axis coefficients:

$$C_X = (-C_D^P u_p + C_L w_p)/V_P \quad (3.83)$$

$$C_Y = C_Y \quad (3.84)$$

$$C_Z = (-C_D^P w_p - C_L u_p)/V_P \quad (3.85)$$

$$C_l = C_{l\beta} + C_{lp} P_p \frac{b}{2V_P} + C_{l\delta a} \delta_a + c_{L\delta e} \delta_e \quad (3.86)$$

$$C_m = \left\{ C_{m_{C/4}}(\alpha_p, \delta_s) + x_{pa} C_z \right\} + C_{mq} q_p \frac{c}{2V_P} + C_{m\delta a} \delta_a + c_{m\delta e} \delta_e \quad (3.87)$$

$$C_n = C_{n\beta}\beta + C_{n_P}P_P \frac{b}{2V_P} + C_{n\delta_a}\delta_a \quad (3.88)$$

$$M_s = -(mga_z + Bb_z) \sin \theta - (mga_x + Bb_x) \cos \phi \cos \theta \quad (3.89)$$

A positive value of δ_e is conventionally associated with a downward deflection. As depicted in Fig. 3.12, the rigging angle μ refers to the angular measurement between the line connecting the mid-baseline point of the hull to joint C and the line parallel to the Zh axis that intersects the mid-baseline point. consequently,

$$z_{cp} = R_{ch} \cos \mu \quad (3.90)$$

$$x_{cp} = R_{ch} \sin \mu \quad (3.91)$$

$$y_{cp} = 0 \quad (3.92)$$

3.3.2 4DOF Longitudinal Dynamic Model with Suspended Payload:

The 4DOF model of hybrid airship with suspended payload has been proposed as shown in Fig. 3.14 by combining the 3DOF longitudinal dynamics of hybrid airship with wing and the 3DOF equation of motions of suspended payload. The idea to develop the 4 DOF model of hybrid airship come from 9 DOF model of hybrid airship [62], the 9DOF and 4DOF model of parafoil payload system [59] and [60]. 4-DOF model of winged hybrid airship flight vehicle -payload system is formed with B vector in Eqns. (3.93a to 3.93f) and A system matrix in Eqn. (3.94) and by deriving dynamic equations for winged hybrid airship flight vehicle. The 4DOF model of a hybrid airship will include a sub model for the canopy of the winged hybrid airship flight vehicle, the link and joint C of the airship wing, and a payload sub model consisting of the payload, payload link, and joint C. This model separates the winged hybrid airship flight vehicle and payload system at the link joint C and takes into account the internal joint forces F_x , F_z , and their respective moments about the airship's center of gravity and the payload's center of gravity. For Level Flight Thrust is equal to drag force for airship dynamic model with velocity assumed to be constant at 20 m/s. In the moment equation the effect of elevator at wing and moment at fin is also considered. Apparent mass is also taken care of in the modelling.

The two body longitudinal dynamics 4DOF model consists of axial force

equation ($\dot{u}_c$), normal force equation ($\dot{w}_c$), pitching moment equation for suspended payload box ($\dot{q}_b$), pitching moment equation for hybrid airship ($\dot{q}_h$), Axial (F_{Cx}) and normal (F_{Cz}). All the 9 DOF non-linear equation of motion is given in body axis frame of reference. The longitudinal 4 DOF non-linear equation of motion for two body dynamics of hybrid airship is proposed here by considering the associated terms used for longitudinal dynamics and dropping out the remaining terms in the given 9 DOF nonlinear equation of motion for hybrid airship from Eqn. (3.72-3.79).

$$b_1 = -m_b g \sin \theta_b - Q_b S_b C_{D_b} \cos \alpha_b \quad (3.93a)$$

$$b_2 = m_b g \cos \theta_b - Q_b S_b C_{D_b} \sin \alpha_b + m_b R_b q_b^2 \quad (3.93b)$$

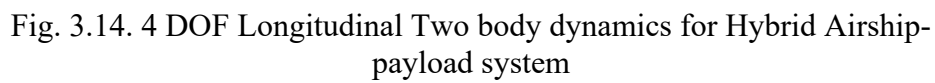
$$b_3 = 0 \quad (3.93c)$$

$$b_4 = -m_p g \sin \theta_p + Q_p S_p C_x + m_{pc} R_p \sin \mu q_p^2 - (C - A)(u_c \sin \theta_p + w_c \cos \theta_p) q_p \quad (3.93d)$$

$$b_5 = m_p g \cos \theta_p + Q_p S_p C_z + m_{pA} R_p \cos \mu q_p^2 - (C - A)(U_c \cos \theta_p - w_c \sin \theta_p) q_p \quad (3.93e)$$

$$b_6 = -x_{pa} Q_p S_p C_z + Q_p S_{pC} C_M \quad (3.93f)$$

The two-body dynamic model as 4DOF is developed for Parafoil payload delivery system and wing payload delivery system cited from [60] and [61]. For hybrid airship with payload delivery system, the two-body dynamic 4DOF model has been proposed here and simulated on MATLAB and verified the result with single body 3DOF result and the result of [28].



The expanded scalar form of Eqn. (3.94) is given below:

$$m_b \cos \theta_b \dot{u}_c + m_b \cos \theta_b \dot{w}_C + \cos \theta_b F_x + \cos \theta_b F_z = m_b g \sin \theta_b - Q_b S_b C_{D_b} \cos \alpha_b = m_b g \cos \theta_b - Q_b S_b C_{D_b} \sin \alpha_b + m_b R_b q_b^2 \quad (3.96)$$

$$I_b \dot{q}_b - R_b \cos \theta_b F_x - R_b \sin \theta_b F_z = 0 \quad (3.97)$$

$$\begin{aligned} m_{pA} \cos \theta_p \dot{u}_c - m_{pA} \sin \theta_p \dot{w}_c + m_{pA} R_p \cos \mu \dot{q}_p - \\ \cos \theta_p F_x + \sin \theta_p F_z = -m_p g \sin \theta_p + Q_p S_p C_x + m_{pc} R_p \sin \mu q_p^2 - \\ (C - A)(u_c \sin \theta_p + w_c \cos \theta_p) q_p \end{aligned} \quad (3.98)$$

$$\begin{aligned} m_{pC} \sin \theta_p \dot{u}_c + m_{pC} \cos \theta_p \dot{w}_c + -m_{pc} R_p \sin \mu \dot{q}_p - \\ \sin \theta_p F_x - \cos \theta_p F_z = m_p g \cos \theta_p + Q_p S_p C_z + m_{pA} R_p \cos \mu q_p^2 - \\ (C - A)(U_c \cos \theta_p - w_c \sin \theta_p) q_p \end{aligned} \quad (3.99)$$

$$I_{PF} \dot{q}_b + z_{cp} F_x - x_{cp} F_z = -x_{pa} Q_p S_p C_z + Q_p S_{pC} C_M \quad (3.100)$$

3.4 VALIDATION AND MODEL COMPARISION:

A set of trim control setting for a specified condition can be achieved and shown in Table 3.4 by adjusting the thrust T and buoyancy force B.

Table 3.4 Longitudinal trim setting:

Trim angle of attack, α	0 degree
Trim pitch angle, θ_1	0 degree
Thrust required for trim, T_1	26.11 N
Buoyancy force, B	50.38 N

Table 3.5 shows the comparative complexity analysis of aircraft, parafoil system and hybrid airship. For the mathematical model development of hybrid airship and for the simulation work, the used aerodynamic data, airship geometry and geometry data, wind tunnel and CFD data is shown in Appendix A3 and in methodology section. Table 3.6 gives an overview of the model comparison in between single body dynamics and two body dynamics modelling of hybrid airship in longitudinal dynamic analysis. In aircraft all masses are within the envelop of aircraft. So single body complete dynamic modelling and analysis is required. In airship payloads are always attached/suspends with gondola. So along with single body, two body longitudinal dynamic modelling and analysis is required in airship.

Fig. 3.15 is the simulation result of 3DOF nonlinear longitudinal HA model. The values of alpha, q, theta, and gamma trim in Fig. 3.15 closely align with the

trim values reported in [28]. The velocity reaches a maximum of 18.5 m/s, starting from an initial value of 16.8 m/s. The trim state is achieved within a time frame of 20 seconds for all output. In the case of q , the time period

Table 3.5. Modelling Complexity

Mathematical Modelling Terms	Aircraft	Parafoil system (parachute/Glider)	Hybrid Airship
Apparent Mass	NO	YES	YES
Buoyancy	NO	YES	YES
Aerodynamic Data	YES	YES	YES
Geometric Data	YES without apparent mass	YES with apparent mass	YES with apparent mass
Aerodynamics	YES	YES	YES
Flight dynamics	EOM is at Centre of gravity	EOM is at Centre of gravity	EOM is at Centre of volume

Table 3.6. Model Comparison:

MODEL	SINGLE BODY DYNAMICS		TWO BODY DYNAMICS	
	6 DOF	3 DOF	9 DOF	4 DOF
Body dynamics	Complete dynamics	Longitudinal dynamics	Complete dynamics	Longitudinal dynamics
States and variables	More states and variables	Comparatively less states and variables	More states and variables	Comparatively less states and variables
CG and CV distance ((r_G^X))	(r_G^X) is present in equation of motion, so complex model	(r_G^X) so is present in equation of motion, complex model	(r_G^X) is not present in equation of motion, so model is comparatively simple	(r_G^X) so is not present in equation of motion, so model is comparatively simple

is measured to be 4.1 seconds, while the oscillation frequency is determined to be 1.5 radians per second. The time cycle of θ is 3.9 seconds, whereas the angular frequency is 1.6 radians per second. The system demonstrates stability as it achieves a settled state within a time frame of 20 seconds. Additionally, the

airship's settling time is analysed and found to be within 20 seconds. Fourier transform techniques with software is used to calculate fundamental frequency.

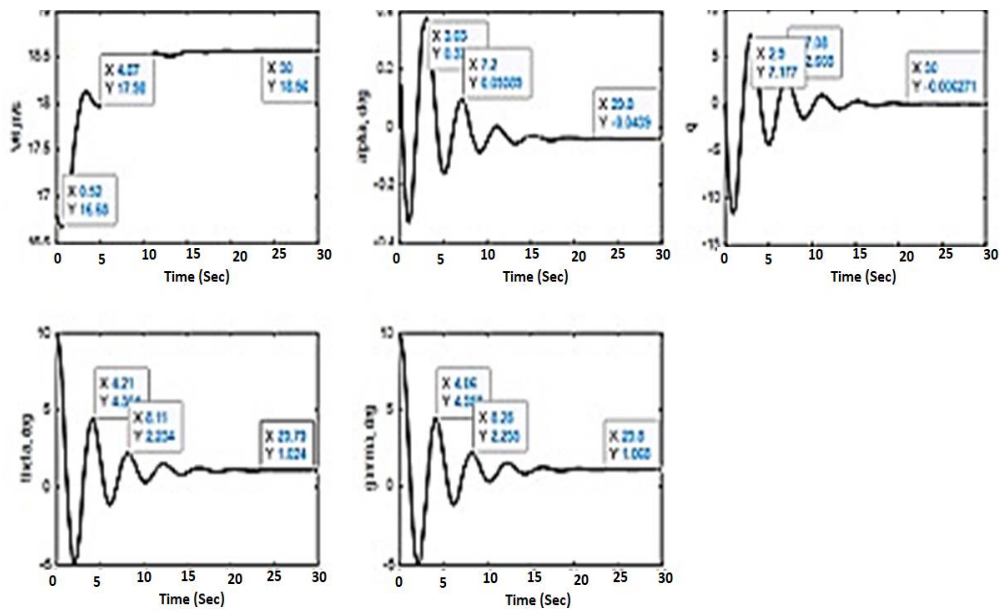


Fig. 3.15. Nonlinear Simulation result of 3DOF longitudinal dynamics of hybrid airship at delfin 0 degree deflection.

As in the work of [28] for winged hybrid airship's longitudinal motion characteristic roots for 3DOF linearized model shown in Table 3.8, consists of a stable first order root, an unstable first order root and a pair of stable oscillatory roots. So, the similar type of result is obtained here in Table 3.7 in the thesis for same geometrical data of the airship. Fig. 3.16 shows bar chart comparison of the proposed longitudinal linear 3DOF model vs a model of [28].

TABLE 3.7 Longitudinal dynamics mode analysis of developed linearized hybrid airship:

System	Pole/longitudinal eigen values	damping	Frequency (rad/s)	Time constant (s)	Stability	mode
System matrix A	-0.177 + 0.6191i	0.275	0.643	5.66	yes	Pendulum mode
	-0.177 - 0.6191i	0.275	0.643	5.66	yes	Pendulum mode

	1.69			-0.59	no	Heave mode
	-2.29			0.436	yes	Surge mode

TABLE 3.8: Longitudinal dynamics mode analysis of linearized hybrid airship for validation [28]:

System	Pole/longitudinal eigen values	damping	Frequency (rad/s)	Time constant (s)	Stability	mode
System matrix A	-0.2164+ 0.6022i	0.338	0.64	4.621	yes	Pendulum mode
	-0.2164 - 0.6022i	0.338	0.64	4.621	yes	Pendulum mode
	1.6296			-0.614	no	Heave mode
	-2.2626			0.442	yes	Surge mode

Simulation of longitudinal hybrid airship in two body dynamic approach developed as 4DOF model is compared with the simulation plot of longitudinal state variables available in two body dynamic as 9DOF model shown in Fig. 3.17. The plotted results are almost near about. 4DOF model is developed from 9DOF model by including only longitudinal dynamic behaviour terms. The results are almost similar. For single body winged hybrid airship in longitudinal motion, eigen value parametric analysis collages of image is done by varying CG and buoyancy effect in Fig A4.26 in Appendix A4.

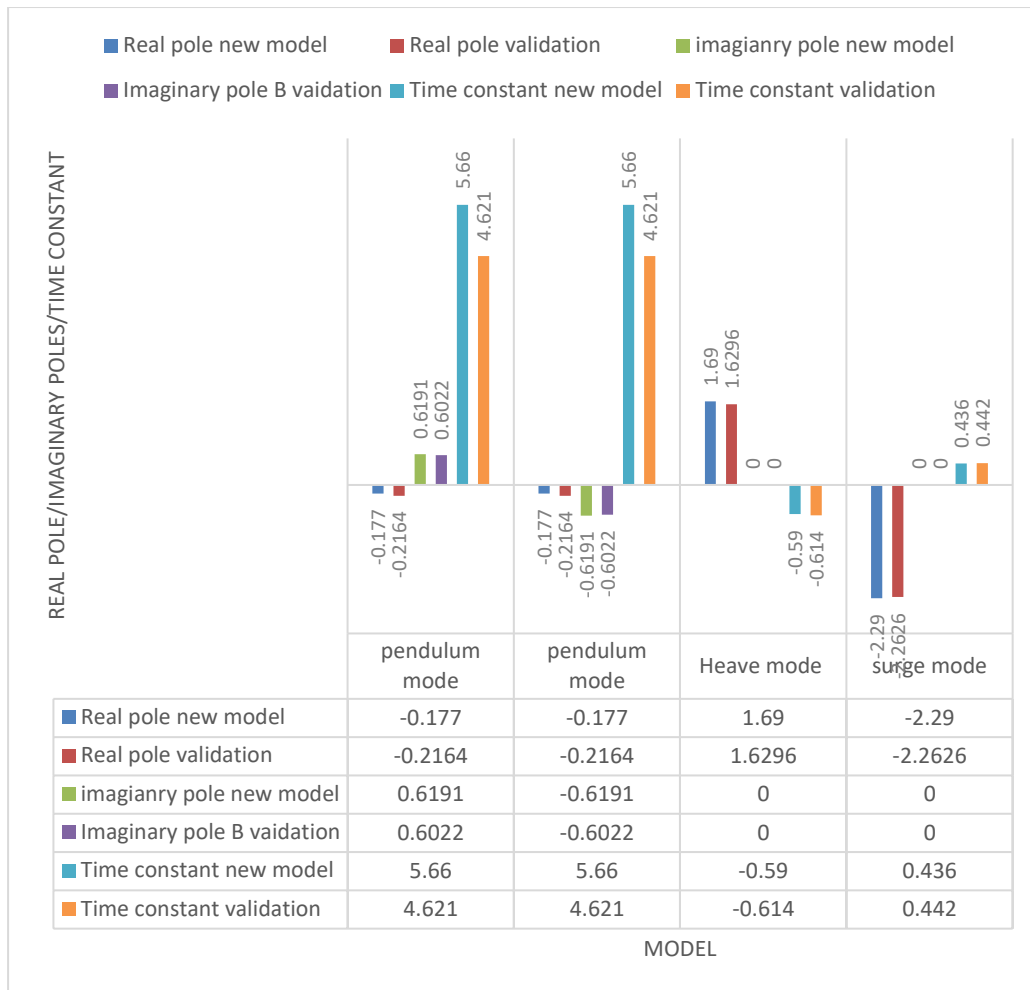


Fig. 3.16 Model comparison of longitudinal dynamic modes (table 3.7 vs table 3.8)

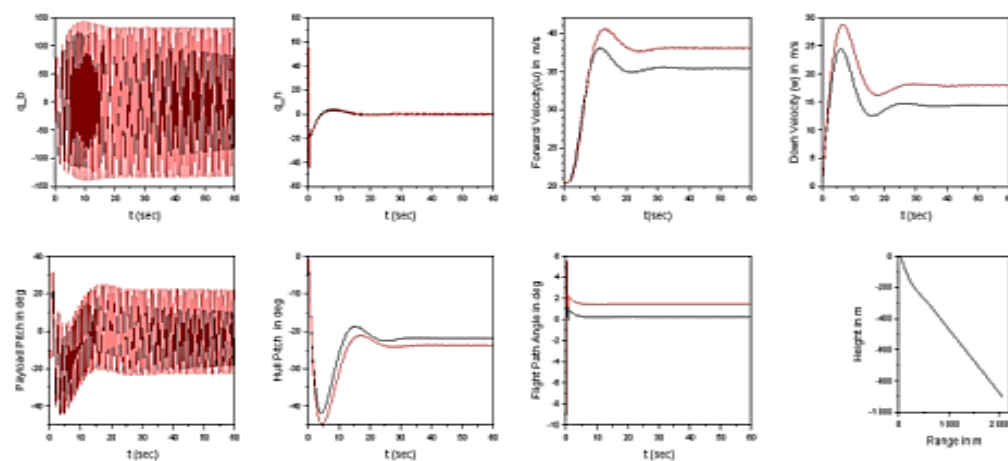


Fig. 3.17. Simulation result of longitudinal dynamics of 9DOF hybrid airship code with suspended payload vs 4DOF code.

3.5 SUMMARY:

Mathematical model of hybrid airship has been developed for longitudinal dynamics of hybrid airship as single body dynamics approach and two body dynamics approach. Hybrid airship consists of HULL, gondola, Clark Y wings, horizontal and vertical fins connected as cross structure, suspended payload with rigging mechanisms and elevator control surface on horizontal tail fins. Hybrid airship is designed for low height and for stationary atmosphere. It is developed with the same concept of aircraft dynamics and dynamics of parafoil payload delivery system. Hybrid airships are considered HTA vehicles with attached HULL and payload. Apparent mass effects including added mass and inertia effect, Buoyancy effect and wing incidence angle with rigging mechanism is taken care in mathematical modelling of hybrid airship. The maximum allowable operational weight for the hybrid airship is 12.47 kg. To ensure a good flight performance, hybrid winged airship is designed to have a total operational weight of 10 kg, which corresponds to the optimum ratio between the buoyancy lift and total airship weight, where the weight compensated by buoyancy is about 6 kg. The calculations are restricted for the flight domain of 20 m/s cruise speed and an altitude of 457 m.

This chapter includes mathematical modelling derivation for longitudinal dynamics of aircraft in body-fixed and wind-fixed frames. Euler transformation matrix is also discussed in this chapter. Mathematical modelling derivation for hybrid airship is done as single body and two body dynamics to study and analyze longitudinal dynamics behavior of the hybrid airship. In this regard 3 DOF model of hybrid airship from 6DOF model and 4DOF model of hybrid airship from 9DOF model has been developed here. For 3DOF nonlinear longitudinal dynamic model of hybrid airship, a linearized model is developed in state space form as well as in transfer function approach for designing a linearized controller. Aerodynamic models are also discussed in the form of equations and data. All geometry data, design data, flight data, Aerodynamic data is taken from other sources and listed here in tabular and pictorial form. 3D model of hybrid airship is also replicated here in XFLR5 software for validation. For, aerodynamic data and geometry data to develop hybrid airship refer Appendix A3 and for two body dynamic model derivation refer Appendix A2.

CHAPTER 4

CONTROLLER DESIGN FOR AIRSHIP

For single body model standard PID controller design practice is used and for multibody model, nonlinear controller (NDI) design is done. State space model is used for single body model (3DOF) as per available literature as a standard practice for stability analysis and PID controller design. Nonlinear differential equations are used for proposed multibody (4DOF and 9DOF) dynamic and trim analysis and NDI controller design. SMC can be extra work in future, when nonlinear aerodynamic flight data is available for the proposed model. MPC is also designed. Our focus is on modeling and in controller PID and NDI design in the thesis as per objectives.

4.1 NDI (NONLINEAR DYNAMIC INVERSION) CONTROLLER:

The trim and stability of flight in similar payload delivery systems are directly influenced by the rigging angle of the suspension lines connecting the payload [27]. Therefore, it is imperative to employ a multibody dynamic model in order to comprehensively analyse the flight dynamics and formulate an effective control legislation for this system. The NDI (Nonlinear Dynamic Inversion) approach is founded upon the principle of nullifying the inherent dynamics of a given system and substituting them with the intended dynamics devised by the designer. The inversion of the governing equations of the system dynamics is accomplished by utilising the measured system states and input commands. The forces and moments are represented mathematically as functions of the measured state variables. Subsequently, opposing forces and moments are instructed to counteract the undesirable contributions, namely the forces and moments exerted on the body. This is followed by the application of the requisite forces and moments to successfully execute the manoeuvre. In the

event of an exact inversion and disregarding actuator saturation and rate constraints, the desired outcome is for the output to precisely correspond to the input. Typically, the application of dynamic inversion necessitates an equivalent quantity of inputs and states. If the number of inputs is smaller than the total number of potential states, it is not possible to exert control over all states. The states that are not under control thereafter exhibit behaviour in accordance with the characteristics of the open-loop dynamics. The concept of multiple timescales refers to the existence of different temporal dimensions or levels inside a given system or phenomenon. These time the utilisation of inner loop inversion for rapid system dynamics and outer loop inversion for sluggish system dynamics enables the control of a greater number of states with a given control input. The dynamic inversion technique has been employed for both longitudinal and lateral control of aircraft at high angle of attack situations. The implemented NDI for heading control of the hybrid-airship/payload system utilises a 9-DOF nonlinear dynamic model that incorporates comprehensive nonlinear longitudinal aerodynamics within the range of -15 deg to +15 deg angle of attack.

In the forward simulation, it is assumed that the hybrid airship/payload system is in a state of trimmed level flight. This is achieved by maintaining a symmetric flap deflection δ_s at zero while using an asymmetric flap command, δ_a , as the variable control. The deflection of the asymmetric flap, denoted as δ_a , is employed to control the angle of turn, denoted as, ψ_h in a Hybrid-Airship. The heading angle ψ_h^C , serves as the external command in the inversion method, requiring conversion into an asymmetric flap deflection demand δ_a^d . The accomplishment of this task is accomplished by following a two-step procedure, which involves an outer loop inversion and an inner loop inversion, as seen in Fig 4.1. The outer loop computes the Hybrid-Airship yaw rate, denoted as r_h^C by utilising the Hybrid-Airship turn command, represented as ψ_h^C . The control demand, δ_a^d is computed in the inner loop using the yaw rate, r_h^C which is determined in the outer loop. It should be noted that the P_h, q_h loops are not closed, resulting in the acceptance of q_h, r_h due to r_h^C . The NDI law is formulated in a manner akin to the approach described in reference

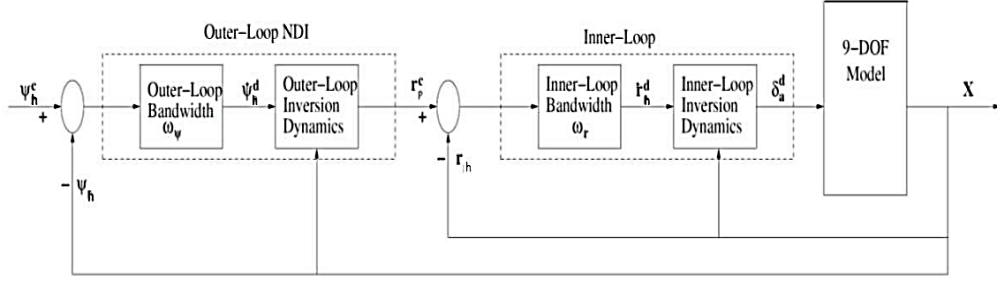


Fig. 4.1 Block diagram of NDI law

[63]. It leverages the multibody 9-degrees of freedom (DOF) equations of motion for the hybrid-airship/payload system, as outlined in reference [27].

These equations are expressed as:

$$A\dot{X} = B \quad (4.1)$$

$$\dot{X} = A^{-1}\{B_0 + B_{\delta_a}\delta_a\} \quad (4.2)$$

Where,

$$B_0 = \begin{bmatrix} F_b^A + F_b^G - \Omega_b \times M_b \Omega_b \times R_{Cb} \\ F_{h0}^A + F_h^G - \Omega_h \times (M_h + M_F) \Omega_h \times R_{ch} + M_F \Omega_h \times T_h V_C - \Omega_h \times M_F T_h V_C \\ \Omega_b \times I_b \Omega_b \\ M_{h0}^A + M_{h0}^T - \Omega_h \times (I_h + I_M) \Omega_h \end{bmatrix} \quad (4.3)$$

$$F_{h0}^A = \bar{q}_h s_h \begin{bmatrix} \{-C_D^h(\alpha_h, \delta_s) U_h + C_L^h(\alpha_h, \delta_s) w_h\} / V_h \\ C_{Y\beta} \beta + C_{Yr} r_h \frac{b}{2V_h} \\ \{-C_D^h(\alpha_h, \delta_s) W_h + C_L^h(\alpha_h, \delta_s) U_h\} / V_h \end{bmatrix} \quad (4.4)$$

$$M_{h0}^T = \bar{q}_h s_h \begin{bmatrix} 0 \\ x_{Ph} T \\ 0 \end{bmatrix} \quad (4.5)$$

$$M_{h0}^A = \bar{q}_h s_h \begin{bmatrix} b \{C_{l\beta} \beta + C_{lp} P_h \frac{b}{2V_h} + C_{lr} r_h \frac{b}{2V_h}\} \\ c \{C_{m_{C/4}}(\alpha_h, \delta_s) + x_{ha} C_z + C_{mq} q_h \frac{c}{2V_h}\} \\ b \{C_{n\beta} \beta + C_{np} P_h \frac{b}{2V_h} + C_{nr} r_h \frac{b}{2V_h}\} \end{bmatrix} \quad (4.6)$$

$$B_{\delta_a} = \begin{bmatrix} O_{3 \times 1} \\ F_h^A \delta_a \\ O_{3 \times 1} \\ M_h^A \delta_a \end{bmatrix} \quad (4.7)$$

$$F_{h\delta a}^A = \bar{q}_h S_h \begin{bmatrix} (-C_{D\delta a} U_h + C_{L\delta a} w_h)/V_h \\ C_{y\delta a} \\ (-C_{D\delta a} W_h + C_{L\delta a} U_h)/V_h \end{bmatrix} \quad (4.8)$$

$$M_{h\delta a}^A = \bar{q}_h S_h \begin{bmatrix} bC_{l\delta a} \\ cC_{m\delta a} \\ bC_{n\delta a} \end{bmatrix} \quad (4.9)$$

yaw rate dynamics of equation of motion is

$$\dot{r}_h = a(12)\{B_0 + B_{\delta a}\delta_a\} \quad (4.10)$$

Let $a(12)$ represent a row vector that corresponds to the 12th row of the inverse matrix A^{-1} in Eqn. (4.2). The yaw rate command, denoted as, r_h^C , and the feedback, denoted as r_h are employed in the computation of the error, represented as, $e_r = (r_h^C - r_h)$. Additionally, the desired yaw acceleration, denoted as $\dot{r}_h^d$ is modelled as such,

$$\dot{r}_h^d = \omega_r(r_h^C - r_h) \quad (4.11)$$

Where $\dot{r}_h^d$ is the derivative of r_h^d with respect to time, ω_r is a constant where ω_r is the bandwidth.

The expression presented below is derived from the inversion function by the manipulation of Eqn. (4.10).

$$\delta_a^C = \frac{\dot{r}_h^d - a(12)B_0}{a(12)B_{\delta a}} \quad (4.12)$$

$$\dot{r}_h = \omega_r(r_h^C - r_h) \quad (4.13)$$

$$\dot{r}_h^C = 0 \text{ as } r_h^C \text{ is a constant.}$$

$$\Delta \dot{r}_h = -\omega_r \Delta r_h \quad (4.14)$$

Inversion of the $\dot{\psi}_h$ equation yields yaw rate command r_h^C in outer loop. The design of the inversion law for the slow state, ψ_h , assumes that the fast state r_h , replies faster. The fast state's transient dynamics are so fast that they have no impact on ψ_h .

The yaw angle dynamics equation is:

$$\dot{\psi}_h = q_h \frac{\sin \phi_h}{\sin \theta_h} + r_h \frac{\cos \phi_h}{\cos \theta_h} \quad (4.15)$$

When comparing the commanded Hybrid-Airship yaw angle, Ψ_h^c , to the feedback, ψ_h , we obtain $\dot{\psi}_h^d$.

$$\dot{\psi}_h^d = \omega_{\psi}(\Psi_h^c - \psi_h) \quad (4.16)$$

where ω_{ψ_h} is the bandwidth. Eqn. (4.15) modification yields the inversion function:

$$r_h^c = \left(\dot{\psi}_h^d - q_h \frac{\sin \phi_h}{\cos \theta_h} \right) / \left(\frac{\cos \phi_h}{\cos \theta_h} \right) \quad (4.17)$$

The inner loop receives the estimated yaw rate. To acquire the yaw rate instantaneously, the inner loop has a higher bandwidth ($\omega_r = 0.5$) and a faster reaction ($\omega_{\psi} = 0.2$), than the outer loop. Inversion with accurate $\frac{\omega_{\phi}}{\omega_r} \sim 0$, results in replacing r_h^c in Eqn. (4.15), yielding the equation:

$$\dot{\psi}_h = \dot{\psi}_h^d = \omega_{\psi_h}(\psi_h^c - \psi_h) \quad (4.18)$$

Using Ψ_h^c is a constant command ($\dot{\psi}_h = 0$), Eqn. (4.18) becomes:

$$\Delta \dot{\psi}_h = -\omega_{\psi_h} \Delta \psi_h \quad (4.19)$$

Let $\Delta \psi_h = (\psi_h^c - \psi_h)$

Eqn. (4.19) shows the outer-loop inversion is a first-order system with $T = \frac{1}{\omega_{\psi}}$.

In practise, the inner and outer loops pair, resulting in a different dynamic than in Eqns. (4.15) and (4.18).

4.2 Tilt Angle Modelling | Side Rigging

Side-wise rigging angle $\delta = 0$ denotes equal length of starboard and port lines, whereas positive δ implies connecting point swinging port side of Z_h axis. Therefore,

$$z_{ch} = R_{ch} \cos \mu \cos \delta$$

$$x_{ch} = R_{ch} \sin \mu$$

$$y_{ch} = R_{ch} \cos \mu \sin \delta$$

Aerodynamic derivative data is used to design a controller based on the different single input single output transfer functions of hybrid airship. Transfer functions obtained from linearized state space model of hybrid airship.

Aerodynamic derivatives [73]

Parameter	Value
C_{m_q}	$-0.0359 (q_h c / 2V_h)^{-1}$
C_{Y_β}	$0.000019 / \text{deg}$
C_{Y_r}	$-0.000013 (r_h b / 2V_h)^{-1}$
C_{l_β}	$-0.0004 / \text{deg}$
C_{l_p}	$-0.0062 (p_h b / 2V_h)^{-1}$
C_{l_r}	$-0.0000016 (r_h b / 2V_h)^{-1}$
C_{n_β}	$0.0047 / \text{deg}$
C_{n_p}	$-0.0083 (p_h b / 2V_h)^{-1}$
C_{n_r}	$0.0000042 (r_h b / 2V_h)^{-1}$

Transfer Function:

G11=

$$\frac{-0.021 s^3 - 0.02646 s^2 - 0.2647 s + 0.2182}{s^4 + 0.9621 s^3 - 2.949 s^2 - 1.282 s - 1.233}$$

G12=

$$\frac{-0.044 s^3 - 2.023 s^2 - 0.5343 s - 0.2737}{s^4 + 0.9621 s^3 - 2.949 s^2 - 1.282 s - 1.233}$$

G13=

$$\frac{-0.0564 s^3 - 0.0543 s^2 - 0.02602 s}{s^4 + 0.9621 s^3 - 2.949 s^2 - 1.282 s - 1.233}$$

G14=

$$\frac{-0.0564 s^2 - 0.0543 s - 0.02602}{s^4 + 0.9621 s^3 - 2.949 s^2 - 1.282 s - 1.233}$$

4.3 PID CONTROLLER:

PID stands for proportional, integral, and derivative. This controller boosts the system stability and reduces the steady-state error. It is used in modern industry as automatic process control for the flight control system. The terms P, I, D effectively control the system dynamics by calculating the error between the measured value and desired value. The gain is tuned as per system design requirements of these three terms. The feedback controller is designed to control the desired output accurately operating a PID controller [20]. Control law $u(t)$ is expressed as

$$U(t) = k_p e(t) + k_i \int e(t) dt + k_d \frac{d}{dt} e(t) \quad (4.20a)$$

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_p \frac{de}{dt} \quad (4.20b)$$

The variables k_p , k_i , and k_d represent the proportional gain, integral gain, and derivative gain, respectively. $u(t)$ is PID Control Variable. $e(t)$ is error Value. The Laplace transform of Eqn. (4.20) in the continuous S-domain is given by Eqn. (4.21)

$$U(s) = \left[k_p + \frac{k_i}{s} + k_d s \right] \quad (4.21)$$

The generalized theoretical control strategy for making a UAV autonomous is shown in Fig. 4.2. The hybrid airship mathematical state space form is decoupled by decomposition methods as diagonalization to get transfer functions. Fig. 4.3 is the linear PID Simulink diagram.

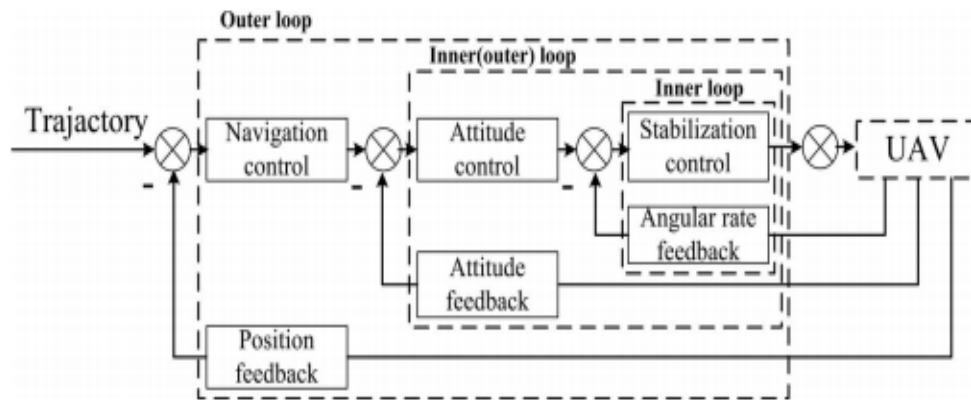


Fig. 4.2 Generalized Control Strategy

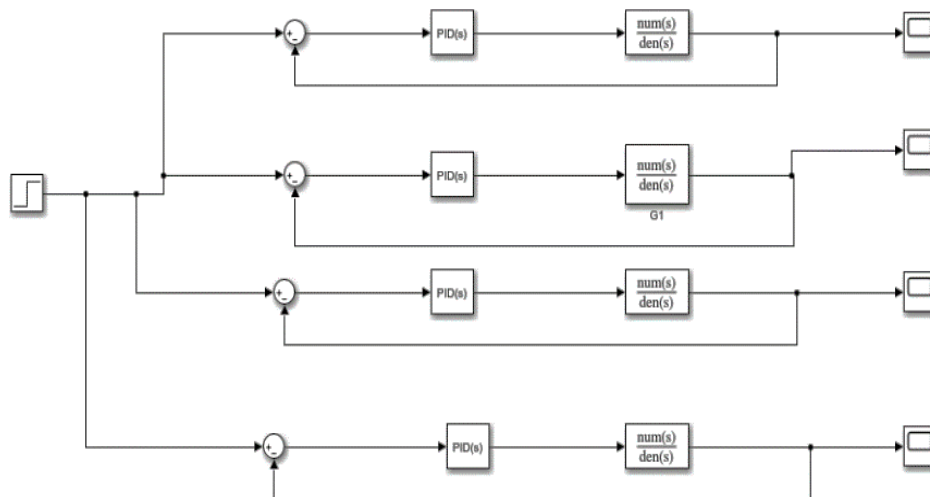


Fig. 4.3 Simulink block diagram for airship with PID

4.3.1 PID Controller Limitation:

One limitation of the PID controller is its inability to effectively control systems with significant time delays or non-linear dynamics. In addition to its reputation as a very effective controller within control action systems, it is important to acknowledge that a PID controller also possesses certain limits. The application of PID control is widespread across various control actions; yet its performance in optimal control scenarios is suboptimal. The primary drawback is in the feedback pathway. The process model does not include the Proportional-Integral-Derivative (PID) controller. Additional limitations include the fact that PID is a linear system and that the derivative component is susceptible to noise.

4.3.2 PID Assumption:

Small amount of noise can cause great change in the output. Small perturbation theory, small angle assumption and neglecting the coupled and higher order derivative assumptions are applied to make a linearized model from it's nonlinear model for controller design.

4.3.3 PID Controller Design:

A controller is an integral component of a control system that provides control signals to minimise the difference between the actual value and the intended value, aiming towards zero. System control ensures correct output. Control action is when the controller generates a control signal. The error signal is the difference between the reached output and the reference input, which the controller must compensate to create the intended output. As shown in Fig. 4.4 and Fig. 4.5, the controller sends control signals to the system to obtain the desired output. The feedback-based proportional-integral-derivative controller (PID controller or three-term controller) is widely used in industrial control systems and other applications that require continuous control modulation. PID feedback control calculates error value $e(t)$ by subtracting the desired setpoint (SP) from the measured process variable. It then modifies the system using proportional, integral, and derivative terms (P, I, and D) according to Fig. 4.5 and the mathematical Eqn. (4.20).

The PID control strategy effectively integrates the benefits of proportional, derivative, and integral control actions. The attributes are listed in Table 4.1 provided above. With the help of semi-empirical method, the mathematical model is developed on MATLAB and the state matrix A and the input matrix B for the winged hybrid airship is computed for longitudinal dynamics as below:

$$A = \begin{bmatrix} -0.3339 & 0.3763 & 0.0633 & -4.8697 \\ -1.2535 & -0.5901 & 34.1954 & 0 \\ -0.1094 & 0.0809 & -0.0286 & 1.1979 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -0.0032 & 0.1113 \\ 0.0056 & 0 \\ -0.0022 & 0.0027 \\ 0 & 0 \end{bmatrix}$$

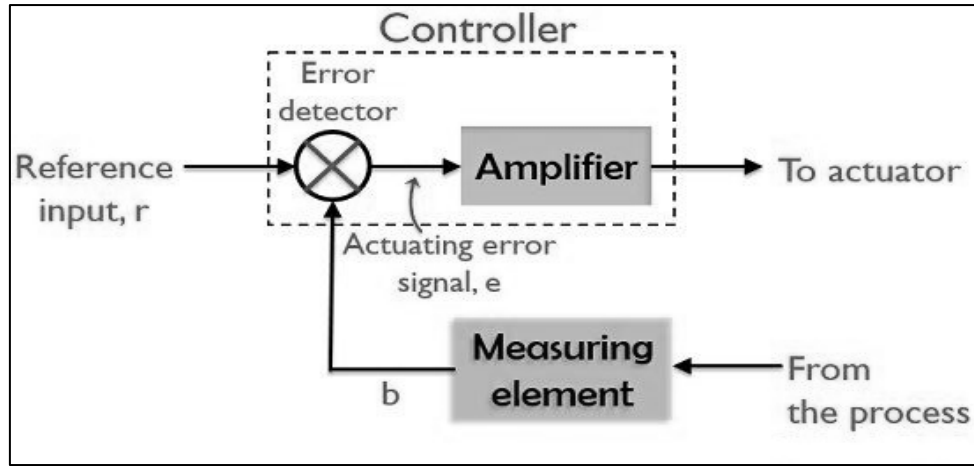


Fig 4.4 Block diagram of an industrial controller.

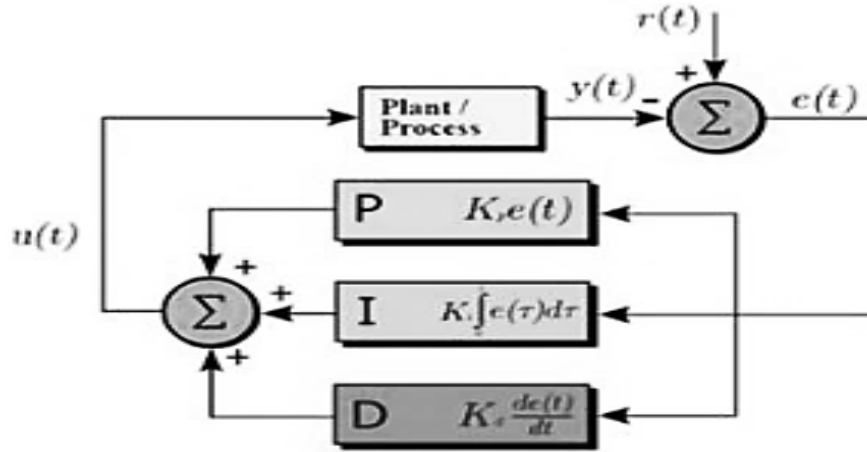


Fig 4.5 PID Controller in Control System.

Table 4.1 PID Controller Parameter Characteristics

Control Response	Rise Time	Settling Time	Overshoot	Steady State Error
K_p	decrease	small change	increase	Decrease
K_d	small change	decrease	decrease	no change
K_i	decrease	increase	increase	Eliminate

The performance, robustness, and stability analysis for transfer functions ($\frac{u(s)}{\delta_e}$, $\frac{w(s)}{\delta_e}$, $\frac{q(s)}{\delta_e}$ and $\frac{\theta(s)}{\delta_e}$) are obtained and listed in Table 4.2. The same analysis is

also done for the system with different controllers as required and is listed in Table 4.2.

Table 4.2: Performance, Robustness, and stability analysis:

Transfer function	GM (dB)	PM (dB)	Rise time	Settling Time	Overshoot	Peak	CL Stability
$G11 = \frac{u(s)}{\delta_e}$	52.4 dB @ 0 rad/s	-	-	-	-	-	NO
$G12 = \frac{w(s)}{\delta_e}$	-	-	-	-	-	-	NO
$G13 = \frac{q(s)}{\delta_e}$	-	-	-	-	-	-	NO
$G14 = \frac{\theta(s)}{\delta_e}$	-	-	-	-	-	-	NO
G11_IMC	0.33 dB @ 1.19 rad/s	- 1.53deg @ 1.52 rad/s	0.0376 s	28.9 s	838.4%	9.38 in 3.83 s	YES
G12_IMC	-2.55 dB @ 0.491 rad/s	18.5 deg @ 1.45 rad/s	0.339 s	8.59 s	236%	3.36 in 2.14 s	YES
G13_PI	-37.8 @ 1.15 rad/s	69 deg @ 31.9 rad/s	0.0433 s	0.264 s	17.8%	1.19	YES
G14_PID	-20.6 dB @ 2.02 rad/s	69 deg @ 20.1 rad/s	0.064 s	1 s	15.7 %	1.16	YES

For pitch control as a trajectory tracking, a PI and PID controller is designed for the transfer function $G13 = \frac{q(s)}{\delta_e}$ and $G14 = \frac{\theta(s)}{\delta_e}$. IMC compensator is designed for stabilizing the transfer function $G11 = \frac{u(s)}{\delta_e}$ and $G12 = \frac{w(s)}{\delta_e}$. The designed controller/compensator transfer function is listed in Table 4.3.

Table 4.3. Controller Transfer Function for Longitudinal Dynamics of Hybrid Airship:

	Controller Transfer function (O/p vs δ_e)
G11_IM C	$\frac{-7241.2 (s + 2.272)(s - 0.8257)(s + 0.09128)(s^2 + 0.397s + 1.948)}{s (s - 19.3)(s - 0.1089)(s^2 + 0.6198s + 1.948)}$
G12_IM C	$\frac{-597.06 (s + 23.98)(s + 3.534)(s + 0.09103)(s^2 + 0.3099s + 1.948)}{s (s^2 + 0.2686s + 0.3322)(s^2 + 47.47s + 646.9)}$
G13_PI	$-13530.16 \left(1 + \frac{12.7037}{s} \right)$
G14_PI D	$-38070.787 - \frac{39504.896}{s} - 8952.545 \frac{109.82}{1 + 109.82s}$

Eigen values of w, the velocity in z body axis of winged hybrid airship is on RHS of pole zero plot, that provides instability in the system. The winged hybrid airship is statically stable and dynamically unstable. An internal model control (IMC) compensator is designed for a SISO mathematical model represented in transfer function form as G11 and G12. The step response of the open loop system is evaluated with state space approach shown in Fig. 4.6. and pole zero plot of the system is shown in Fig. 4.7. The bode plot and step response of IMC compensator for G11 and G12 are shown in Fig.4.8 and Fig. 4.9 respectively and it gives closed loop stability in the system.

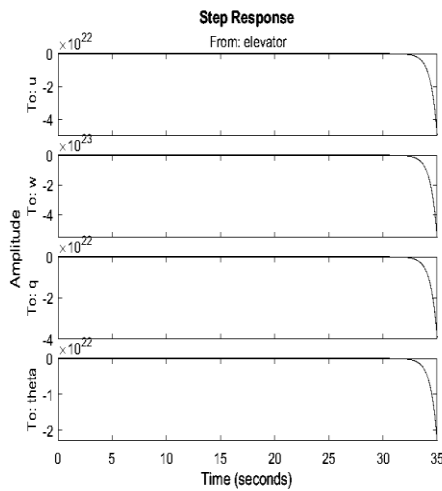


Fig. 4.6 Open loop step response

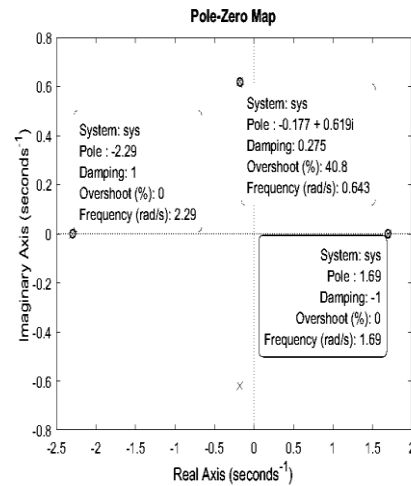


Fig. 4.7 pole zero plot of system

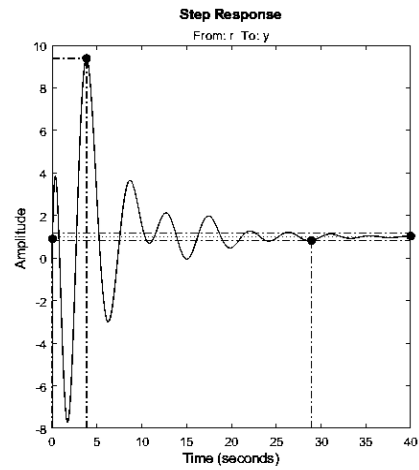
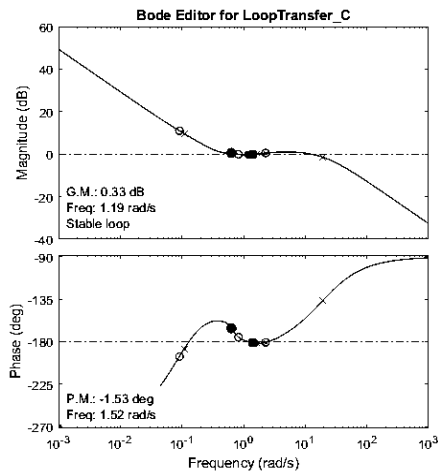


Fig. 4.8 G11_IMC Compensator with system response

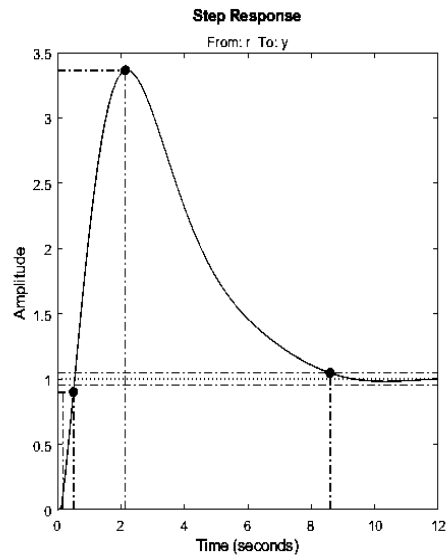
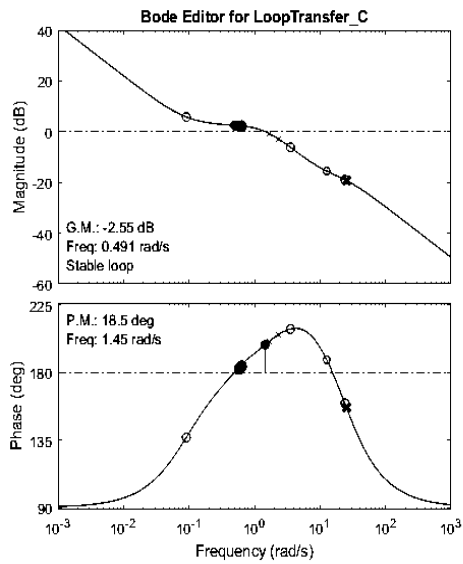


Fig. 4.9 G12_IMC Compensator with system response

PI (proportional integral) controller and PID (proportional integral derivative) controller is designed as pitch control and trajectory tracking for G13 and G14 transfer function. The reference tracking response of G13 and G14 is shown in Fig 4.10 and Fig. 4.11.

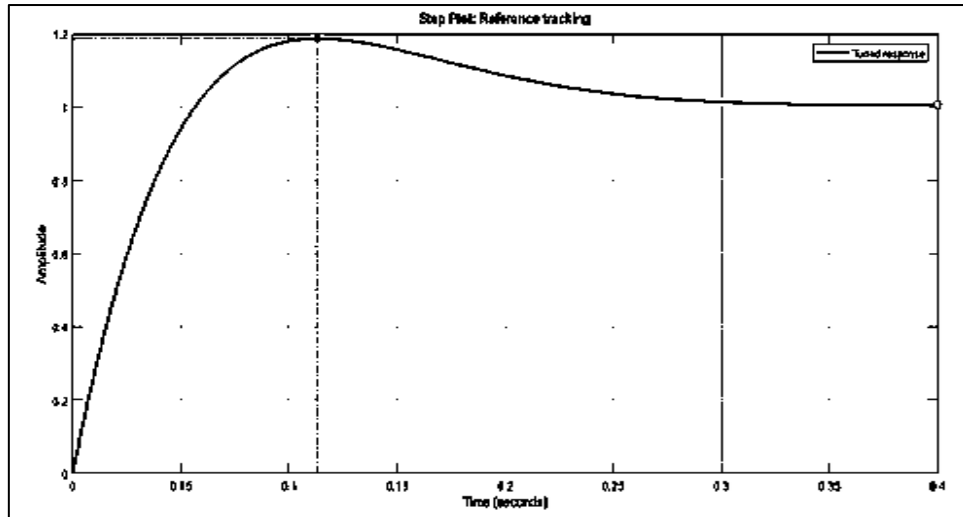


Fig. 4.10 G13_PI Controller reference tracking

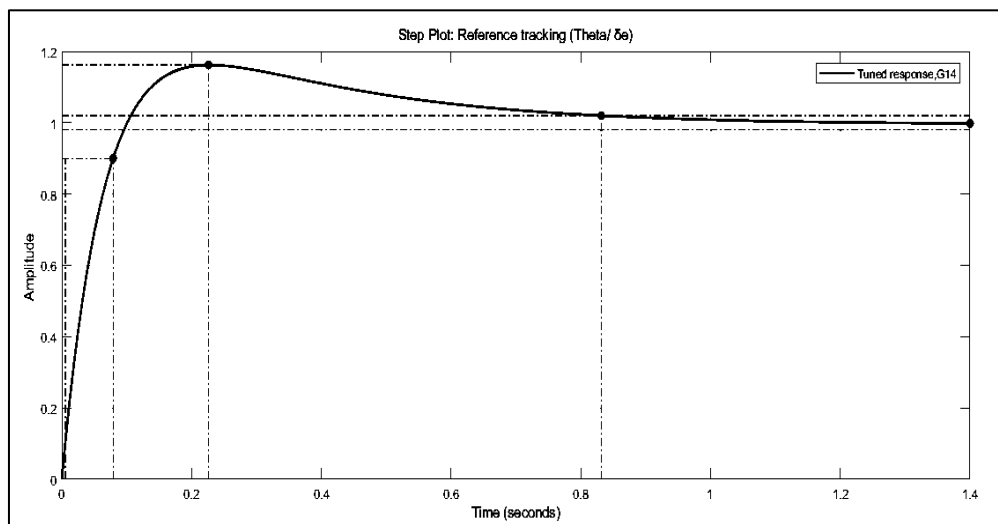


Fig. 4.11 G14_PID Controller reference tracking

Finally, a complete longitudinal dynamics model with controller/compensator for winged hybrid airship is designed as Simulink model and is shown in Fig. 4. 12. u and w with compensator overshoots at higher value, so HIL solution for the same is not possible here in this case. PID controller for pitch control is fabricated on UNO board with ALTAIR Embed software shown in Fig. 4.13. The movement of elevator for pitch control is represented here with paper attached to servo motor. This is a small step to make hybrid airship autonomous with navigation, guidance, and control.

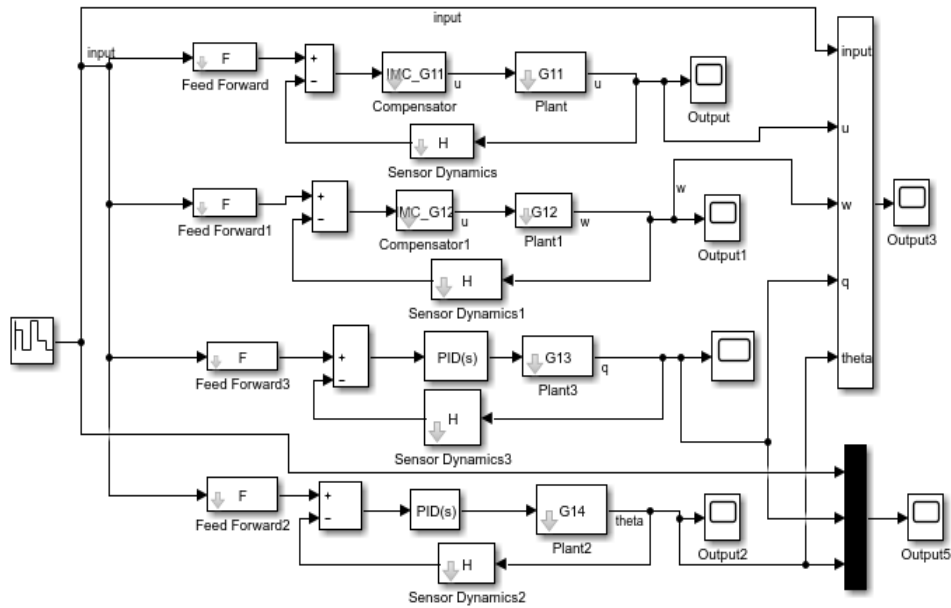


Fig. 4.12 Simulink Longitudinal Dynamic Model for Winged Hybrid Airship

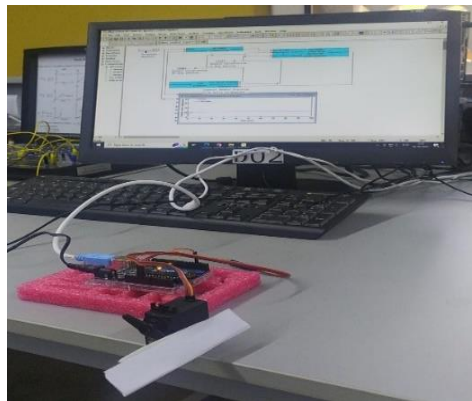


Fig. 4.13 HIL for G14 as pitch control



Fig.4.14 Development of airship in campus

The airship shown in Fig. 4.14 is under development as a SEED funded project under Dr Om Prakash in UPES, Dehradun campus.

For pitch control different controllers are designed, and comparisons are given below in table 4.4.

TABLE 4.4 Response Analysis of Controllers for Pitch Control Designed for Hybrid Airship:

System	Controllers	GM (dB)	PM (dB)	Rise time	Settling Time	Over shoot	Peak	Stability
$G14 = \frac{\theta(s)}{\delta_e}$	PID	-20.6 dB @ 2.02 rad/s	69 deg @ 20.1 rad/s	0.064 s	1 s	15.7 %	1.16	YES
	IMC	-2.51 dB	19.8 deg	0.377 s	10.1 s	232	3.32	YES
	LQG	-0.98 dB	5.16 deg	0.346 s	6.49 e+03 s	1.01e +03	11.1	YES
	LQR	1.0747e +03	inf	2.043 4s	16.587 1 s	19.89 60	0.001 1	YES

4.4 OTHER NONLINEAR CONTROLLERS:

Sliding mode control (SMC) uses a discontinuous, set-valued control signal to change the behavior of a nonlinear system. This signal makes the system "slide" along a typical behavior cross-section. State-feedback control legislation is discontinuous. Alternatively, the system can switch continuous structures depending on its state space location. Thus, sliding mode control is changeable structure control. Numerous control structures ensure that trajectories continually approach an adjacent region with a separate control structure. Thus, the end trajectory will not be contained in one control structure. Alternatively, it will travel around control structures. Variable control structures include sliding mode control. Robustness, finite time convergence, and lower order compensated dynamics are advantages of sliding mode control. Sliding mode controller is the future work scope of the thesis.

Model predictive controller is also designed for aircraft lateral dynamics and similar work has been carried out for hybrid airship in longitudinal dynamics mode shown as extra work in Appendix A5.

Further Nonlinear Controllers preliminary study as model predictive controller and sliding mode controller is carried out in Appendix A5 is further explored in future. LQG and LQR is also designed for the system as an extra preliminary work shown in Appendix A5. The all-extra controller works can be further explored as a future scope.

4.5 SUMMARY:

The aim to design different controller for one body and two body dynamic model of hybrid airship is to make a hybrid airship autonomous. In this direction a Nonlinear dynamic inversion (NDI) controller has been designed to achieve the direction control for hybrid airship in two body dynamic modelling approach. NDI is designed based on the obtained design parameter as rigging angle or elevator deflection after trim analysis. Table 4.2 discuss about the performance and stability analysis of hybrid airship. PI (proportional integral) controller and PID (proportional integral derivative) controller for single body dynamic model is designed as pitch control and trajectory tracking. Compensators are designed to make u and w transfer function stable. Finally, a complete longitudinal dynamics model with controller/compensator for winged hybrid airship is designed as Simulink model. Closed loop and open loop stability analysis is also done in this chapter. Hardware in loop solution on Uno board with Altair Embed software is provided as a small prototype replica to make hybrid airship autonomous. SMC model can be designed as an alternate solution for better trajectory tracking for navigation and guidance. For controller part extra simulation is shown in appendix A5.

CHAPTER 5

TRIM AND STABILITY ANALYSIS USING BIFURCATION TECHNIQUE

Nonlinear differential equation is used for single body (3DOF) stability analysis using bifurcation method. For two body (4DOF) trim and stability analysis is done. Bifurcation parameters used here as control inputs like rigging angle variation and elevator deflection one by one.

5.1 INTRODUCTION TO BIFURCATION ANALYSIS:

A bifurcation of a dynamical system is a qualitative change in its dynamics produced by varying parameters. A bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden qualitative or topological change in its behaviour. The tangible, dynamic mathematics of chaos and fractals are a representation of this. Dynamics is an even-handed topic that includes chaos and fractals for example. Differential equations and iterated maps (sometimes called difference equations) are the two basic categories of dynamical systems. When time is discrete, difficulties involving iterated maps occur, while differential equations explain the evolution of systems in continuous time. The formula for harmonic oscillator with damped harmonics is in Eqn. (5.1).

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \quad (5.1)$$

It involves the ordinary derivatives $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$, making it an ordinary differential equation. This means that the time t is the only independent variable. The heat equation Eqn. (5.2), in comparison,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad (5.2)$$

is a partial differential equation for which space x and time t are independent variables. Here are the procedures to write the damped harmonic ordinary differential equation in state space form:

$x_1 = x$ and $x_2 = \dot{x}$ are new variables that we introduce.

Thus,

$$\dot{x}_1 = x_2 \quad (5.3)$$

$$\dot{x}_2 = \ddot{x} = -\frac{h}{m}\dot{x} - \frac{k}{m}x = -\frac{h}{m}x_2 - \frac{k}{m}x_1 \quad (5.4)$$

The fact that every x_i on the right appears only to the first power indicates that this system is said to be linear. A nonlinear system would result from this. Pros, powers, and functions of the x_i such as x_1x_2 , $(x_1)^3$ or $\cos x_2$ are examples of typical nonlinear expressions.

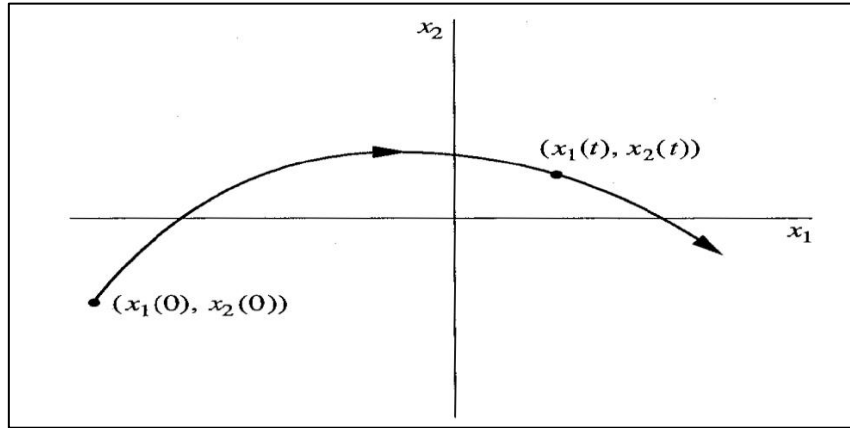


Fig. 5.1: A trajectory curve

The space is referred to as the system's phase space, and the curve seen in Fig. 5.1 is known as a trajectory. Trajectories fill the phase space because any point might be an initial condition. Every time we increase the phase space dimension from $n=1$ to $n=3$, we come across new phenomena: at $n=1$, fixed points and bifurcation are encountered; at $n=2$, nonlinear oscillation is observed; and at $n=3$, chaos and fractals are encountered. In every situation, a geometric method works extremely well and provides us with the majority of the data we need. Any equation with nonlinearity is extremely challenging to solve analytically. The typical method to get around this is to fudge, using the small angle approximation for $x \ll 1$ as $\sin x = x$. This makes the problem linear, which

makes it easier to solve. But we are discarding some of the physics by limiting it to small x . We wish to sketch the trajectories given by the system in order to gather details about the solutions. Geometric reasoning can often be used to sketch the trajectories without the need to solve the system! Most nonlinear systems are impossible to solve analytically, as we have already discussed. Why is it so much more difficult to analyze nonlinear systems than linear ones? The primary distinction is that linear systems are segmental. The solution can then be obtained by solving each component independently and then recombining them. This principle underpins techniques like normal modes, Laplace transforms, superposition arguments, and Fourier analysis and enables an amazing reduction of complicated situations. A linear system is exactly equal to the sum of its parts in this sense. However, this is not how many things in nature behave. Nonlinear interactions occur whenever components of systems collaborate, compete, or interfere with one another.

5.2 TYPES OF BIFURCATION

Local Bifurcations: which can be analyzed entirely through the changes in the local stability properties of equilibrium, periodic orbits or other invariant sets as parameters cross through critical thresholds.

Global Bifurcations: Which often occur when larger invariant sets of the system collide with each other, or with equilibrium of the system. They cannot be detected purely by a stability analysis of the equilibrium (fixed points).

In general, in a dynamical system, a parameter is allowed to vary, then the differential system may change. An equilibrium can become unstable, and a periodic solution may appear or a new stable equilibrium may appear making the previous equilibrium unstable. The point at which these alterations take place is referred to as the "bifurcation value," while the variable that is manipulated is referred to as the "bifurcation parameter."

The initial three categories of bifurcation manifest in both scalar and systems of differential equations. Hopf bifurcation, the fourth type of bifurcation, is not observed in scalar differential equations due to its characteristic association with the emergence of periodic solutions. Periodic solutions cannot exist for scalar autonomous differential equations. The phenomenon of Hopf bifurcation

manifests in systems of differential equations comprising two or more equations. Bifurcations are generally classified into several types, including saddle-node, pitchfork, Hopf, and trans-critical bifurcations. Each type represents a different mechanism by which system stability is lost, leading to new behavior patterns. For instance, Hopf bifurcations involve the transition from a stable fixed point to a limit cycle, a concept extensively studied in the context of oscillatory systems like predator-prey models and electrical circuits.

- **Saddle-Node Bifurcation:** The simplest bifurcation, where two fixed points (one stable and one unstable) collide and annihilate each other.
- **Pitchfork Bifurcation:** Involves symmetry-breaking, where a single stable solution splits into multiple branches.
- **Hopf Bifurcation:** Leads to the emergence of periodic solutions from a fixed point.
- **Trans-critical Bifurcation:** Involves the exchange of stability between fixed points.

1. Saddle node bifurcation.

- Fixed points are created, collided and destroyed shown in Fig. 5.2.

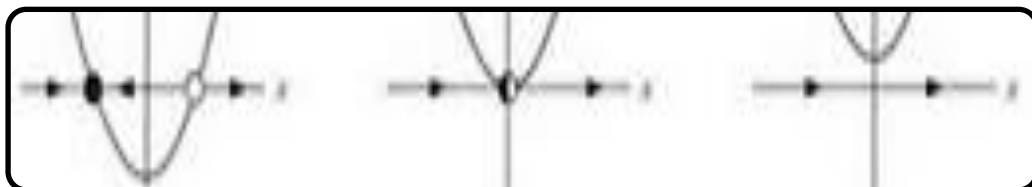


Fig. 5.2 saddle node

2. Transcritical Bifurcation

- Fixed points must exist for all values of a parameter as shown in Fig. 5.3.

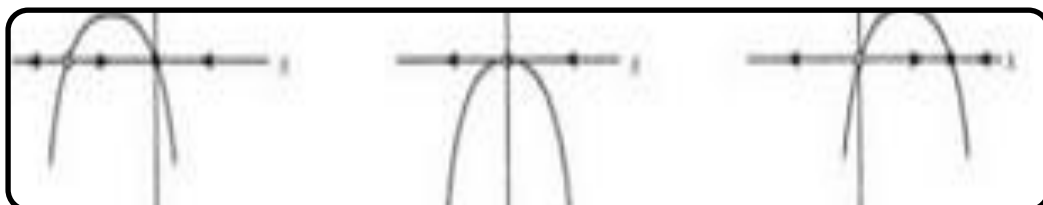


Fig. 5.3 Transcritical bifurcation

3. Pitchfork

- This is common in physical problems.

- That have a symmetry as shown in Fig. 5.4.

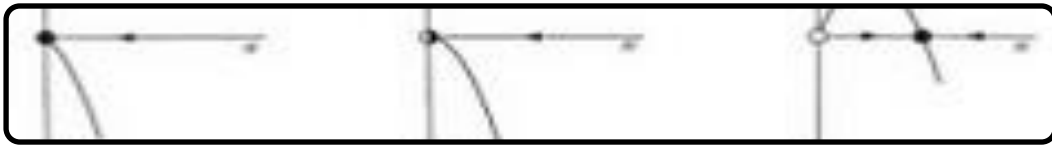


Fig. 5.4 Pitchfork bifurcation

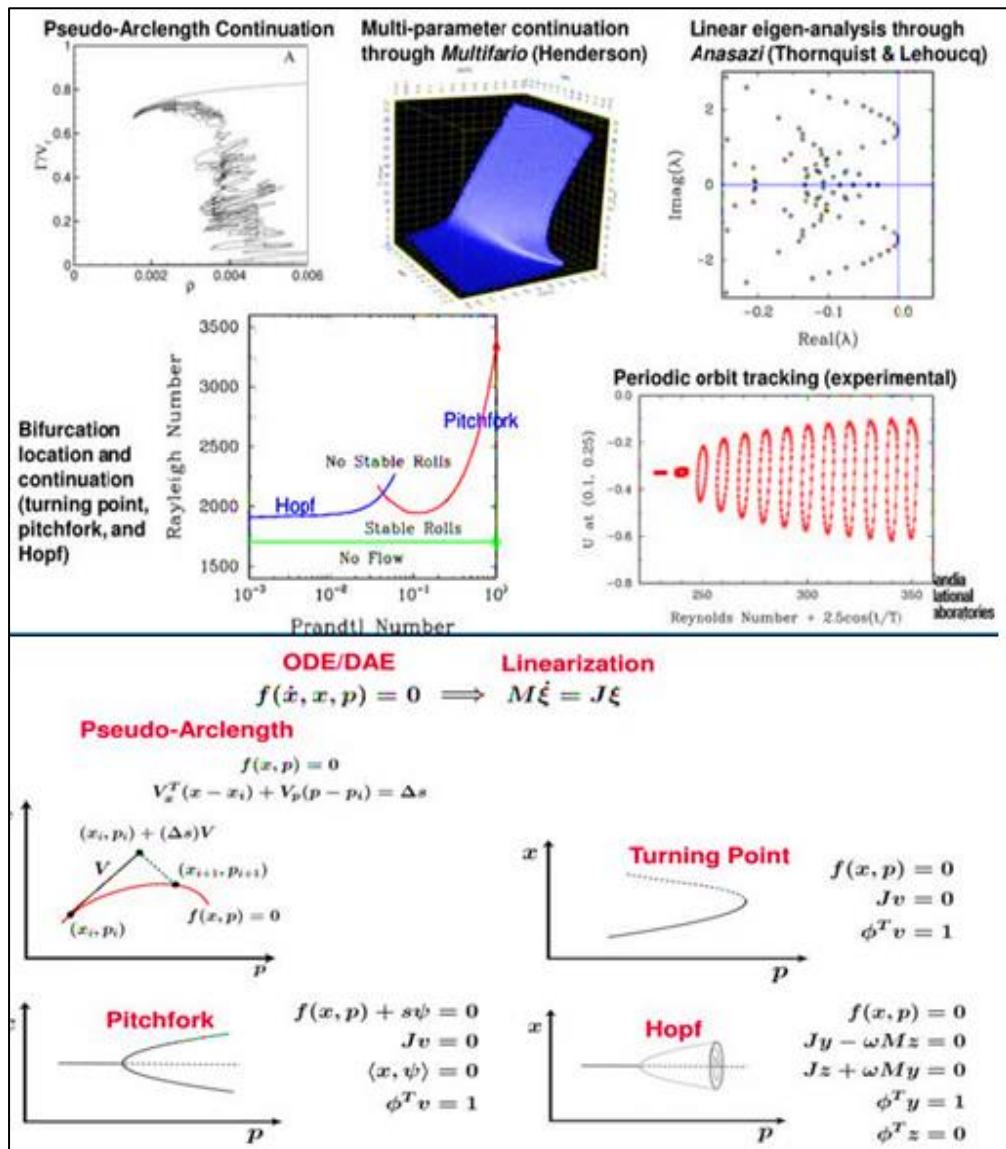


Fig. 5.5 Bifurcation analysis techniques and methods.

Fig. 5.5 clearly visualize the types of bifurcation and methods to apply it.

5.3 METHODOLOGIES IN BIFURCATION ANALYSIS OF AIRCRAFT

Bifurcation analysis in aircraft involves several methodologies, each suited to different aspects of aircraft behavior:

- **Analytical Techniques:** For simple models, bifurcations can often be studied analytically using methods like perturbation theory or normal form analysis. These techniques provide insights into the nature of bifurcations and the stability of resulting flight modes.
- **Numerical Continuation Methods:** For more complex models, numerical continuation methods are commonly used. These methods allow for the systematic exploration of how equilibrium points and periodic orbits change as parameters are varied. Software tools like AUTO or MATCONT are frequently employed in this context.
- **Computational Fluid Dynamics (CFD):** CFD plays a crucial role in providing accurate aerodynamic data, especially at high angles of attack or in transonic regimes where nonlinear effects are pronounced. The results from CFD simulations can be integrated into bifurcation analysis to predict stability boundaries more accurately.
- **Experimental Validation:** Wind tunnel tests and flight tests are often used to validate predictions made by bifurcation analysis. These experiments are crucial in verifying the existence of bifurcation points and the associated nonlinear behaviors in real-world conditions.

5.4 LONGITUDINAL TRIM ANALYSIS

The software MATLAB is utilized for the development of subroutines that facilitate the simulation of the model. The equations (5.1 to 5.3) represent the lift force, drag force, and pitching moment as $L = \bar{q}SC_L$, $D = \bar{q}SC_D$ and $M = \bar{q}cSCm$ respectively. These equations define the lift coefficient, drag coefficient, and pitching moment coefficient. The linear relationships between aerodynamic coefficients, as described in Eqn. (5.5 - 5.7), are utilized in the modelling of the 3DOF longitudinal model of a hybrid airship. These relationships are also employed in the implementation of the bifurcation approach, where the elevator deflection δ_e is a key parameter.

$$C_L = \left\{ C_{L_0} + C_{L_\alpha} \cdot \alpha + C_{L_q} \cdot \frac{q\bar{c}}{2V_1} + C_{L_{\delta_e}} \cdot \delta_e \right\} \quad (5.5)$$

$$C_D = \left\{ C_{D_0} + C_{D_\alpha} \cdot \alpha + C_{D_q} \cdot \frac{q\bar{c}}{2V_1} + C_{D_{\delta_e}} \cdot \delta_e \right\} \quad (5.6)$$

$$C_m = \left\{ C_{m_0} + C_{m_\alpha} \cdot \alpha + C_{m_q} \cdot \frac{q\bar{c}}{2V_1} + C_{m_{\delta_e}} \cdot \delta_e \right\} \quad (5.7)$$

To achieve static longitudinal stability in a vehicle, it is necessary for the coefficient of pitching moment (C_m) to have a negative value. The positive and negative values of C_m correspond to nose-up and nose-down pitching moments, respectively. It should be noted that C_m varies with changes in the angle of attack (α).

The trim or equilibrium state is obtained by utilizing the longitudinal nonlinear dynamic hybrid airship model, as stated in Eqn. (3.62 to 3.69). The states V , γ , q , θ , and α are significant in this context. Specifically, the states V , γ , and θ are assumed to be zero, indicating that the model is assumed to be flying with constant values for V , γ , and θ . In the above scenario, the flight trajectory follows a linear path, with potential conditions represented by γ . A value of $\gamma=0$ indicates level flight, a positive value of γ signifies ascent, and a negative value of γ denotes descent. Additionally, it is important to note that the orientation of the aircraft's nose, represented by θ , remains constant during the flight. In order to get the desired trim conditions for the flight of the hybrid airship, the trim states are determined by equating the left-hand side of Eqn. (3.62 to 3.69) to zero and solving for the right-hand side. This relationship is expressed by Eqn. (5.8 to 5.11).

$$0 = (T \cos(\alpha) - \bar{q}S C_D - (mg - B) \sin(\gamma)) / (m - X_{ud}) \quad (5.8)$$

$$0 = (T \sin(\alpha) + \bar{q}S C_L + z_q - (mg - B) \cos(\gamma)) / \{(m - z_{wd}) \cdot V\} \quad (5.9)$$

$$0 = M_z / (I_{yy} - M_{qd}) \quad (5.10)$$

$$\dot{\theta} = 0 \quad (5.11)$$

The trim states are represented by (*) and are given by the following Eq. (5.12

– 5.14).

$$C_D^* = (T \cos(\alpha) - (mg - B) \sin(\gamma)) / \bar{q}s \quad (5.12)$$

$$C_L^* = ((mg - B) \cos(\gamma) - T \sin(\alpha) - z_q) / \bar{q}s \quad (5.13)$$

$$C_m^* = \frac{M_s - (Mq - ma_x uu - ma_z ww) \cdot q - T \cdot 0.1}{\bar{q}Sc} \quad (5.14)$$

From Eqn. (5.8) we can write the thrust relation given by Eqn. (5.15).

$$T = (mg - B) \sin(\gamma) + C_D^* \bar{q}^* s / \cos(\alpha) \quad (5.15)$$

The initial values for the simulation are also used as the initial values to implement the bifurcation method.

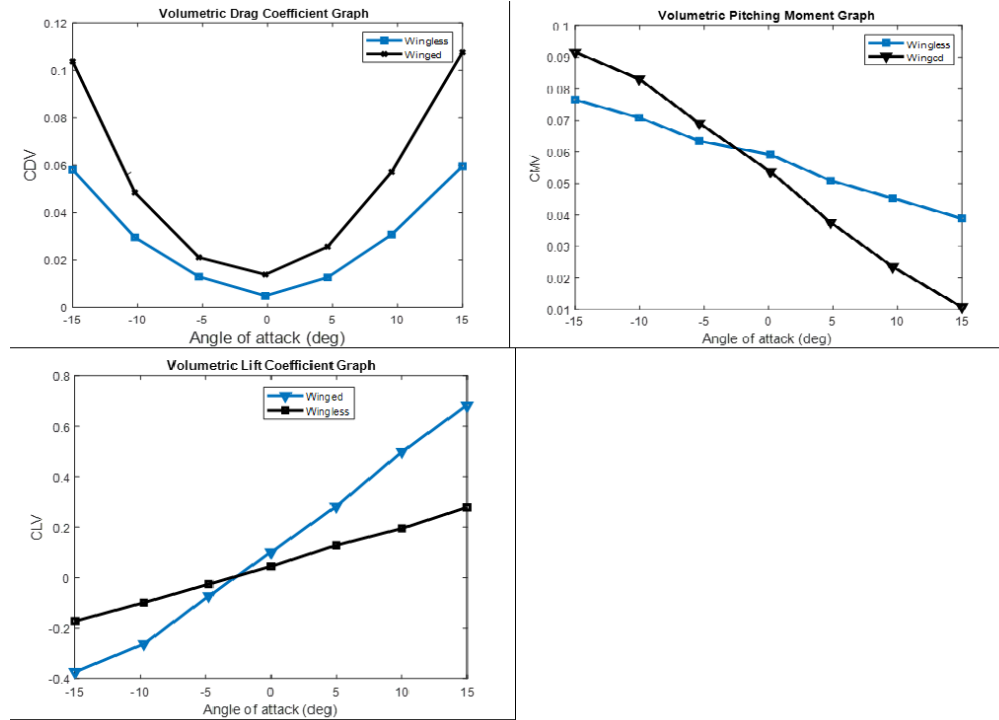


Fig. 5.6. Aerodynamic data [96]

Aerodynamic data shown in Fig. 5.6, geometry data and flight data is discussed in appendix A3 and in methodology section.

5.5 BIFURCATION METHOD ANALYSIS

The utilization of the Bifurcation Method offers an opportunity to optimize the design parameters for vehicle control through the application of the bifurcation

technique, hence demonstrating global stability. The approach yields numerical data pertaining to the overall stability of the aircraft and presents tactics for mitigating the nonlinear dynamics of the system. The approach employed in this study involves the utilization of a continuation-based algorithm (CBA) for the exploration of a nonlinear dynamic model. This exploration is conducted through the implementation of first-order ordinary differential equations (ODEs) and the utilization of AUTO-07p software [104]. The primary objective of this approach is to identify steady-state and various equilibrium points within the model. Ordinary Differential Equations (ODEs) can be expressed as a function of state variables and control parameters, denoted as $\dot{x}=f(x, u)$. The solution of the nonlinear equation is defined by $\dot{x}=f(x, u, p)=0$, while maintaining the constraint p at a constant value and adjusting the subsequent parameter u . This process leads to the discovery of all trim states. The implementation of the Standard Bifurcation Analysis (SBA) approach involves the utilization of continuation-based algorithm (CBA). In this methodology, the continuation algorithm is utilized to calculate all solutions with trim states of the system, considering the variable and fixed parameters. Additionally, it defines the specific characteristics of local stability at each trim state. The alterations in the stability of the trim state solutions are referred to as "Bifurcation Points". The occurrence or behavior of an unstable dynamical system is prominently indicated by the migration of the system's eigenvalues from the left half to the right half of the complex plane, specifically at the bifurcation points. These points mark the moment when the eigenvalues lead to instability in the stability plane.

The implemented approach involves the application of the Standard Bifurcation Analysis (SBA) to the nonlinear dynamic model of a 3-degree-of-freedom (3DOF) longitudinal hybrid airship model, as described in references [26], [61] and [62]. The state variables are denoted as $x=[V, \gamma, \alpha, \theta, q, h]^T$. In order to simplify the analysis, the states are reduced by assuming constant velocity (V), constant height (h), and negligible variations in the parameter gamma (γ). Consequently, the state variable evaluated for the study is decreased and denoted as $x=[\alpha, \theta, q]^T$. The Standard Bifurcation Analysis (SBA) has been utilized to

develop the hybrid airship, resulting in the acquisition of a bifurcation diagram that displays the Eigen values for each scenario. This diagram effectively illustrates the many flight dynamics behaviors of the hybrid airship. Fig. 5.7 displays the bifurcation diagram illustrating the relationship between velocity, specifically at a magnitude of 20 m/s, and the mass of the vehicle, which is specified as 10 kg. The examination of stability information for the longitudinal dynamic model can be conducted by analyzing the Eigenvalues derived from the bifurcation analysis. The stability of the system's behavior is contingent upon the characteristics of its Eigenvalues. Table 5.1 displays the distinct Eigenvalues acquired by the bifurcation approach for the various scenarios involving lift deflection. The resulting Eigenvalues are utilized for the purpose of generating a pole-zero map, which serves to provide insights into the stability behavior of the system. This representation is visually depicted on Fig. 5.8. The pole plot for the various simulated cases reveals that Cases 1, 2, 3, 5, and 6 exhibit stable behavior, while Cases 4 and 7 demonstrate unstable behavior as a result of the poles being located on the right-hand side (RHS) plane.

Table 5. 1 Eigen values obtained using Bifurcation Method

SimulationCases	Elevator deflection (delfin , deg)	Eigenvalues	Stability
1	0.0	-0.195248, -11.5970, -44591.4	Stable
2	1.5	-0.168959, -12.2705, -44591.2	stable
3	3.13672	-0.00909272, -12.8659, -44591.0	Stable
4	3.15437	0.000000092, -12.8647, -44591.0	Unstable
5	-1.5	-0.168893, -10.8492, -44591.6	stable
6	-3.13478	-0.00923439, -10.0340, -44591.8	stable
7	-3.15253	0.0000000976266, -10.0296, -44591.8	Unstable

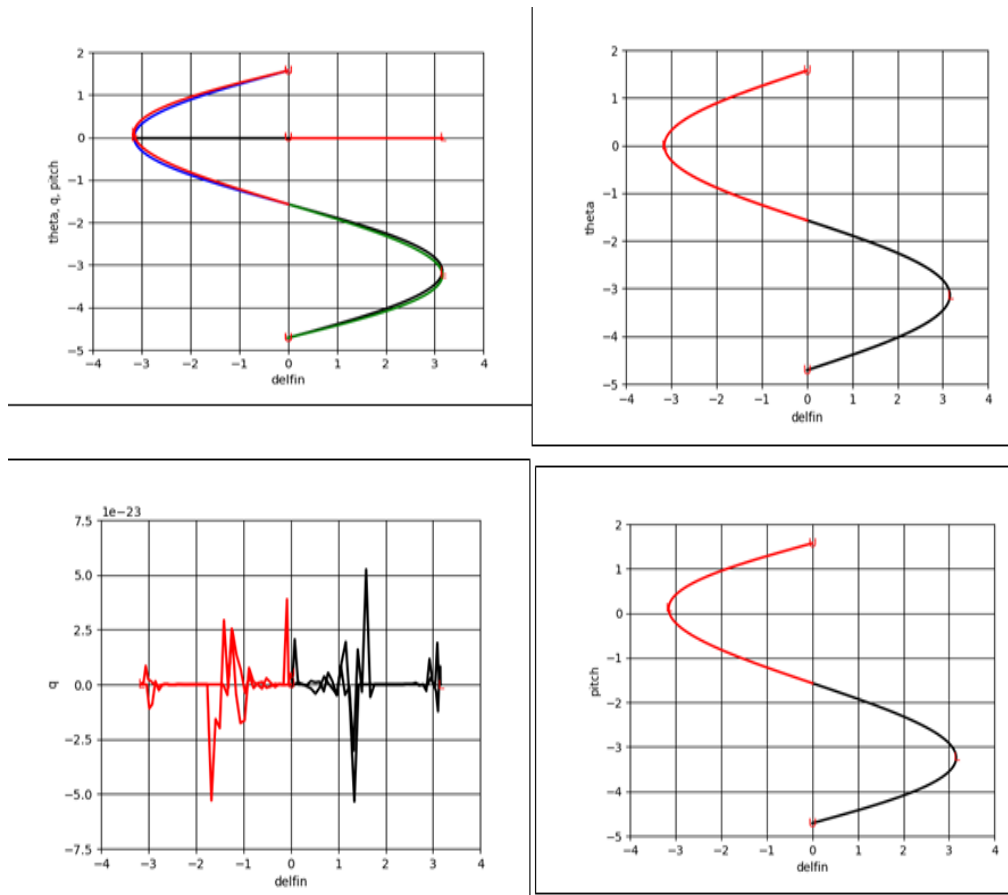


Fig. 5.7. Bifurcation Diagram ($theta, q, pitch$ (combined), θ in degree, q in degree and pitch Vs dele deflection in degree)

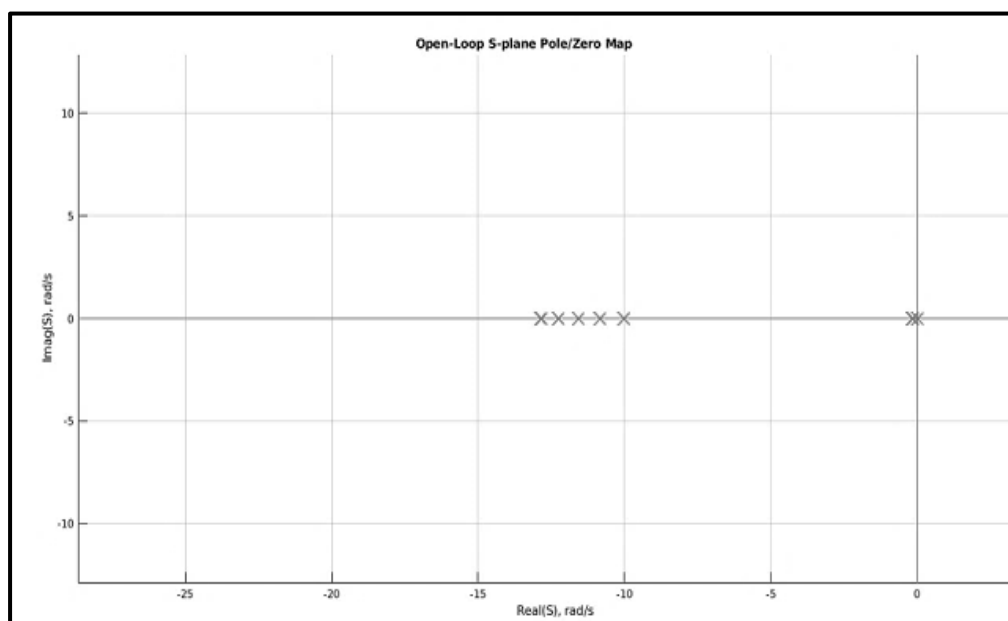


Fig. 5.8. System Poles plot for the simulation cases

Additional Bifurcation results are in appendix A4 with bifurcation parameters as rigging angle variation and elevator deflection to study the complete behavior of the system.

5.6 SUMMARY:

This work examines the longitudinal trim and stability evaluation of a hybrid airship model with both 3 degrees of freedom (3DOF) and 4 degrees of freedom (4DOF). The analysis is conducted using dynamic simulation and stability analysis utilizing the bifurcation approach. The analysis and modelling of the 3-degree-of-freedom (3DOF) hybrid airship model are conducted, considering various scenarios involving lift deflections. The stable simulation of the hybrid airship model is conducted for seven different cases, involving varying elevator deflections. Specifically, cases 1, 2, 3, 5, and 6 exhibit stable behavior for elevator deflections of 0 degrees, 1.5 degrees, -1.5 degrees, 3.13672 degrees, and -3.13672 degrees, respectively. However, cases 4 and 7 demonstrate unstable behavior for elevator deflections of 3.15437 degrees and -3.15437 degrees, respectively. The validation of the cases is conducted by using the bifurcation method on the 3DOF hybrid airship model, assuming a constant velocity and level flight. By employing the bifurcation approach, the eigenvalues are computed for various scenarios, enabling the identification of related poles. This analysis serves to validate and corroborate the observed outcomes in both stable and unstable instances. The initial longitudinal mode is commonly referred to as the surge mode, which is distinguished by the fluctuation in drag as a consequence of changes in velocity. The stability of the surge mode exhibited by the winged hybrid airship is attributed to the higher drag variation resulting from changes in speed. This stability is characterized by a tiny time constant of 0.442 seconds. The longitudinal dynamics of a hybrid airship yield two stable modes and one unstable mode. The offered bifurcation analysis results serve as a means to potentially ascertain the stability of various trim points. Bifurcation parameters taken as elevator deflection and rigging angle in level flight condition.

CHAPTER 6

SIMULATION & RESULT ANALYSIS

Simulation results are obtained and discussed below in section wide according to the objective.

6.1 SIMULATION RESULT FOR SINGLE BODY LONGITUDINAL DYNAMICS (3 DOF) OF HYBRID AIRSHIP:

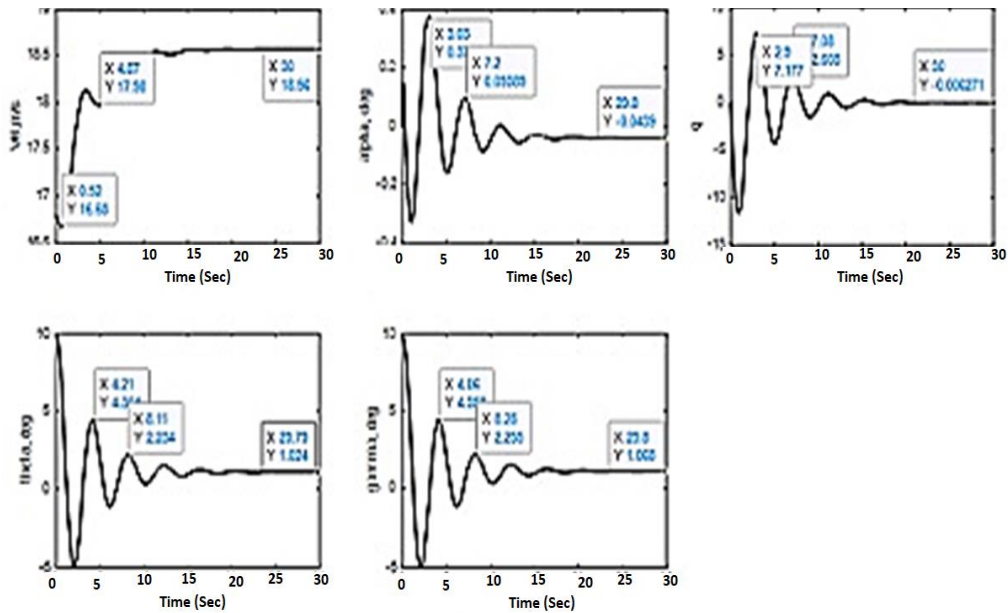


Fig. 6.1. Simulation result of 3DOF longitudinal dynamics of hybrid airship at delfin (elevator) 0 degree deflection.

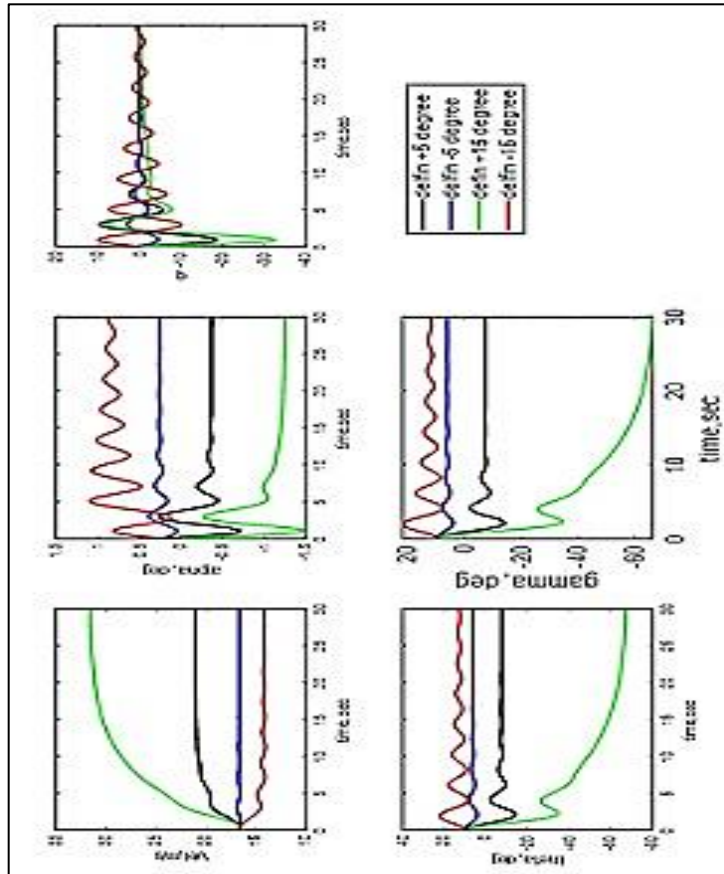


Fig. 6.2 Simulation result of 3 DOF longitudinal dynamics of hybrid airship at different delfin (elevator) deflection.

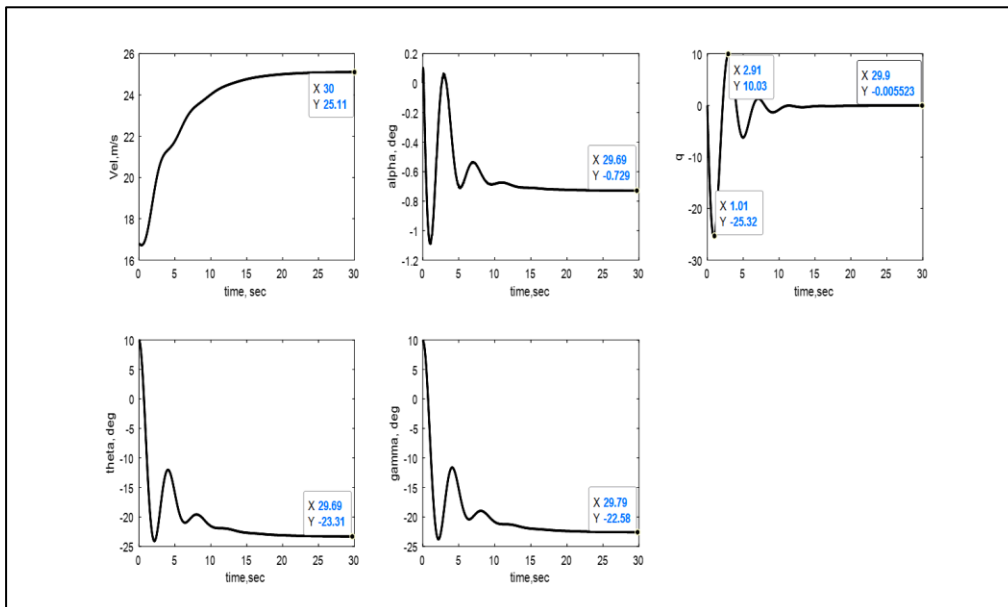


Fig. 6.3. Simulation result of 3DOF longitudinal dynamics of hybrid airship at delfin (elevator)10 degree deflection.

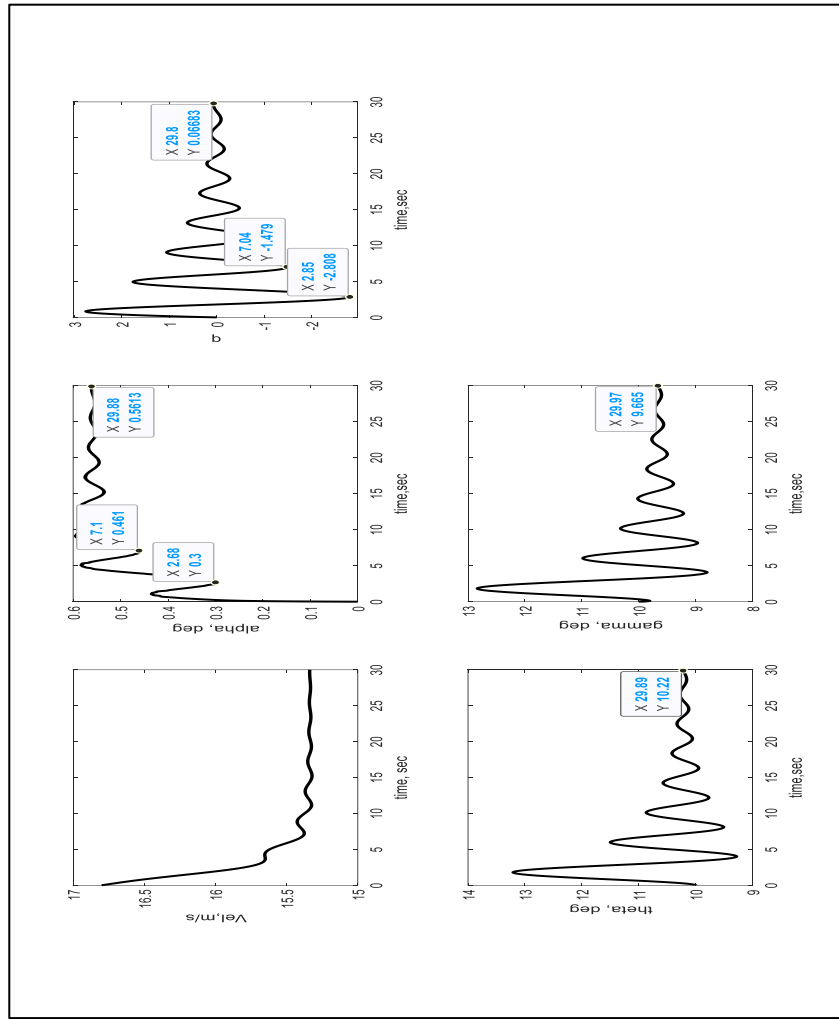


Fig. 6.4. Simulation result of 3DOF longitudinal dynamics of hybrid airship at delfin (elevator) -10 degree deflection.

The values of α , q , θ , and γ trim in Fig. 6.1 closely align with the trim values reported in [28]. The velocity reaches a maximum of 18.5 m/s, starting from an initial value of 16.8 m/s. The trim state is achieved within a time frame of 20 seconds for all output. In the case of q , the time period is measured to be 4.1 seconds, while the oscillation frequency is determined to be 1.5 radians per second. The time cycle of θ is 3.9 seconds, whereas the angular frequency is 1.6 radians per second. The system demonstrates stability as it achieves a settled state within a time frame of 20 seconds. The trim value of the longitudinal dynamics of the hybrid airship is recorded by observing the deflection of the delfin (elevator control surface). Additionally, the airship's settling time is analyzed and found to be within 20 seconds, as depicted in Fig.

6.2, 6.3, and 6.4.

The dynamic stability analysis of the hybrid airship model at different elevator deflection is performed in level flight for obtaining bifurcation results for single body dynamics (3DOF) model.

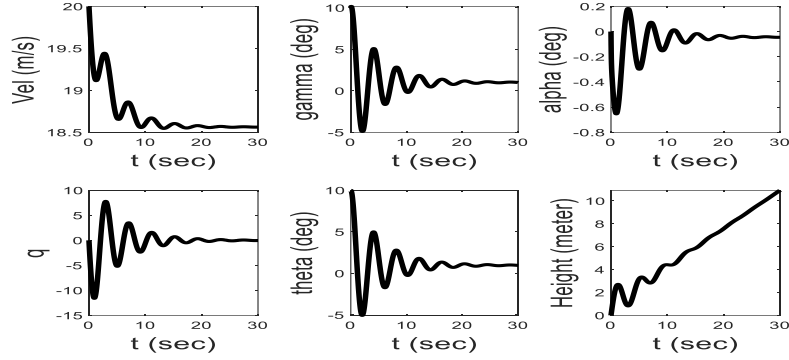


Fig 6.5. longitudinal dynamics of 3DOF model at dele 0 degree

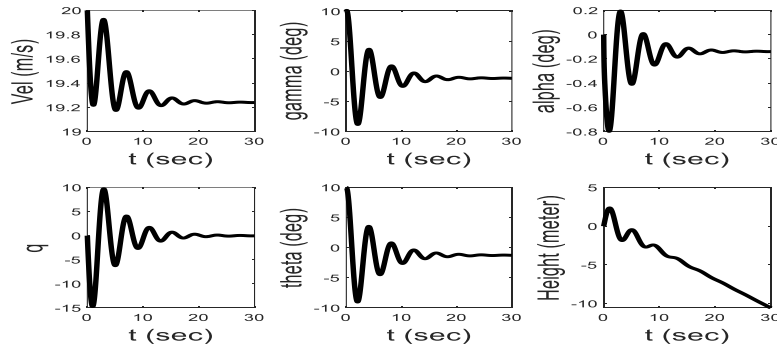


Fig. 6.6. longitudinal dynamics of 3DOF model at dele 1.5 degree

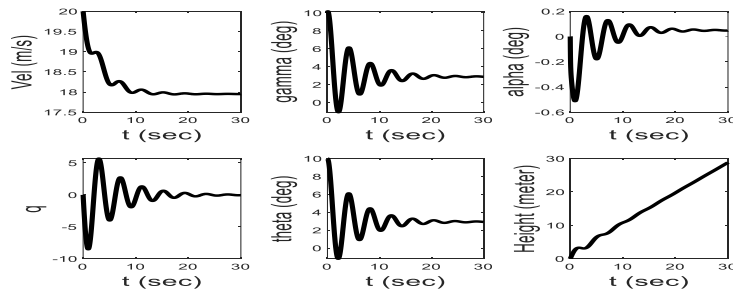


Fig. 6.7. longitudinal dynamics of 3DOF model at dele -ve 1.5 degree

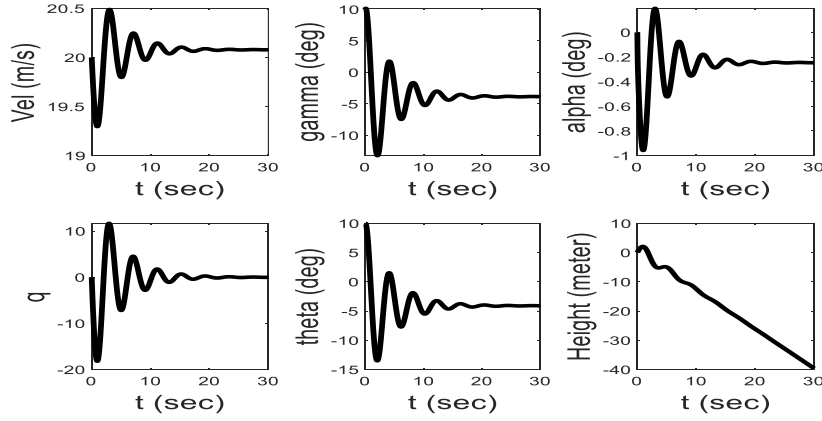


Fig. 6.8. longitudinal dynamics of 3DOF model at dele 3.154 degree

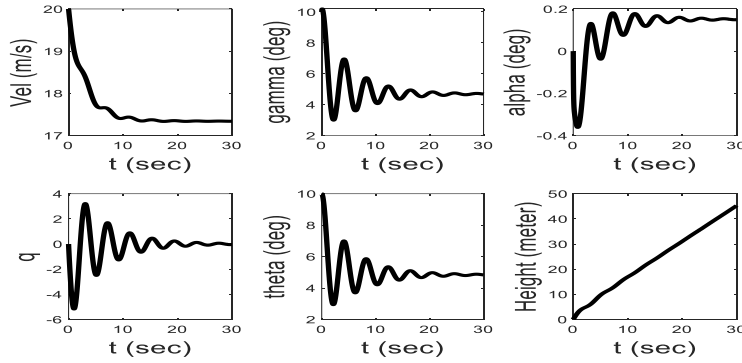


Fig. 6.9. longitudinal dynamics of 3DOF model at dele -3.154 degree

Case 1 simulation shown in Fig. 6.5, shows that the angle of attack α , q , theta and gamma is stabilizing 0 degrees with the decrease in the hybrid airship speed for elevator deflection, $\delta_e = 0$ degree, and with the gain in height. The overall dynamic simulation for the case shows the stable short period dynamic behaviour and the angle of attack α shows converging nature, near the equilibrium or trim point. Some more simulation time will show long period dynamic behaviour. Case 2 simulation shown in Fig. 6.6 shows that the angle of attack α , pitch rate q , and theta is stabilizing 0 degrees with the decrease in the hybrid airship speed in oscillatory nature for elevator deflection, $\delta_e = 1.5$ degree, and with the oscillatory gain in height in opposite direction. The flight path angle gamma maintains the flight of $\gamma = -0.2$ degrees and settles after 25 sec. The overall dynamic trim simulation for the case shows a stable behaviour and the angle of attack, α shows a divergence nature, near to the

equilibrium or trim point. Case 3 simulation shown in Fig. 6.7, shows that the angle of attack, α is stabilizing near $\alpha=-0.22$ degrees with the decrease in the hybrid airship speed for elevator deflection, $\delta_e = -1.5$ degree, and with height gain sharply. The flight path angle, γ increases from $\gamma=0$ to 4.6 degrees and the pitch rate, q is stabilized near $q=0$. Theta stabilizes near 4.36 degree. The overall dynamic simulation for the case shows the stable behaviour and the angle of attack, α shows the convergent nature, near to the equilibrium or trim point. Case 4 simulation shown in Fig. 6.8, shows that the angle of attack, α is stabilizing near $\alpha=-0.44$ degrees after 25 sec with the airship speed settles at trim value of 19.67 m/s for elevator deflection, $\delta_e = 3.154$ degree, and with height gain in opposite direction. The flight path angle, γ maintains the flight at -2.6 degree and the pitch rate, q is maintained near to $q=0$. The overall dynamic simulation for the case shows the stable behaviour and the angle of attack, α shows convergent nature, near the equilibrium or trim point. Case 5 simulation shown in Fig. 6.9, shows that the angle of attack, α settles near $\alpha=-0.15$ degrees with the breaking down the airship speed at 17.26 m/s after 15 sec for elevator deflection, $\delta_e = -3.154$ degree, and with the gain in altitude in positive direction. The flight path angle, γ maintains the flight of $\gamma=6.5$ degrees and the pitch rate, q also settles down to 0 with less oscillation. The overall dynamic simulation for the case shows stable behaviour and the angle of attack, α shows divergence nature, near to the equilibrium or trim point.

Simulation of hybrid airship is also done with wing and without wing and it is shown in Fig. 6.10. Black colour denotes with wing hybrid airship and blue colour denotes without wing HA. Velocity of HA with wing settles at 18 m/s and HA without wing settles at 21.5 m/s. Both starts from 20 m/s and settles in almost 10 sec. The flight path angle gamma maintains the flight of $\gamma=-1$ degrees and settles after 6 sec for wingless HA and $\gamma=4$ degrees and settles after 10 sec for winged HA.

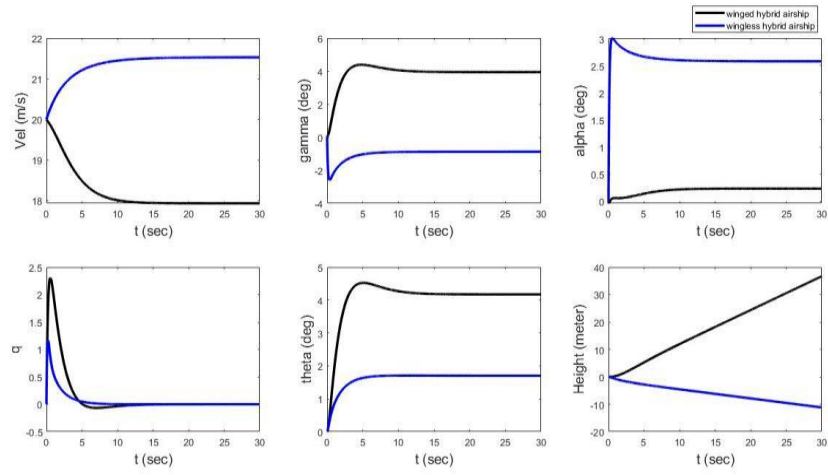


Fig. 6.10. A Simulation result of 3DOF model of hybrid airship with wing and without wing

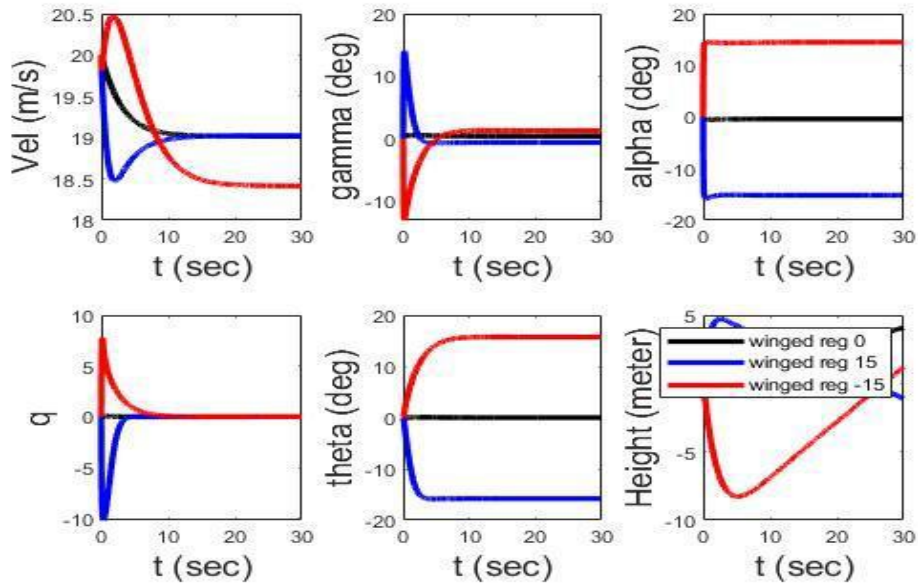


Fig. 6.11 Simulation result of 3DOF model of hybrid airship with wing by varying wing incidence angle (reg).

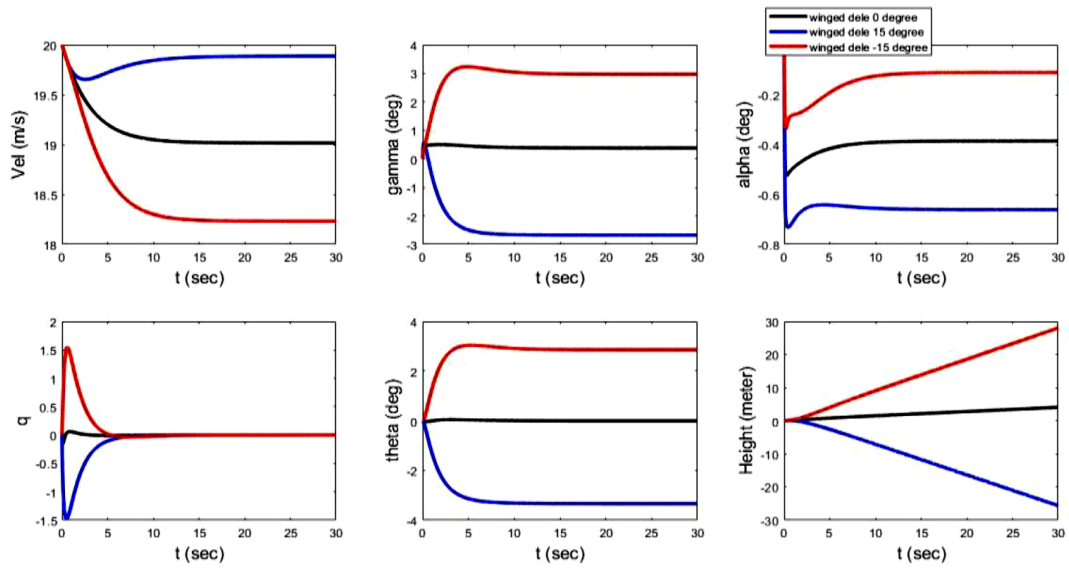


Fig. 6.12. Simulation Result of 3DOF model of Hybrid Airship with wing with Three different Level of elevator

Fig. 6.11 and Fig. 6.12 show the simulation result of single body longitudinal dynamics (3 DOF) of winged hybrid airship by varying wing incidence angle and by varying elevator control surface respectively.

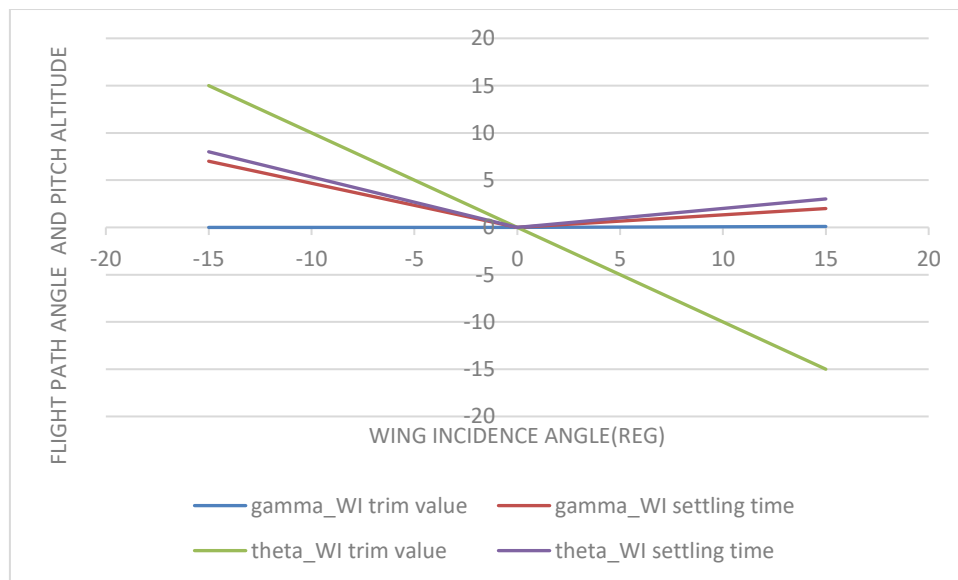


Fig. 6.13. Analysis of Simulation result of 3DOF model of hybrid airship with wing by varying wing incidence angle (reg).

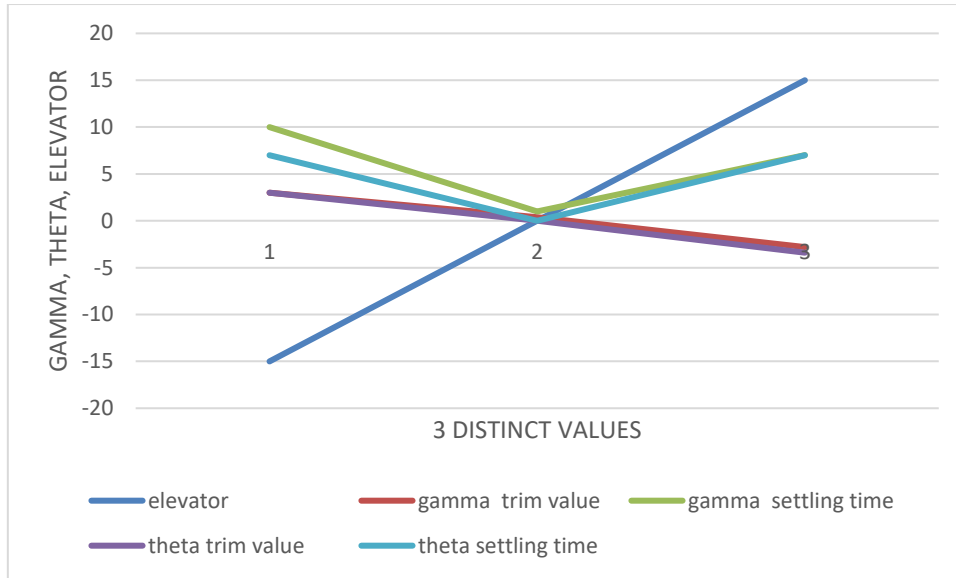


Fig. 6.14. Analysis of Simulation Result of 3DOF model of Hybrid Airship with wing with Three different Level of elevator.

Analysis of the simulation for Fig. 6.11 and 6.12 is done as trim analysis and settling time for flight path angle (gamma) and pitch angle (theta) with variation of wing incidence angle and elevator respectively in Fig. 6.13 and 6.14.

6.2 BIFURCATION RESULT AND TRIM ANALYSIS FOR 3 DOF LONGITUDINAL DYNAMICS OF HYBRID AIRSHIP:

The trim value of the longitudinal dynamics of the hybrid airship is recorded from Fig. 6.1 to 6.4 by observing the deflection of the delfin (elevator). This data is presented in Table 6.1.

The longitudinal dynamics exhibit trajectory variation through the examination of the flight path angle and theta angle trajectories, specifically when a deflection occurs in the elevator (delfin) control surface, as depicted in Fig. 6.15.

The bifurcation results obtained for 3DOF single body dynamics model of hybrid airship in level flight is done by varying elevator as input parameter and then by varying rigging angle or wing incidence angle as input parameter.

Table 6.1: Trim analysis table for 3DOF longitudinal dynamics of hybrid airship

Sr. No.	delfin deflection (degree)	gamma (degree)	theta (degree)
1	-15	12.06	12.92
2	-10	9.665	10.22
3	-5	6.193	6.446
4	0	1.068	1.024
5	5	-7.162	-7.529
6	10	-22.58	-23.31
7	15	-66	-67.13

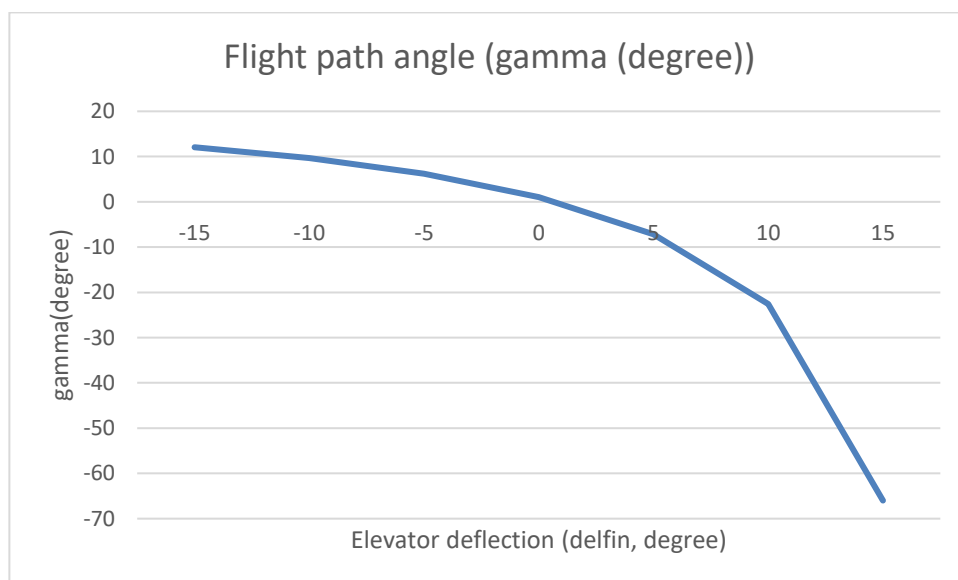


Fig. 6.15. Flight path trajectory with elevator (delfin) deflection.

Simulation work is also done from Fig. 6.5 to Fig. 6.9 in lined with the values obtained from bifurcation methods in Fig. 6.18. Fig. 6.16 is the bifurcation plot between theta vs pitch (q). The values are in degree. Black region is the stable region. Fig 6.17 is the bifurcation plot of pitch, theta and q with the deflection of control parameter elevator (delfin). 3 bifurcation points are obtained at delfin 0 degree reference crossing point. At delfin 3.14 degree two bifurcation points are there, at -3.12 degree one bifurcation point is there. It

generates eigen values also. Fig. 6.19 and 6.20 is also a bifurcation plot with wing incidence angle variation and elevator deflection. Black region is the stable region,

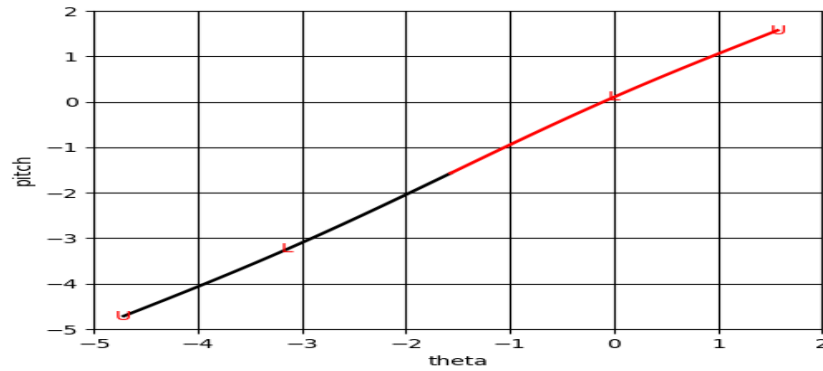


Fig. 6.16 Bifurcation result of of 3DOF longitudinal dynamics of hybrid airship with pitch vs theta

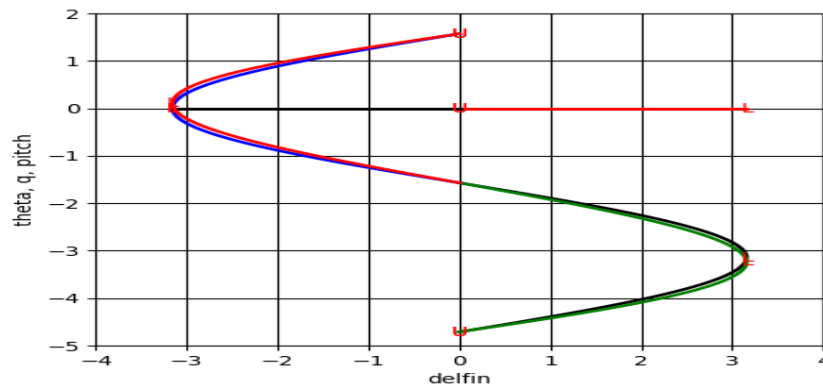


Fig. 6.17 Bifurcation result of of 3DOF longitudinal dynamics of hybrid airship with pitch, theta and q vs delfin

BR	PT	TY	LAB	delfin	L2-NORM	theta	q	pitch
1	1	EP	1	0.00000E+00	2.22128E+00	-1.57068E+00	0.00000E+00	-1.57068E+00
1	20		2	1.41194E+00	2.90698E+00	-2.03315E+00	1.59980E-23	-2.07770E+00
1	40		3	3.01189E+00	4.07716E+00	-2.83551E+00	2.49382E-26	-2.92969E+00
1	45	LP	4	3.15775E+00	4.51270E+00	-3.14125E+00	-3.34961E-25	-3.23991E+00
1	60		5	2.22442E+00	5.60960E+00	-3.93150E+00	1.49891E-27	-4.00137E+00
1	80		6	4.27005E-01	6.48197E+00	-4.57666E+00	-1.90036E-24	-4.59023E+00
1	85	UZ	7	2.51964E-07	6.66375E+00	-4.71198E+00	-1.65197E-25	-4.71199E+00

Fig. 6.18. Bifurcation result of 3DOF model of hybrid airship

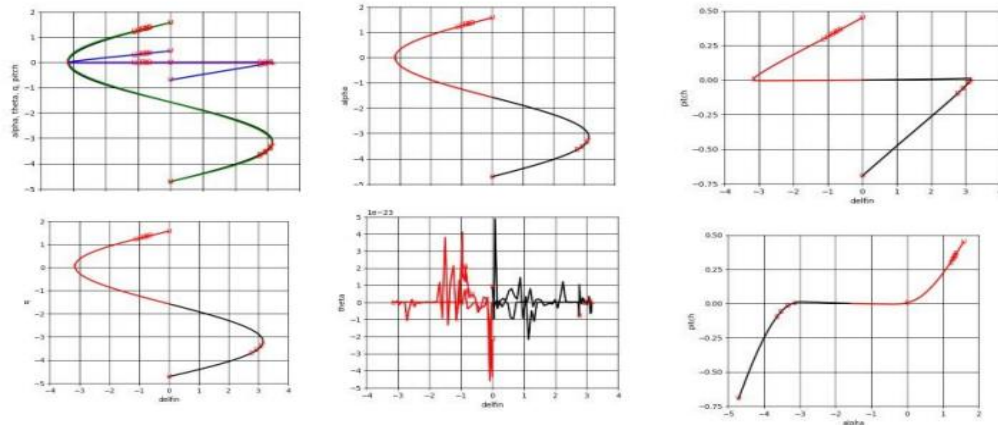


Fig. 6.19. Bifurcation result HA winged with elevator deflection

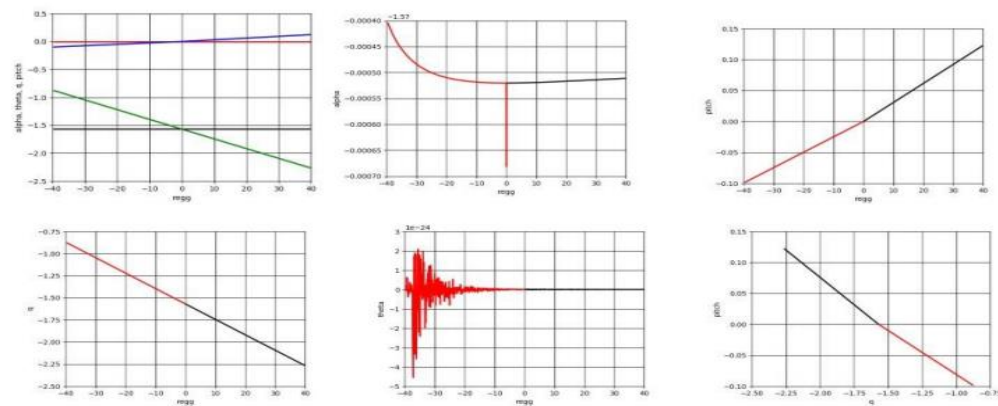


Fig. 6.20 Bifurcation result HA winged with different rigging angle

6.3 SIMULATION RESULT OF TWO BODY DYNAMICS (4 DOF) OF HYBRID AIRSHIP WITH SUSPENDED PAYLOAD:

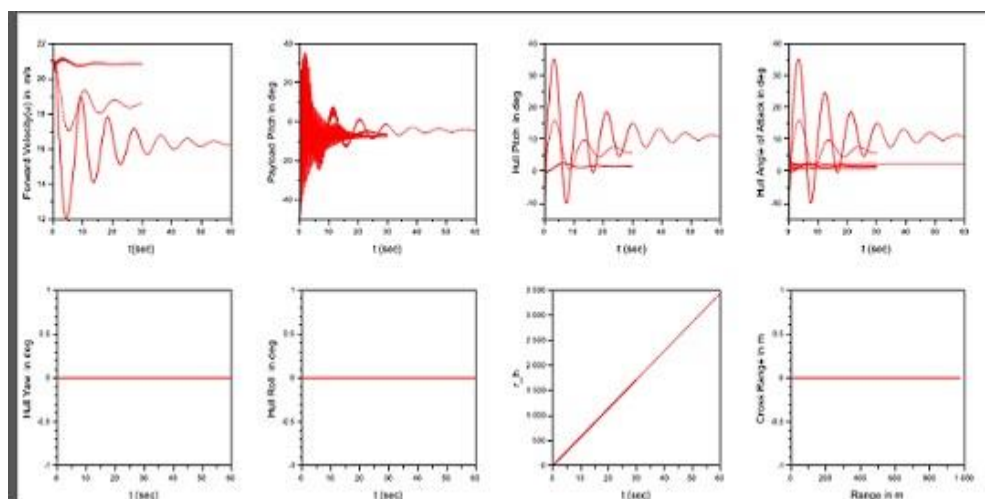


Fig. 6.21 Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload.

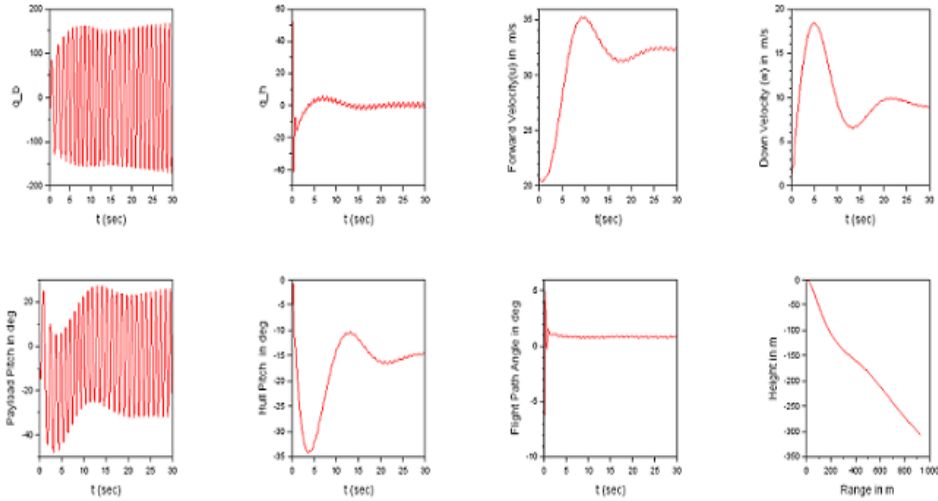


Fig. 6.22 Simulation result of longitudinal dynamics of 9DOF hybrid airship code with suspended payload

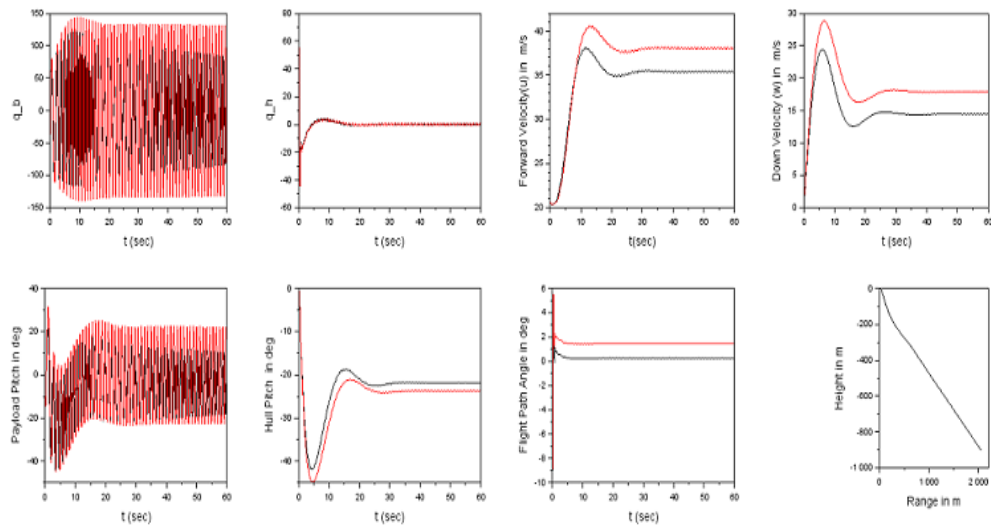


Fig. 6.23 Simulation result of longitudinal dynamics of 9DOF hybrid airship code with suspended payload vs 4DOF code

As in Fig. 6.21, 6.22 and 6.23, By changing the rigging angle μ , the payload attached with airship moves backward and forward. If the payload movement is in backward side, then the airship noses up and vice-versa. So, from changing the rigging angle also we can control the pitching moment of the airship. The simulation is done for two body dynamics, so in the simulation, dynamics of payload box and dynamics of hybrid airship both are studied here.

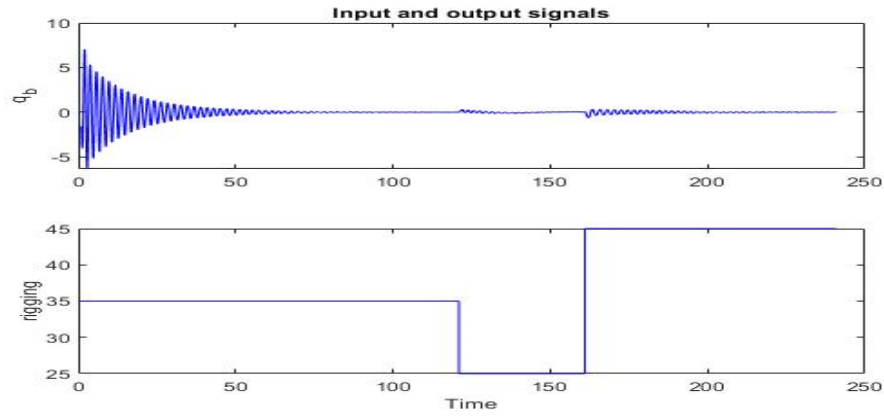


Fig 6.24. q of suspended payload vs rigging angle of longitudinal dynamics of 4 DOF hybrid airship with suspended payload.

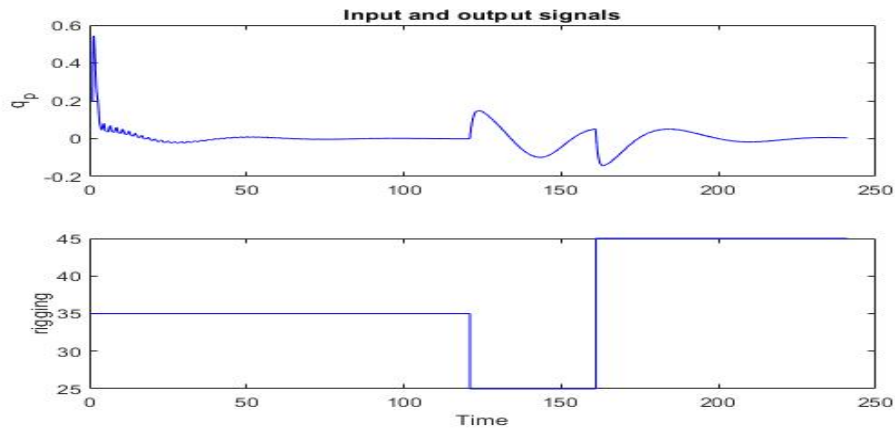


Fig 6.25 q of airship vs rigging angle of longitudinal dynamics of 4 DOF hybrid airship with suspended payload.

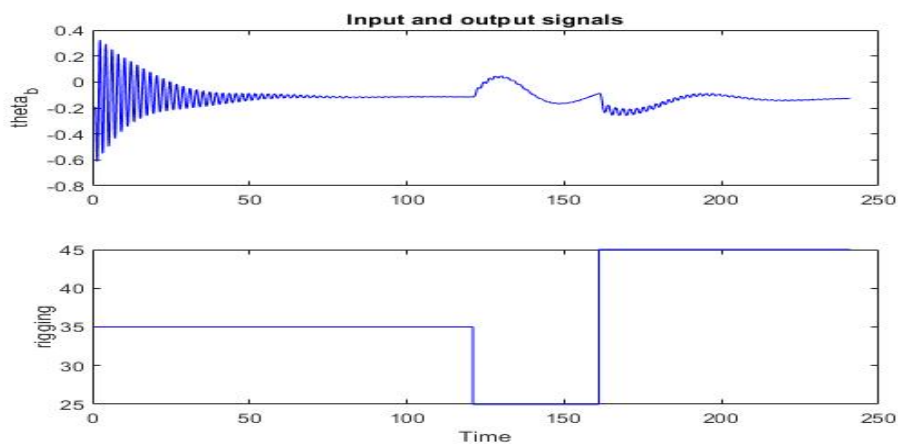


Fig 6.26 θ of payload vs rigging angle of longitudinal dynamics of 4DOF hybrid airship with suspended payload.

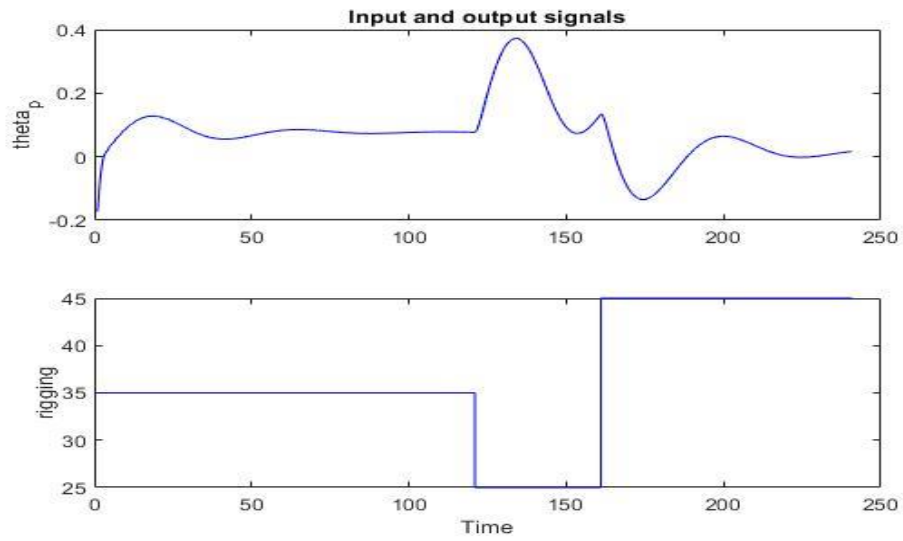


Fig 6.27. theta of airship vs rigging angle of longitudinal dynamics of 4DOF hybrid airship with suspended payload.

The response of q of suspended payload, q of airship, theta of payload and theta of airship is obtained in Fig. 6.24 to 6.27, by varying the input as rigging angle or by sliding the attached payload position.

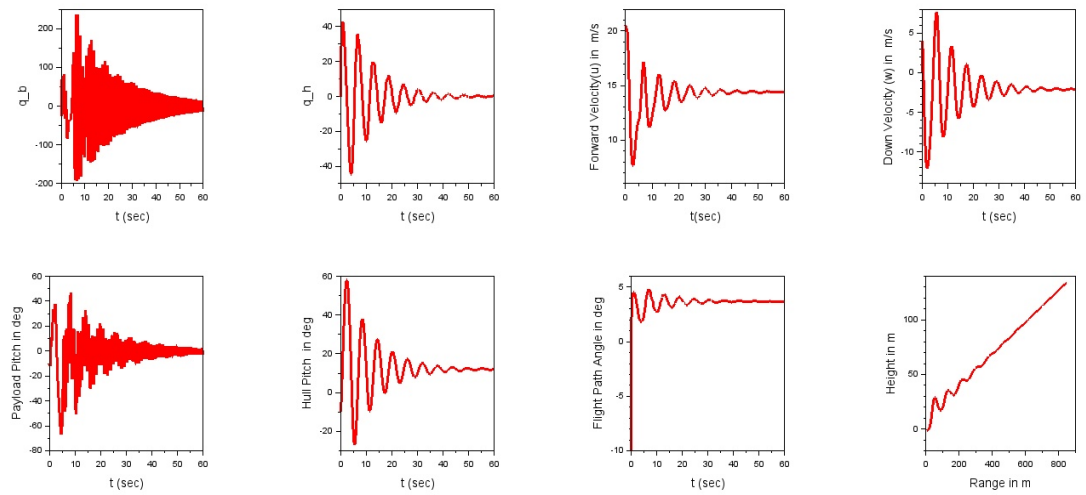


Fig. 6.28. Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload dele 0, reg +20 deg.

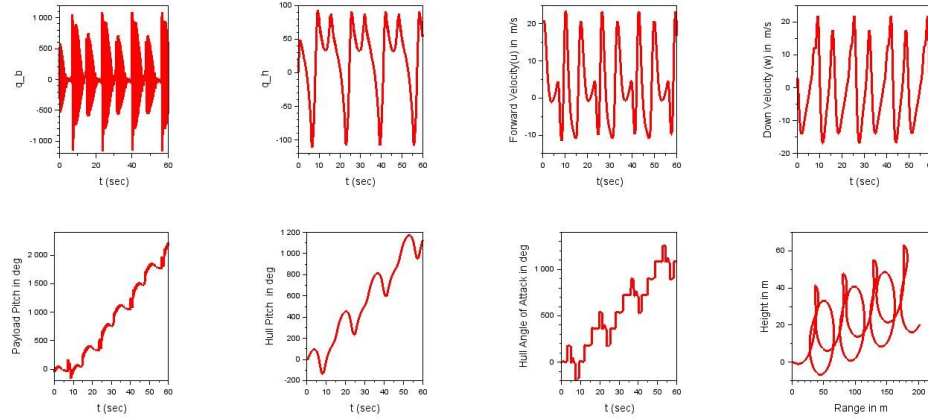


Fig. 6.29. Reg=0 deg dele=0 deg for 4DOF model

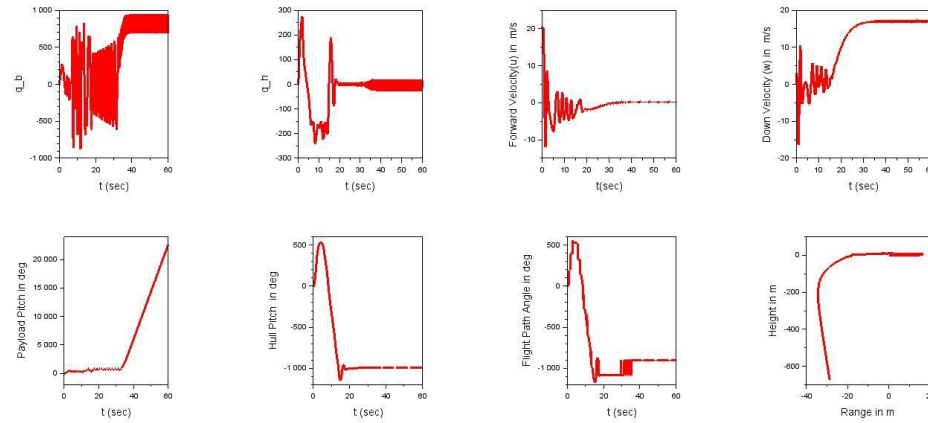


Fig. 6.30. Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload dele 0 ,reg -20 deg.

Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload is obtained in Fig. 6.28 to 6.30 by keeping elevator as 0 degree and by varying rigging angle. After comparing the results of all these cases shown in Fig. 6.28 to 6.30, it can be analyzed that at a rigging angle 16 degrees to 20 degrees, the system responds fast, settles quickly and have a trim value. So, the designed parameter for this case is rigging angle as 16 degrees or 20 degrees.

6.4 SIMULATION RESULT OF NDI CONTROLLER FOR HEADING TRACKING AND TRAJECTORY TRACKING OF HYBRID AIRSHIP:

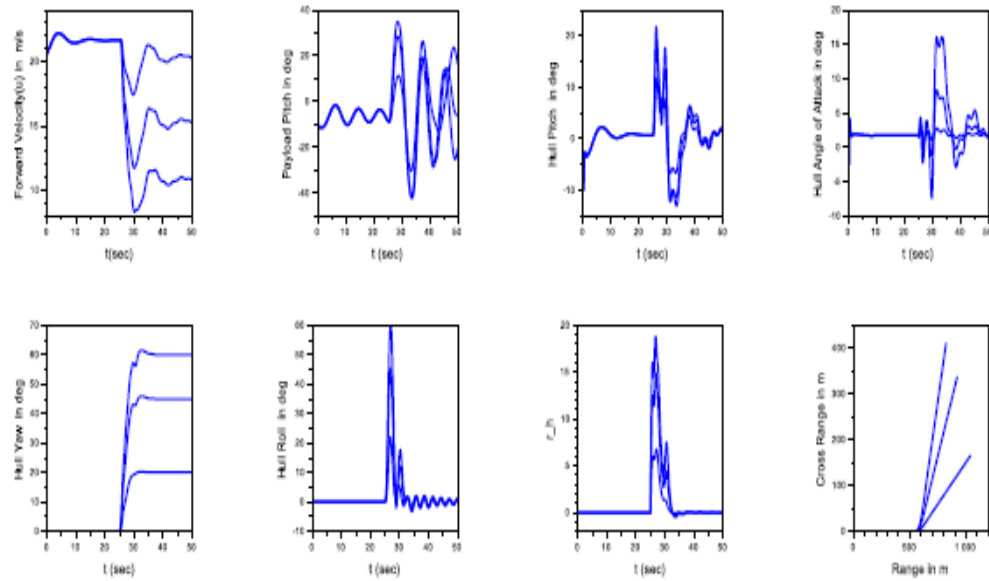


Fig. 6.31. Tracking capability of closed loop hybrid airship with suspended payload system with NDI controller

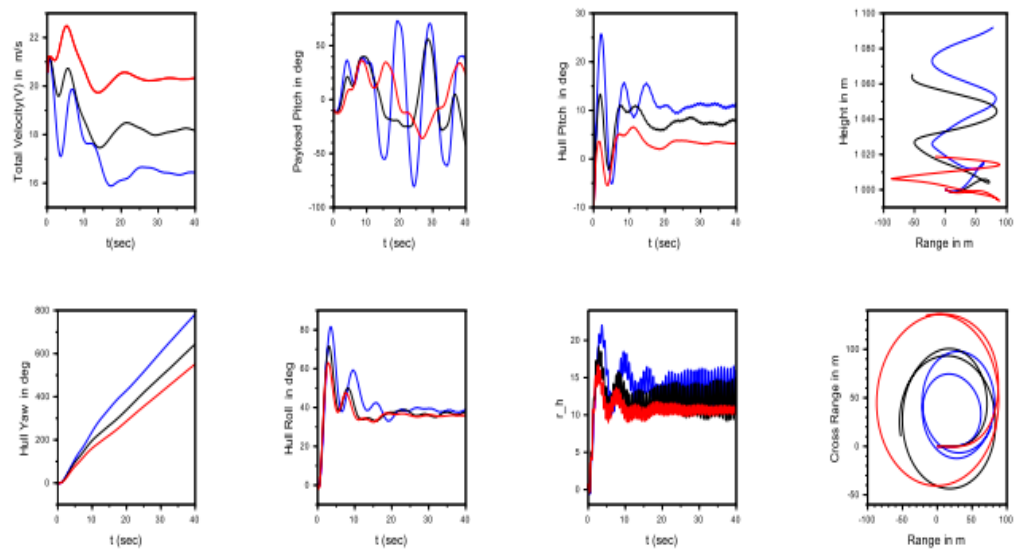


Fig. 6.32 Varying line length effect (Rch) (red-1.0, blue-1.2 and black-1.8 m) on 10 deg side tilt (δ) of Airship-payload system open loop Simulation with tracking capabilities

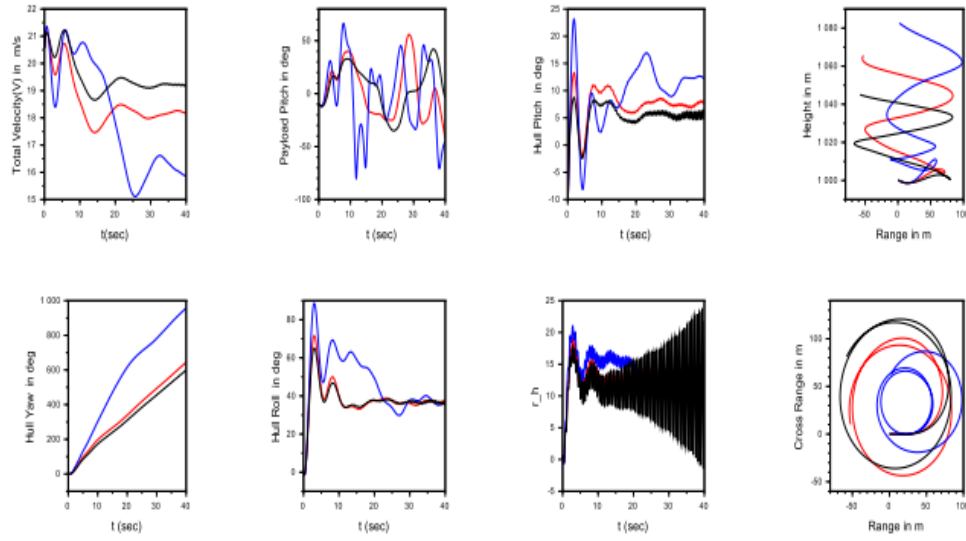


Fig. 6.33 Open loop simulation of two body dynamic model with varying rigging (μ) (black-15, blue-10, red-0 degs) angles and 10 deg side tilt (δ) effect on turning flight

Fig. 6.31 illustrates the functionality of a hybrid-airship controlled by NDI (Nonlinear Dynamic Inversion) in terms of its ability to accurately follow specified heading commands, denoted as Ψ_h . This is achieved by the generation of the necessary r_p , which in turn leads to the appropriate deflection of the asymmetric control surface δ_a situated on the wing. The simulation of the effect on turning flight manoeuvres resulting from the rigging angle of suspension lines is conducted using a two-body dynamic 9-degree-of-freedom (DOF) model. This model incorporates nonlinear longitudinal aerodynamic data. Additionally, a recent study has investigated the impact of longitudinal trims on the stability of Winged Hybrid airships in relation to their ability to transfer payloads. A viable winged Hybrid airship can be conceptualised, using specific rigging configurations to facilitate efficient cargo transportation. By carefully determining the appropriate line length, the desired turning flight manoeuvre can be achieved by the implementation of a side tilt angle. A comprehensive analysis of the various scenarios depicted in Figure 6.32 and Fig 6.33 is presented. The increase in line length of the Airship-payload system at 1.0, 1.2, and 1.8 metres, while experiencing a 10-degree side tilt, results in a drop in forward velocity by 20, 18, and 16 metres per second, respectively.

Additionally, there is an increase in pitch by 4, 8, and 12 degrees, respectively. These changes in line length and tilt allow the airship payload system to execute tighter turn loops, with an approximate 35-degree bank, while the payload goes forward relative to the system. To design an NDI controller the turning dynamic response of the hybrid airship in Fig. 6.32 and 6.33 is analysed. Then a NDI controller is designed.

NDI controller is able to track step response in yaw directional tracking for 20 deg., 40 deg. And 60 deg. showcasing good tracking capabilities of the controller for practical implementation of directional tracking capability of airship and suspended payload system.

The result of the system of hybrid airship with wing in two body dynamic approach shown in Fig. 6.32, Fig. 6.33 and Fig. A4.25 (taken from appendix A4). With analysis, it is observed that similar results follow flight behaviour as highlighted through single body eigen value analysis in Table A4.1 in appendix A4. In two body dynamic model of winged hybrid airship, all modes are stable which is an added advantage in two body system.

6.5 CLOSED LOOP AND OPEN LOOP RESPONSE WITH PID CONTROLLER FOR PITCH CONTROL AND TRAJECTORY TRACKING OF HYBRID AIRSHIP:

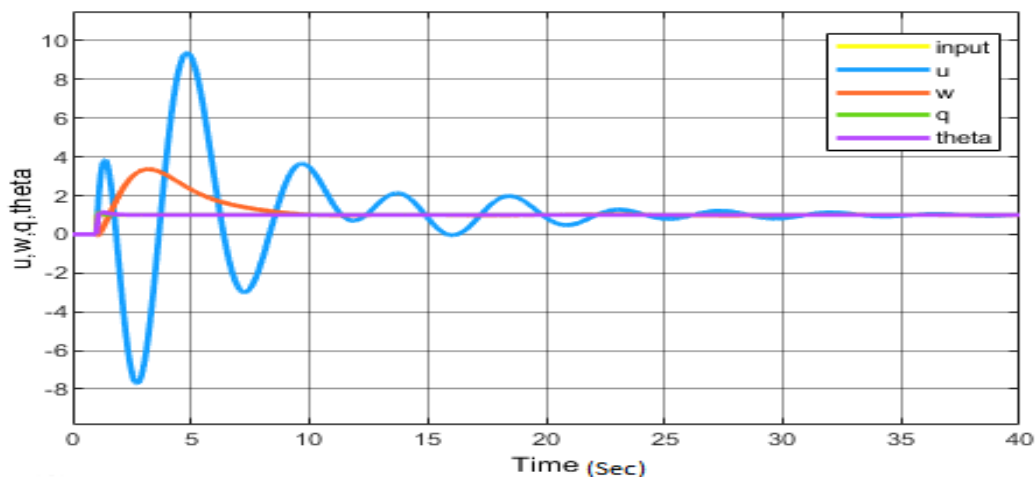


Fig. 6.34 Step response of Simulink model

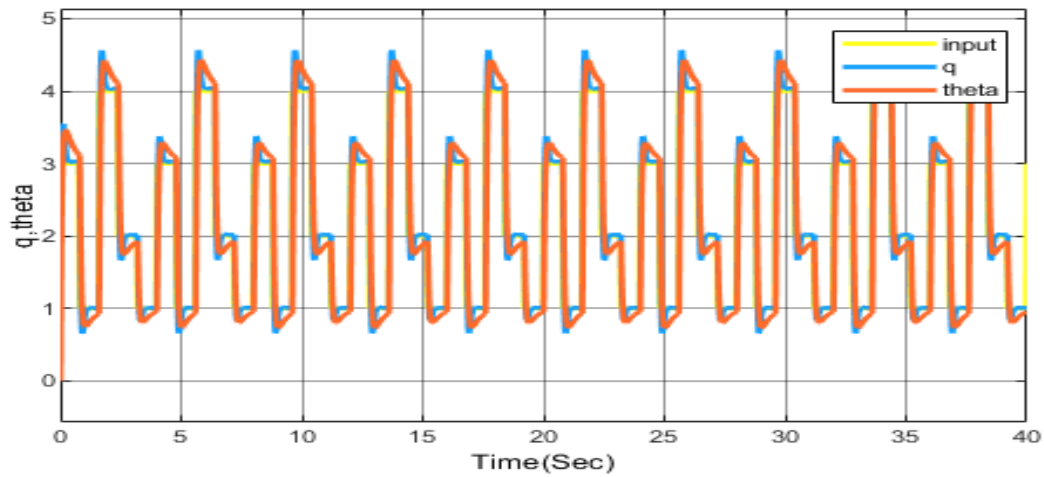


Fig. 6.35 Trajectory Tracking Response

Simulink step response and trajectory tracking pitch control response is shown in Fig. 6.34 and Fig. 6.35 respectively. In Fig. 6.35 system response is shown in red colour and it is tracking input shown in blue colour.

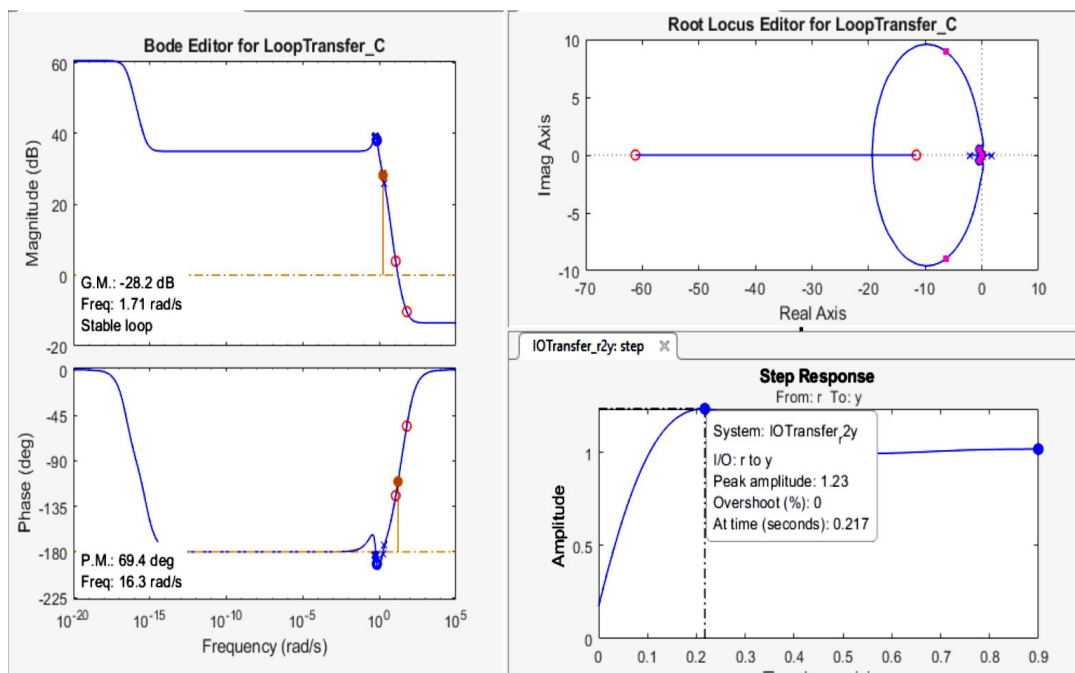


Fig. 6.36 Closed Loop and Open Response of G13 with PID Design

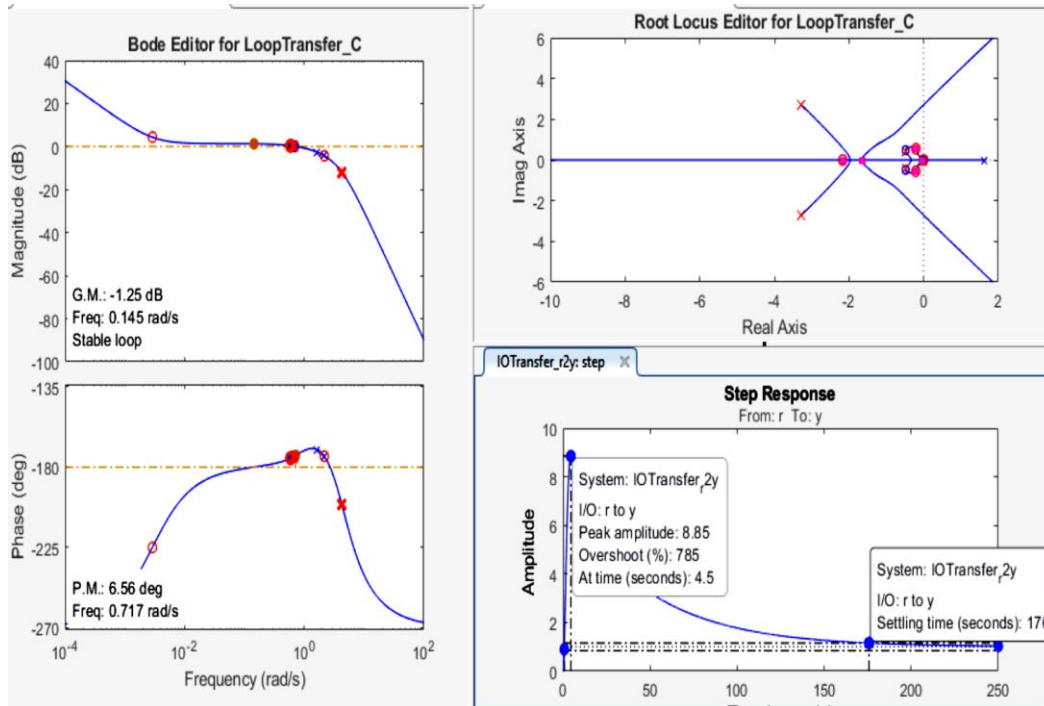


Fig. 6.37 Closed Loop and Open Response of G14 With LQG Compensator Design

Analysis of Fig. 6.36 and 6.37 is done in Table 4.2. Additional Simulation results are in Appendix A4.

6.6 SUMMARY:

Dynamic simulation is done for the following flight conditions:

- Level flight trim condition with no elevator deflection and is used as validation with literature as ($V = 20$ m/s, $\alpha = \theta = \gamma = 0$ degree).
- With elevator deflection -15 degree to +15 degree for both 3 DOF and 4 DOF.
- With rigging angles variation for the same model.
- Simulation done for winged and wingless.

Reducing DOF from 9 DOF to 4 DOF and from 6 DOF to 3 DOF, is to analyze longitudinal dynamics of the vehicle and to compare the result with literature for different flight conditions before stall angle of attack to access the safety of the flight.

The thesis topic is based on longitudinal dynamics, so, it is developed and analyzed the longitudinal behavior of the hybrid airship model for single body dynamics approach and for two body dynamics approaches as 3DOF and 4DOF. For validation, we used 6DOF and 9DOF models to develop 3DOF and 4DOF

model respectively. Thesis topic is aimed on longitudinal dynamic analysis, which is sufficient to study flight safety from unstable stall condition which can result in loss of aircraft. 3DOF model is used to aim objective 1 and 3 of the thesis and 4DOF model is required to aim objective 2 and 4 of the thesis. Additional Simulation Results are in appendix A4.

CHAPTER 7

CONCLUSION

The 3DOF nonlinear longitudinal dynamic model for hybrid airship is developed and the simulation result is verified with trim values of [27] obtained for linearized longitudinal model for the same. The 4 DOF nonlinear longitudinal dynamic model for hybrid airship with suspended payload has been developed and the simulated result is almost behaving like the result obtained from single body 3 DOF longitudinal model. The new thing in the 3 DOF hybrid model is included and that is fin deflection control maneuvers and in 4DOF, the new finding is by changing the rigging angle also pitch control of airship can be done. More accurate results will come if wind tunnel data for the system is available. The simulations employ the NDI control law, as previously mentioned, which incorporates both inner and outside inversion loops, for the purpose of analysing the Hybrid airship payload delivery system. The simulations were conducted for the Airship Hull, as presented in Table 3.1, respectively, sourced from [27]. A prototype of a winged Hybrid airship with a multi-wing configuration is currently being constructed on the university campus for the purpose of conducting a demonstration flight. The payload is suspended beneath the airship hull through the utilisation of riser lines connected to the payload and suspension lines attached to the airship hull, both converging at a same point. In the forward simulation, it is anticipated that the Hybrid Airship will carry an attached cargo while maintaining trimmed powered flight. This flight configuration is achieved with a thrust of 27 N and a buoyancy force of 50 N. The NDI based controller designed for Hybrid-Airship/payload system uses closed loop in yawing motion keeping longitudinal and lateral loops left open due to natural pendulum stability. The analysis of the heading tracking capabilities of the Hybrid-Airship/payload system is conducted by employing the Nonlinear Dynamic Inversion control law. The

control rule described is applied to a nine-degree-of-freedom (9-DOF) model representing the dynamics of a two-body system. This model includes nonlinear longitudinal aerodynamic characteristics. Additionally, a recent study has unveiled the potential for collision avoidance capabilities in winged Hybrid airships, specifically designed for autonomous payload distribution. The analysis will be useful for developing an airship to fly in an autonomous way. The study presents the longitudinal trim with stability assessment of the 3DOF and 4DOF longitudinal hybrid airship model with dynamic simulation and stability analysis using the bifurcation method. Trim analysis and simulation of the 3DOF hybrid airship model are performed under the different cases of elevator deflections. For the different 7 cases, trim simulation is performed with the hybrid airship model resulting in the stable simulation for cases 1, 2, 3, 5 and 6 for elevator deflection $\delta_e = 0$ degree, $\delta_e = 1.5$ degree, $\delta_e = -1.5$ degree, $\delta_e = 3.13672$ degree and $\delta_e = -3.13672$ degree and for cases 4 and 7 for for elevator deflection $\delta_e = 3.15437$ and $\delta_e = -3.15437$ show unstable behavior. The validation of the cases is carried out with the bifurcation method implementation using the 3DOF hybrid airship model with constant velocity and for level flight. Using the bifurcation method, for all the different cases, the eigenvalues are obtained, and corresponding poles are determined, and this shows and confirms the same findings for stable cases and unstable cases. The first longitudinal mode is the surge mode, and it is characterized by drag variation due to speed. Since the drag variation due to speed is increased, the surge mode demonstrated by the winged hybrid airship is stable with small time constant of 0.442s. Two stable and one unstable mode obtained in longitudinal dynamics of a hybrid airship. The bifurcation analysis results presented here to demonstrate a potential way for determining the stability of different trim points. The simulation of the effect on turning flight maneuvers resulting from the rigging angle of suspension lines is conducted using a two-body dynamic model with nine degrees of freedom (9-DOF), which incorporates nonlinear longitudinal aerodynamic data. Additionally, a recent study has examined the impact of longitudinal trims on the stability of Winged Hybrid airships in relation to their ability to transfer payloads. A feasible winged Hybrid airship can be conceptualized, using certain rigging arrangements to facilitate efficient cargo

transportation. By carefully determining the appropriate line length, the desired turning flying maneuver can be achieved through the implementation of a side tilt angle. A comprehensive analysis of the many instances depicted in Fig. 6.32 and Fig. 6.33 is warranted. The forward velocity of the Airship-payload system reduces by 20, 18, and 16 m/s when the line length is increased to 1.0, 1.2, and 1.8 m, respectively, while the system is tilted at a 10-degree side tilt. Additionally, an increase in pitch of 4, 8, and 12 degrees is seen respectively with airship payload system making tighter turn loops with approximately 35 deg bank as payload moves forward relatively. Decreasing rigging angles 15, 10, and 0 deg, and 10 deg side tilt of airship payload system decreases forward velocity 19, 18, and 16 m/s, increases pitch approximately 5, 8, 11 deg, with airship payload system making tighter turn loops with approximately 30 deg bank as payload moves rearwards causing pitch up and drop in forward velocity for same thrust condition. In two body dynamic model of winged hybrid airship, all modes are stable which is an added advantage in two body system.

Bifurcation for 4DOF, capturing real time flight test data of the system and making an autonomous winged hybrid airship are the scope of future work. Bifurcation analysis has emerged as a critical tool in understanding the complex stability dynamics of airships. The nonlinear nature of airship dynamics necessitates a thorough exploration of bifurcation scenarios to ensure safe and efficient operation. With ongoing advancements in computational methods and the integration of interdisciplinary approaches, bifurcation analysis will continue to play a pivotal role in the design and operation of the next generation of airships.

Sliding mode controller design for the hybrid airship is the future work. It works fine when nonlinear aerodynamic flight and control derivative data is available beyond stall cases. As the hybrid airship considered in the thesis is a new development system, so if the aerodynamic data (flight data, CFD data and wind tunnel data) beyond stall is available then nonlinear behavior can be captured in more advanced manner. As SMC is a nonlinear controller, it will be designed after having nonlinear data available. Here PID controller is designed for simplicity and easy to give hardware solution. In future, the hybrid airship can be made an autonomous vehicle.

REFERENCES

- [1] BL. Nagabhushan, & NP. Tomlinson. (1982). Dynamics and control of a heavy lift airship hovering in a turbulent cross wind. *Journal of Aircraft*, Vol.19(10), pp.#826–830#. <https://doi.org/10.2514/3.61564>.
- [2] Li Y, & Nahon M. (2007). Modelling and simulation of airship dynamics. *Journal of Guidance, Control and Dynamics*. 30(6) #1691–700#. <https://doi.org/10.2514/1.29061>.
- [3] Cai Z-L, Qu W-D, & Xi Y-G. (2005). Dynamic modelling for airship equipped with ballonets and ballast. *Applied Mathematics and Mechanics*, Volume 26(8): #1072–82#. <https://doi.org/10.1007/BF02466421>.
- [4] De Paiva, E. Bueno, S., Gomes, S.B.V., Ramos, J.J.G. & Bergerman M. (1999). A Control System Development Environment for AURORA Semi-Autonomous Robotic Airship. *Proceedings of the International Conference on Robotics and Automation*, Demit, MI, USA. DOI:10.1109/ROBOT.2001.933267.
- [5] Amann JH. (1987). A comparison of a nonlinear flight dynamic simulation of an airship with flight test results. *7th AIAA lighter-than-air technology conference*, Monterey, CA, p. #78–86#. <https://doi.org/10.2514/6.1987-2507>.
- [6] Jones SP, & DeLaurier JD. (1983). Aerodynamic estimation techniques for aerostats and airships. *Journal of Aircraft* ,20(2), #120–6#. <https://doi.org/10.2514/3.44840>.
- [7] Carvalho J., de Paiva E., Azinheira J., Ferreira P., Ramos J., Bueno, S. Bergerman M., Maeta S., Mirisola L., & Faria, B. (2001). Classic and Robust PID Heading Control of an Unmanned Robotic Airship. *Proceedings of the 9th International Symposium on Intelligent Robotic Systems*, Toulouse, France, #1820#. <https://pascalfrancis.inist.fr/vibad/index.php?action=getRecordDetail&idt=14194811>.
- [8] DK. Schmidt. (2012). Modeling and near-space station-keeping control of a large high-altitude airship. *Journal of guidance, control, and dynamics*. Vol. 30(2), #540–547#. <https://doi.org/10.2514/1.24865>.
- [9] Hygounenc, E., & Souères, P. (2002). Automatic airship control involving

- backstepping techniques. *In Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, Vol. 6, #1–6#.* IEEE. <https://doi.org/10.1109/ICSMC.2002.1175632>.
- [10] S. Lacroix and Il-Kyun Jung. (2002). High resolution terrain mapping with an autonomous blimp. *IEEE/RSJ International Conference on Intelligent Robots and Systems, Lausanne, Switzerland, vol.1, #781–786#*, doi: 10.1109/IRDS.2002.1041485.
- [11] Hygounenc, E., & Souères, P. (2003). Lateral path following GPS-based control of a small-size unmanned blimp. *In Proceedings of the IEEE International Conference on Robotics and Automation, Vol. 1, #540–545#.* IEEE. <https://doi.org/10.1109/ROBOT.2003.1241650>.
- [12] Hygounenc E, Jung I-K, Souères P & Lacroix S. (2004). The Autonomous Blimp Project of LAAS-CNRS: Achievements in Flight Control and Terrain Mapping. *The International Journal of Robotics Research. Vol 23(4-5), #473–511#*. doi:10.1177/0278364904042200.
- [13] F Fukao, T., Fujitani, K., & Kanade, T. (2003). Image-based tracking control of a blimp. *In Proceedings of the 42nd IEEE Conference on Decision and Control, Vol. 5, #5414–5419#*, IEEE. <https://doi.org/10.1109/CDC.2003.1272498>.
- [14] Fukao, T., Fujitani, K., & Kanade, T. (2003). An autonomous blimp for a surveillance system. *In Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2003), Vol. 2, #1820–1825#.* IEEE. <https://doi.org/10.1109/IROS.2003.1248908>.
- [15] Fukao, T., Kanzawa, T., & Osuka, K. (2005). Inverse optimal tracking control of an aerial blimp robot. *In Proceedings of the Fifth International Workshop on Robot Motion and Control (RoMoCo 2005), #193–198#*, IEEE. <https://doi.org/10.1109/ROMOCO.2005.201423>.
- [16] Moutinho, A., & Azinheira, J. R. (2005). Stability and robustness analysis of the AURORA airship control system using dynamic inversion. *In Proceedings of the IEEE International Conference on Robotics and Automation, #2265–2270#*. <https://doi.org/10.1109/ROBOT.2005.1570450>.
- [17] J. R. Azinheira & A. Moutinho. (2008). Hover Control of an UAV With Backstepping Design Including Input Saturations. *IEEE Transactions on*

Control Systems Technology, vol. 16(3), #517-526#. doi: 10.1109/TCST.2007.908209.

[18] Y. Bestaoui. (2005). Characterization of non-trim trajectories of an autonomous underactuated airship in a low velocity flight. *Proceedings of the IEEE International Conference on Robotics and Automation, Barcelona, Spain*, #2259-2264#. Doi: 10.1109/ROBOT.2005.1570449.

[19] S. Hima and Y. Bestaoui. (2006). Trim Trajectories Characterization for an Unmanned Autonomous Airship. *IEEE/RSJ International Conference on Intelligent Robots and Systems, Beijing, China*, #137-142#. Doi: 10.1109/IROS.2006.281930.

[20] Takaya, T., Kawamura, H., Minagawa, Y., Yamamoto, M., & Ohuchi, A. (2006). PID landing orbit motion controller for an indoor blimp robot. *Artificial Life and Robotics*, 10(2), #177–184#. <https://doi.org/10.1007/s10015-006-0385-9>.

[21] Takaya, T., Kawamura, H., Minagawa, Y., Yamamoto, M., & Ouchi, A. (2006). Motion control in three-dimensional round system of blimp robot. *Proceedings of the SICE-ICASE International Joint Conference* #1291–1294#. *IEEE*. <https://doi.org/10.1109/SICE.2006.315440>.

[22] Fukushima, H., Saito, R., Matsuno, F., Hada, Y., Kawabata, K., & Asama, H. (2006). Model predictive control of an autonomous blimp with input and output constraints. *Proceedings of the IEEE Conference on Computer Aided Control System Design*, #2184–2189#. <https://doi.org/10.1109/CACSD-CCA-ISIC.2006.4776979>.

[23] Fukushima, H., Kon, K., Matsuno, F., Hada, Y., Kawabata, K., & Asama, H. (2006). Constrained model predictive control: Applications to multi-vehicle formation and an autonomous blimp. *Proceedings of the SICE-ICASE International Joint Conference*, #4515–4520#. *IEEE*. <https://doi.org/10.1109/SICE.2006.315039>.

[24] H. Fukushima, K. Kon, Y. Hada, F. Matsuno, K. Kawabata and H. Asama. (2007). State-Predictive Control of an Autonomous Blimp in the Presence of Time Delay and Disturbance. *IEEE International Conference on Control Applications, Singapore*, #188-193#, Doi: 10.1109/CCA.2007.4389228.

[25] T. Kampke and A. Elfes. (2003). Optimal aerobot trajectory planning for

wind-based opportunistic flight control. *Proceedings IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2003)*, Las Vegas, NV, USA, vol.1, #67-74#, DOI: 10.1109/IROS.2003.1250607.

[26]] C. Weisbin et al. (2008). Human-Robot Lunar Exploration: Pressurized vs. Unpressurized Rovers. *19th International Conference on Systems Engineering*, Las Vegas, NV, USA, #8-12#, Doi: 10.1109/ICSEng.2008.10.

[27] Prakash O. & Purohit S. (2020). Multibody Dynamics of winged Hybrid Airship Payload Delivery System. *AIAA Aviation 2020 Forum*, DOI: 10.2514/6.2020-3200.

[28] Ghaffar, A. F. A. (2012). The Development of a Mathematical Model of a Hybrid Airship [Master's thesis, *University of Southern California*].

[29] Tiwari, A., Vora, A., & Sinha, N. K. (2016). Airship trim and stability analysis using bifurcation techniques. *Proceedings of the 7th International Conference on Mechanical and Aerospace Engineering (ICMAE)*, #471–475#. *IEEE*. <https://doi.org/10.1109/ICMAE.2016.7549586>.

[30] Ringland RF, Tischler MB, Jex HR, Emmen RD, & Ashkenas IL. (1982). Flight dynamics analysis and simulation of heavy lift airships. *NACA CR-16647*. <https://ntrs.nasa.gov/api/citations/19830013937/downloads/19830013937.pdf>.

[31] Lacroix, S., & Il, K. J. (2002). High Resolution Terrain Mapping with an Autonomous Blimp. In *Proceedings of the International Conference on Intelligent Robots and Systems (Lausanne, Switzerland)*. DOI: 10.1109/IRDS.2002.1041485.

[32] Hygounenc, E., Il, K.J., Soueres, P. & Lacroix, S. (2004). The Autonomous Blimp Project of LAAS-CNRS: Achievements in Flight Control and Terrain Mapping. *Int. J. Robot. Res.* 23(4-5), #473–511#. <https://doi.org/10.1177/0278364904042200>.

[33] Waishek, J., Dogan, A. & Bestaoui, Y. (2009). Investigation into the Time Varying Mass Effect on Airship Dynamics Response. In *Proceedings of the Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, Orlando, FL, USA*. <https://doi.org/10.2514/6.2009-735>.

[34] Waishek, J. & Dogan, A. (2010). Comprehensive Characterization of Airship Response to Wind and Time Varying Mass. In *Proceedings of the*

Atmospheric Flight Mechanics Conference, Toronto, ON, Canada.
<https://doi.org/10.2514/6.2010-7626>.

[35] S. B. V. Gomes. (1990). An Investigation of the Flight Dynamics of Airships with Application to the YEZ-2A (PhD thesis). *Cranfield Institute of Technology, College of Aeronautics, Cranfield.*
<http://hdl.handle.net/1826/3015>.

[36] Khoury, G. A. (2012). Airship technology. *Cambridge University Press*. ISBN 1107019702, 9781107019706.

[37] Elfes, A., Bueno, S., Bergerman, M., & Ramos, J. G. (1998). A semi-Autonomous Robotics Airship for Environmental Monitoring Missions. *In Proceedings of the IEEE International Conference on Robotics and Automation (Vol. 4), #3449–3455#*. DOI: 10.1109/ROBOT.1998.680971.

[38] Kulczycki, E. A., Joshi, S. S., Hess, R. A., & Elfes, A. (2006). Towards Controller Design for Autonomous Airship Using SLC and LQR Methods. *AIAA Guidance, Navigation and Control Conference and Exhibit, AIAA, Reston, VA*. <https://doi.org/10.2514/6.2006-6778>.

[39] Schmidt D. K. (2006). Dynamic Modeling, Control and Station-Keeping Guidance of a Large High-Altitude ‘Near-Space’ Airship. *AIAA Guidance, Navigation and Control Conference and Exhibit, AIAA, Reston, VA, #21–24#*. <https://doi.org/10.2514/6.2006-6781>.

[40] S.S. Bueno, J.R. Azinheira, J.G. Ramos, E. C. de Paiva, P. Rives, A. Elfes, J.R.H. Carvalho, & G. F. Silveira. (2002). Project aurora-towards an autonomous robotic airship. *In Workshop on Aerial Robotics, IEEE/RSJ International Conference on Intelligent Robots and Systems – IROS*. https://www.researchgate.net/publication/228518304_Project_AURORA_Towards_an_autonomous_robotic_airship.

[41] Ely Carneiro de Paiva, Jose Raul Azinheira, Josue G. Ramos, Alexandra Moutinho, & Samuel Siqueira Bueno. (2006). Project aurora infrastructure and flight control experiments for a robotic airship. *Journal of Field Robotics, 23(3-4)*. <https://doi.org/10.1002/rob.20111>.

[42] Elfes, A. *et al.* (2002). Modelling, Control and Perception for an Autonomous Robotic Airship. In: Hager, G.D., Christensen, H.I., Bunke, H., Klein, R. (eds) *Sensor Based Intelligent Robots. Lecture Notes in Computer*

Science, vol 2238. Springer, Berlin, Heidelberg. https://doi.org/10.1007/3-540-45993-6_13.

[43] Moutinho, Alexandra. (2007). Modeling and Nonlinear Control for Airship Autonomous Flight (phd thesis). *Institute of Mechanical Engineering (IDMEC) in Instituto Superior Tecnico, Technical University of Lisbon*. DOI: 10.13140/2.1.1333.8568.

[44] Lacroix, S., Jung, I.-K., Souères, P., Hygounenc, E., & Berry, J.-P. (2003). The autonomous blimp project of LAAS/CNRS: Current status and research challenges. In B. Siciliano & P. Dario (Eds.), *Experimental Robotics VIII* , #422–443#. Springer. DOI:10.1007/3-540-36268-1_44.

[45] G. Y. Lee, D. M. Kim, & C. H. Yeom. (2006). Development of korean high altitude platform systems. *International Journal of Wireless Information Networks*, 13(1), #31–42#. <https://doi.org/10.1007/s10776-005-0018-6>.

[46] Lee, S.-J., Kim, S.-P., Kim, T.-S., Kim, H.-K., & Lee, H.-C. (2004). Development of Autonomous Flight Control System for 50 m Unmanned Airship. In *Proceedings of the 2004 Intelligent Sensors, Sensor Networks and Information Processing Conference*, #457–462#. IEEE. DOI: 10.1109/ISSNIP.2004.1417504.

[47] A Moutinho & Jose Raul Azinheira. (2005). Stability and robustness analysis of the aurora airship control system using dynamic inversion. In *Proceedings of the 2005 IEEE International Conference on Robotics and Automation, ICRA*, #2265–2270#. DOI: 10.1109/ROBOT.2005.1570450.

[48] J. S. Santos, L. C. S. Goes, and R. S. Pant. (2015). Design and flight testing of an autonomous airship. In 22nd AIAA Lighter-Than-Air Systems Technology Conference, USA. <https://doi.org/10.2514/6.2015-3352>.

[49] J S Santos, S Stevanovic, K Kondak, F Holzapfel, L C S Goes & R S Pant. (2016). Stability augmentation system for a tethered airship. In *Proceedings from the 16th AIAA Lighter-Than-Air Systems Technology Conference, AIAA, Washington, DC*. <https://doi.org/10.2514/6.2016-4224>.

[50] F. Ben Abdallah, N. Azouz, L. Beji, S. Hima & A. Abichou. (2018). Modeling and control of an aerial robocrane using a wire driven system. Annual American Control Conference (ACC), #508-513#. DOI:

10.23919/ACC.2018.8431168.

[51] M. Gouttefarde & C. M. Gosselin. (2006). Analysis of the wrench-closure workspace of planar parallel cable-driven mechanisms. *IEEE Transactions on Robotics*, 22(3) #434-445#. DOI: 10.1109/TRO.2006.870638.

[52] C. Gosselin & M. Grenier. (2011). On the determination of the force distribution in over constrained cable-driven parallel mechanisms. *Meccanica*, 46(1), #3-15#. <https://doi.org/10.1007/s11012-010-9369-x>.

[53] Pott, A., Mütterich, H., Kraus, W., Schmidt, V., Miermeister, P., Verl, A. (2013). IPAnema: A family of Cable-Driven Parallel Robots for Industrial Applications. In: Bruckmann, T., Pott, A. (eds) *Cable-Driven Parallel Robots. Mechanisms and Machine Science*, vol 12. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-31988-4_8.

[54] Saber O & Zohoor H. (2013). Workspace Analysis of a Cable-Suspended Robot with Active/Passive Cables. *ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference Vol 6A*. <https://doi.org/10.1115/DETC2013-12646>.

[55] Ke-shi Zhang, Z.-h. H., & feng Song, B. (2010). Flight Performance Analysis of Hybrid Airship: Revised Analytical Formulation. *Journal of Aircraft*, Vol. 47(4). doi:10.2514/1.47294.

[56] P. A. Mackrodt. (1980). Further Studies in the Concept of Delta-Winged Hybrid Airships. *Journal of Aircraft*, Vol. 17(10). doi:10.2514/3.57960.

[57] Aman Raj Verma, K. K. S., & Priyadarshi, P.. (2014). Optimum Buoyant and Aerodynamic Lift for a Lifting-Body Hybrid Airship. *Journal of Aircraft*, Vol. 51(5), doi:10.2514/1.C032038.

[58] Nagabhushan, B. L. (1983). Dynamic stability of a buoyant quad-rotor aircraft. *Journal of Aircraft*, Vol. 20(3). doi:10.2514/3.44859.

[59] Slegers, N., and Costello, M. (2003). Aspects of Control for a Parafoil and Payload System. *Journal of Guidance, Control and Dynamics*, Vol. 26(6), #898–905#. doi:10.2514/2.6933.

[60] Prakash, O., and Ananthkrishnan, N. (2006). Modeling and Simulation of 9-DOF Parafoil-Payload System Flight Dynamics. *AIAA Atmospheric Flight Mechanics Conference and Exhibit*, AIAA, Keystone, Colorado, USA. doi:10.2514/6.2006-6130.

- [61] Prakash O., Daftary A. & Ananthkrishnan N. (2005). Bifurcation Analysis of Parafoil-Payload System Flight Dynamics. AIAA Atmospheric Flight Mechanics Conference and Exhibit, AIAA, San Francisco, California, USA, <https://doi.org/10.2514/6.2005-5806>.
- [62] Prakash O., Daftary A. & Ananthkrishnan N. (2005). Trim and Stability Analysis of Parafoil/Payload System Using Bifurcation Methods. 18th AIAA Aerodynamic Decelerator Systems Technology Conference and Seminar, AIAA, Munich, Germany. doi:10.2514/6.2005-1666.
- [63] Prakash O. & Ananthkrishnan N. (2020). NDI based Generic Heading Tracking Control Law for Parafoil/payload System. *AIAA Aviation 2020 Forum*, DOI: 10.2514/6.2020-3195.
- [64] BL. Nagabhushan, & NP. Tomlinson. (1982). Dynamics and control of a heavy lift airship hovering in a turbulent cross wind. *Journal of Aircraft*, Vol.19(10), pp.#826–830#. <https://doi.org/10.2514/3.61564>.
- [65] S. V. D. Zwaan, A. Bernardino, & J. Santos-Victor. (2000). Vision based station keeping and docking for an aerial blimp. *International conference on Intelligent Robots and Systems*, #614–619#. DOI:10.1109/IROS.2000.894672.
- [66] Y Wu, M Zhu, Z Zuo, & Z Zheng. (2011). Adaptive Trajectory Tracking of Stratospheric Airship Based on Input-output Stability Theory. *IEEE Xplore, Computational Intelligence and Design (ISCID), Fourth International Symposium*, Hangzhou, 2011, pp. #301-305#. DOI: 10.1109/ISCID.2011.83.
- [67] L. Sun, & Z. Zheng. (2015). Nonlinear adaptive trajectory tracking control for a stratospheric airship with parametric uncertainty. *Nonlinear Dynamics*, Vol. 83(3), pp #1419–1430#. DOI:10.1007/s11071-015-2248-1.
- [68] B. Miripour Farad, & P. Abdollahzadeh. (2016). Adaptive and robust control of a stratospheric airship. *Modares Mechanical Engineering*, Vol.16(2), pp.#111-119#. URL: <http://mme.modares.ac.ir/article-15-2383-en.html>.
- [69] EC. De. Paiva, F. Benjovengo, & SS. Bueno. (2007). Sliding mode control for the path following of an unmanned. *IFAC Proceedings*. Volumes Vol. 40(15), pp #221-226#. <https://doi.org/10.3182/20070903-3-FR-2921.00040>.
- [70] JR. Azinheira, A. Moutinho, & EC. De. Paiva. (2006). Airship hover stabilization using a backstepping control approach. *Journal of guidance, control, and dynamics*, Vol. 29 (4), #903–914#.

<https://doi.org/10.2514/1.17334>.

[71] Park, C.S., Lee, H., Tahk, M.J. & Bang, H. (2003). Airship Control Using Neural Network Augmented Model Inversion. *In Proceedings of the Conference on Control Applications, Istanbul, Turkey*. DOI: 10.1109/CCA.2003.1223497.

[72] Moutinho, A. & Azinheira, J.R. (2004). Path Control of an Autonomous Airship Using Dynamic Inversion. *Proceedings of the 5th IFAC/EURON Symposium on Intelligent Autonomous Vehicles, Lisbon, Portugal*. *IFAC-PapersOnLine*, 37(8),#633-638#. [https://doi.org/10.1016/S1474-6670\(17\)32049-9](https://doi.org/10.1016/S1474-6670(17)32049-9).

[73] Yan, Z., Weidong, Q., Yugeng, X. & Zili, C. (2008). Stabilization and Trajectory Tracking of Autonomous Airship's Planar Motion. *J. Syst. Eng. Electron.* 19, # 974–981#. DOI: 10.1016/S1004-4132(08)60184-X.

[74] Repoulas, F. & Papadopoulos, E. (2008). Robotic Airship Trajectory Tracking Control Using a Backstepping Methodology. *Proceedings of the International Conference on Robotics and Automation, Pasadena, CA, USA*. DOI: 10.1109/ROBOT.2008.4543207.

[75] Peter Dorato, Vito Cerone, & Chaouki Abdallah. (1994). Linear-Quadratic Control: An Introduction. *Publisher: Simon & Schuster, Inc., USA*. ISBN:978-0-02-329962-9.

[76] Masar, I. & Stohr, E. (2011). Gain Scheduled LQR Control for an Autonomous Airship. *Proceedings of the 18th International Conference on Process Control, Bratislava, Slovakia*, <https://www.uiam.sk/pc11/data/papers/097.pdf>.

[77] Nakpam, J. (2011). *Control of airship motion in presence of wind* (Master's thesis). Department of Mechanical and Aerospace Engineering, The University of Texas at Arlington, Arlington, TX, United States. https://mavmatrix.uta.edu/mechaerospace_theses/279.

[78] Mishra, A. (2019). Autonomous obstacle avoidance maneuvering of thrust-vectorized airship with neural network control. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, Vol. 234(3)*, #689-708#. <https://doi.org/10.1177/0954410019879821>.

[79] P. Ordaz *et al.* (2018). Nonlinear Control and Trajectory Tracking of

- Unmanned Aircraft System Based on of a Complete State Space Representation. *ScienceDirect, IFAC*. <https://doi.org/10.1016/j.ifacol.2018.07.339>.
- [80] G. Atmeh & K. Subbarao. (2016). Guidance, Navigation and Control of Unmanned Airships under Time-Varying Wind for Extended Surveillance. *Aerospace, Vol 3(1)*, DOI:10.3390/aerospace3010008.
- [81] Meuller JB, Zhao YJ, & Garrard WL. (2009). Optimal ascent trajectories for stratospheric airships using wind energy. *Journal of Guidance, Control, and Dynamics*, Volume 32(4): #1232–45#. <https://doi.org/10.2514/1.41270>.
- [82] Thomasson PG. (2000). Equations of motion of a vehicle in a moving fluid. *Journal of Aircraft*.37(4):630–9. <https://doi.org/10.2514/2.2645>.
- [83] Liao L, & Pasternak I. (2009). A review of airship structural research and development. *Progress in Aerospace Sciences*, 45(4–5), #83–96#. DOI:10.1016/j.paerosci.2009.03.001.
- [84] Azinheira JR, de Paiva EC, & Bueno SS. (2002). Influence of wind speed on airship dynamics. *Journal of Guidance, Control and Dynamics*. 25(6), #1116–24#. <https://doi.org/10.2514/2.4991>.
- [85] Evans JR, & DeLaurier JD. (1981). The Shenandoah flies again: a computer simulation. *AIAA lighter-than-air systems technology conference*, Annapolis, MD, p. #62–73#. <https://doi.org/10.2514/6.1981-1325>.
- [86] Li Y, Nahon M, & Sharf I. (2009). Modelling and simulation of flexible airships. *AIAA Journal*, Vol 47(3), #592–605#. <https://doi.org/10.2514/1.37455>.
- [87] Gomes SBB. (1980). An investigation into the flight dynamics of airships with application to the YEZ-2A. *PhD thesis, Cranfield Institute of Technology*. <http://hdl.handle.net/1826/3015>.
- [88] F. Ben Abdallah, N. Azouz & L. Beji. (2019). Modeling of a heavy-lift airship carrying a payload by a cable-driven parallel manipulator. *International Journal of Advanced Robotic Systems Vol 16(4)*, #1–17#. <https://doi.org/10.1177/1729881419861769>.
- [89] Avenant, G. (2010). Autonomous Flight Control System for an Airship. *M.S. Thesis, Stellenbosch University, Stellenbosch, South Africa*. <https://scholar.sun.ac.za/server/api/core/bitstreams/9956fbcd-e74c-4ab7-8c14-120ee7b3861e/content>.

- [90] Funke, J., Brown, M., Erlien, S. M., & Gerdes, J. C. (2017). Collision avoidance and stabilization for autonomous vehicles in emergency scenarios. *IEEE Transactions on Control Systems Technology*, Vol 25(4), #1204–1216#. <https://doi.org/10.1109/TCST.2016.2599783>.
- [91] W. Gao and F. Lu. (2015). Simulation of proportional guidance law with an airship. *The 27th Chinese Control and Decision Conference (2015 CCDC)*, Qingdao, China, #3523-3526#. DOI: 10.1109/CCDC.2015.7162533.
- [92] Amelda Dianne Andan, Waqar Asrar, Ashraf A. Omar (2012). Aerodynamics of a hybrid airship. *AIP Conf. Proc.* 1440 (1), #154–161#. <https://doi.org/10.1063/1.4704214>.
- [93] Duraisamy, G., & Sinha, N. K. (2019). Nonlinear simulation framework based on bifurcation analysis for controlled airship maneuvering. *AIAA Scitech 2019 Forum*. <https://doi.org/10.2514/6.2019-1564>.
- [94] Zheng, Zewei & Sun, Liang. (2017). Adaptive Sliding Mode Trajectory Tracking Control of Robotic Airships with Parametric Uncertainty and Wind Disturbance. *Journal of the Franklin Institute*. DOI: 355. 10.1016/j.jfranklin.2017.11.004.
- [95] Ramesh Hun, Nandan K Sinha, Debasish Chakraborti, & P. Balasubramanian. (2016). Dynamic Modeling and Stability Analysis of a High-Altitude Airship. 30th Congress of the International Council of Aeronautical Sciences. At: Daejeon, South Korea Volume: ISBN:978-3-932182-85-3. https://www.icas.org/icas_archive/ICAS2016/data/papers/2016_0700_paper.pdf.
- [96] Andan, A. D., Asrar, W., & Omar, A. A. (2012). Investigation of aerodynamic parameters of a hybrid airship. *Journal of Aircraft*, 49(2), #658–662#. <https://doi.org/10.2514/1.C031491>.
- [97] M. Z. Ashraf & M. A. Choudhry. (2013). Dynamic Modeling of the Airship with MATLAB using Geometrical Aerodynamic Parameters. *Aerospace Science and Technology*, vol. 25(1), #56-64#. <https://doi.org/10.1016/j.ast.2011.08.014>.
- [98] Y. Li, M. Nahon & I. Sharf. (2011). Airship dynamics modeling: A literature review. *Progress in Aerospace Sciences*, vol. 47(3), #217–239#. <https://doi.org/10.1016/j.paerosci.2010.10.001>.

- [99] Melbous, A., Tami, Y., Guessoum, A. & Hadjsadok, M. (2010). UAV Controller Design and Analysis Using Sliding Mode Control. J. Autom. Syst. Eng. (4). doi:10.1016/j.isatra.2014.03.010.
- [100] Ramesh Hun, Nandan K Sinha, Debasish Chakraborti, & P. Balasubramanian. (2016). Dynamic Modeling and Stability Analysis of a HighAltitude Airship. 30th Congress of the International Council of Aeronautical Sciences. At: Daejeon, South Korea Volume: ISBN: 978-3-932182-85-3.
https://www.icas.org/icas_archive/ICAS2016/data/papers/2016_0700_paper.pdf
- [101] Y. Li & M. Nahon. (2007). Modeling and simulation of airship dynamics. Journal of Guidance, Control, and Dynamics, vol. 30 (6), #1691-1700#. <https://doi.org/10.2514/1.29061>.
- [102] Bernard Etkin, Lloyd Duff Reid. (1995). Dynamics of Flight: Stability and Control, 3rd Edition. *John Wiley & Sons, ISBN: 978-0-471-03418-6*.
- [103] Puranik, A. S. (2011). Dynamic Modeling, Simulation and Control Design of a Parafoil-Payload System for Ship Launched Aerial Delivery System (SLADS) [Doctoral dissertation, Michigan Technological University]. Digital Commons @ Michigan Tech. <https://doi.org/10.37099/mtu.dc.ets/401>.
- [104] Doedel, Eusebius & Oldeman, Bart. (2009). AUTO07P: Continuation and Bifurcation Software for Ordinary Differential Equations. Concordia University Montreal, Canada.
- [105] J. Gordon Leishman. (2022). Introduction to Aerospace Flight Vehicles. Creative Commons Attribution-Noncommercial-No Derivative Works 3.0 License. ISBN : 979-8-9852614-0-0.

LIST OF PUBLICATIONS

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ORCID ID: <https://orcid.org/0000-0003-1256-4850>

- [1] Kumar, A., & Prakash, O. (2023). Longitudinal Trim and Stability Analysis of Hybrid Airship with Suspended Payload for Single Body and Two Body Dynamics using Bifurcation Method. *Mathematical Modelling of Engineering Problems, Vol. 10(2)*, #671–680#. <https://doi.org/10.18280/mmep.100238>.
- [2] Kumar, A., & Prakash, O. (2024). Designing a PID Pitch Controller with HIL Solution for Maintaining Stability and Controllability of a Hybrid Airship. *Mathematical Modelling of Engineering Problems, Vol. 11(4)*, #931–942#. <https://doi.org/10.18280/mmep.110410>.
- [3] Kumar, A., & Prakash, O. (2024). Analysis of hybrid airship longitudinal dynamics with model predictive controller. *Proceedings of the 2024 Second International Conference on Microwave, Antenna and Communication (MAC) # 1–6#. IEEE*. <https://doi.org/10.1109/MAC61551.2024.10837547>.
- [4] Prakash, O., & Kumar, A. (2022). NDI-based heading tracking of hybrid airship for payload delivery. *AIAA Scitech 2022 Forum, AIAA 2022*, #1432#. <https://doi.org/10.2514/6.2022-1432>.
- [5] Prakash, O., & Kumar, A. (2023). Modeling and simulation of turning flight maneuver of winged airship-payload system using 9-DOF multibody model. *AIAA Scitech 2023 Forum, AIAA 2023* #1681#. <https://doi.org/10.2514/6.2023-1681>.
- [6] Kumar, A., & Prakash, O. (2022). Nonlinear modelling and analysis of longitudinal dynamics of hybrid airship. In S. Banerjee & A. Saha (Eds.), *Nonlinear Dynamics and Applications* #965–976#, Springer. https://doi.org/10.1007/978-3-030-99792-2_82.

ADDITIONAL PUBLICATION:

[7] Kumar, A., & Prakash, O. (2022). Analysis of lateral dynamics of ARX model of an aircraft with model predictive controller. *In Proceedings of the 2022 International Conference for Advancement in Technology (ICONAT) #1–6#*, IEEE. <https://doi.org/10.1109/ICONAT53423.2022.9726117>.

PATENT (PUBLISHED)

[8] Kumar, A., & Prakash, O. (2024). Control mechanism for orientation of hybrid airship with suspended payload. Indian Patent Office, Journal No. 38/2024, Page No. 84488.

APPENDIX A1

EULER ANGLE TRANSFORMATION AND 6DOF

DERIVATION OF AIRCRAFT

A1.1 Transformation (body-fixed frame):

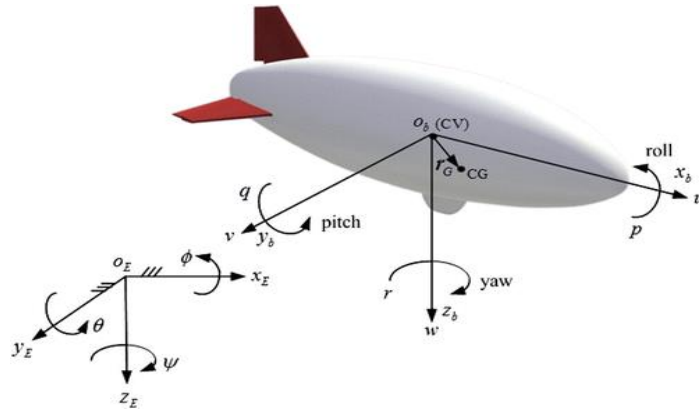


Fig. A1.1 Coordinate System of an Airship

Three angles can be used to depict an object's orientation in three dimensions using Euler angles. The three angles, roll ϕ , pitch θ , and yaw ψ , define the orientation of the body frame relative to the inertial frame, as depicted in Fig. A1.1. In Euler's rotation theorem, the angles associated with the phenomenon are referred to as follows: the angle of self-rotation is denoted as ϕ , the angle of nutation is denoted as θ , and the angle of precession is denoted as ψ . The roll angle ϕ is considered positive when the aircraft is in a right-wing down orientation. The pitch angle θ is considered positive when the aircraft is in a nose-up position. Additionally, the yaw angle ψ is regarded as positive when the aircraft rotates to the right in a clockwise direction. The aforementioned angles are commonly referred to as Euler angles, serving as a representation of the airship's attitude. Therefore, the rotation matrix $R1$, which transforms vectors represented in the body frame to coordinates in the inertial frame, is composed of the three Euler angles. $R1$ is formed by three

rotations that convert the inertial frame $G\{i, j, k\}$ to the body frame $O\{e1, e2, e3\}$ [55]. The rotation around point $G - k$ with a yaw angle ψ may be observed in the diagram presented in Fig. A1.2 (a). This rotation results in the transformation of point $G - i$ to $G - x1$ and point $G - j$ to $G - y1$. The rotation around point G with a pitch angle θ can be described as follows: The point $G - x1$ is rotated to point $O - e1$, and the point $G - k$ is rotated to point $G - z1$. This rotation is illustrated in Fig. A1.2(b). The rotation around point O with a roll angle ϕ can be described as follows: The point G , initially aligned with the $y1$ axis, is rotated to align with the $e2$ axis, while the point G , initially aligned with the $z1$ axis, is rotated to align with the $e3$ axis. This rotation is illustrated in Fig. A1.2 (c).

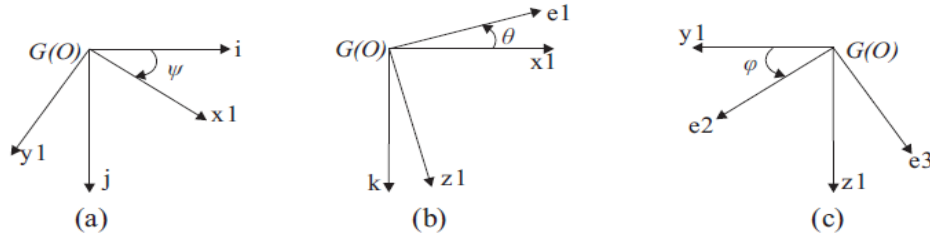


Fig. A1.2 Euler angle rotation

To combine the three coordinate transformations shown above in Fig. A1.2, the following relation is obtained:

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} e1 \\ e2 \\ e3 \end{bmatrix} \quad (A1.1)$$

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = R1 \begin{bmatrix} e1 \\ e2 \\ e3 \end{bmatrix} \quad (A1.2)$$

$$R_1 = \begin{bmatrix} \cos \phi \cos \theta & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \sin \phi + \sin \psi \sin \phi \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \quad (A1.3)$$

Where

$$R_1^{-1} = R_1^T \quad (A1.4)$$

A1.2 WIND FRAME AND THE TRANSFORMATION:

The rotation matrix that transforms from the wind frame to the body frame, as depicted in Fig. A1.3, is denoted as R_2 and is defined in ref [70] as follows:

$$R_2 = \begin{bmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{bmatrix} \quad (A1.5)$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = R_2 \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad (A1.6)$$

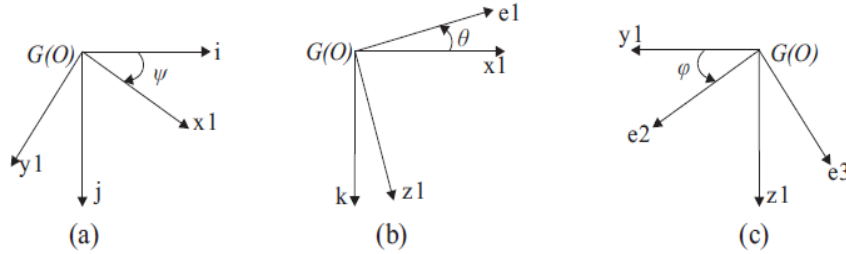


Fig. A1.3 Euler angle rotation with wind frame

The 6-DOF airship motion equations assume that the Centre of Gravity (CG) is below the CV and the Centre of Buoyancy (CB) is in the same location. We ignore airship volume and mass changes.

Airship 6-DOF kinematic model equations:

$$\dot{\epsilon} = R(\zeta)v \quad (A1.7)$$

$$R(\zeta) = \begin{bmatrix} R_1(\zeta) & 0_{3 \times 3} \\ 0_{3 \times 3} & R_2(\zeta) \end{bmatrix} \quad (A1.8)$$

$$R_1(\zeta) = \begin{bmatrix} \cos \phi \cos \theta & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \sin \phi + \sin \psi \sin \phi \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \quad (A1.9)$$

$$R_2(\zeta) = \begin{bmatrix} 1 & \sin \phi \tan(\theta) & \cos \phi \tan(\theta) \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \sec(\theta) & \cos \phi \sec(\theta) \end{bmatrix} \quad (A1.10)$$

Additionally, the matrix $0_{3 \times 3}$ contains no entries. The rotation matrices $R_1(\sigma)$ and $R_2(\sigma)$ are utilised to convert the angular and linear velocities of the body reference frame, respectively, into an inertial reference frame.

A1.3 Transformations and Euler Angles:

An aircraft's attitude or orientation can be ascertained using the Euler angles, ψ, θ, ϕ , relative to an inertial frame of reference that is fixed at Earth. By rotating the Earth fixed inertial frame $X^E Y^E Z^E$ by the angles ψ, θ, ϕ , one can obtain the aircraft body-fixed axis system $X^B Y^B Z^B$. As illustrated in Fig. A1.6, the rotations of the Earth-fixed axis system adhere to the 3–2–1 rule in the subsequent order:

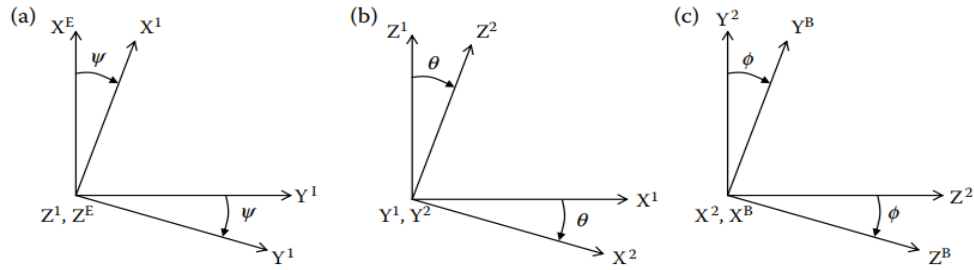


Fig. A1.6 (a) Rotation ‘3’ about the Z^E axis, (b) rotation ‘2’ about Y^1 axis, (c) rotation ‘1’ about X^2 axis.

$$\text{First rotation: } X^E - Y^E - Z^E \xrightarrow{\text{rotation } \psi \text{ about } Z^E} X^1 - Y^1 - Z^1$$

The relation between the two axis systems can be found from Fig. A1.6a as

$$\begin{bmatrix} X^E \\ Y^E \\ Z^E \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X^1 \\ Y^1 \\ Z^1 \end{bmatrix} = R_\psi \begin{bmatrix} X^1 \\ Y^1 \\ Z^1 \end{bmatrix} \quad (A1.11)$$

Second rotation: $X^1 - Y^1 - Z^1 \xrightarrow{\text{rotation } \theta \text{ about } Y^1} X^2 - Y^2 - Z^2$

The second rotation results in the relation as shown in Fig. A1.6b is

$$\begin{bmatrix} X^1 \\ Y^1 \\ Z^1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} X^2 \\ Y^2 \\ Z^2 \end{bmatrix} = R_\theta \begin{bmatrix} X^2 \\ Y^2 \\ Z^2 \end{bmatrix} \quad (\text{A1.12})$$

Third rotation: $X^2 - Y^2 - Z^2 \xrightarrow{\text{rotation } \phi \text{ about } X^2} X^B - Y^B - Z^B$

The Third rotation results in the relation as shown in Fig. A1.6c is

$$\begin{bmatrix} X^2 \\ Y^2 \\ Z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} = R_\phi \begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} \quad (\text{A1.13})$$

R_ψ , R_θ and R_ϕ are known as the rotation (or transformation) matrices and are orthogonal. The relation between the axis systems $X^E Y^E Z^E$ and $X^B Y^B Z^B$ is given by the Eqn. (A1.14).

$$\begin{bmatrix} X^E \\ Y^E \\ Z^E \end{bmatrix} = R_\psi R_\theta R_\phi \begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} \quad (\text{A1.14})$$

Multiplying both sides of the Eqn. (A1.14) by $R_\psi^T R_\theta^T R_\phi^T$ and using orthogonality condition.

$$\begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} = R_\psi^T R_\theta^T R_\phi^T \begin{bmatrix} X^E \\ Y^E \\ Z^E \end{bmatrix} \quad (\text{A1.15})$$

Using transformation, we are now in a position to write components of aircraft weight along the body-fixed axes as follows in Eqn. (A1.16):

$$\begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} = \begin{bmatrix} X^G \\ Y^G \\ Z^G \end{bmatrix} = R_\psi^T R_\theta^T R_\phi^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = \begin{bmatrix} -mg \sin \theta \\ mg \sin \phi \cos \theta \\ mg \cos \phi \cos \theta \end{bmatrix} \quad (\text{A1.16})$$

A1.4 Relation between Body and Wind-Fixed Coordinates

(Rotation Triplet $(-\beta) - \alpha - O$)

Another useful transformation is that between the wind-fixed axis system and the body-fixed axis system given by the following rotations:

First rotation: $X^W - Y^W - Z^W \xrightarrow{\text{rotation } -\beta \text{ about } Z^W} X^S - Y^S - Z^S$

The subscript 'S' above refers to what is called the stability axis system. Following Fig. A1.7, the relation between the two axis systems can be written as

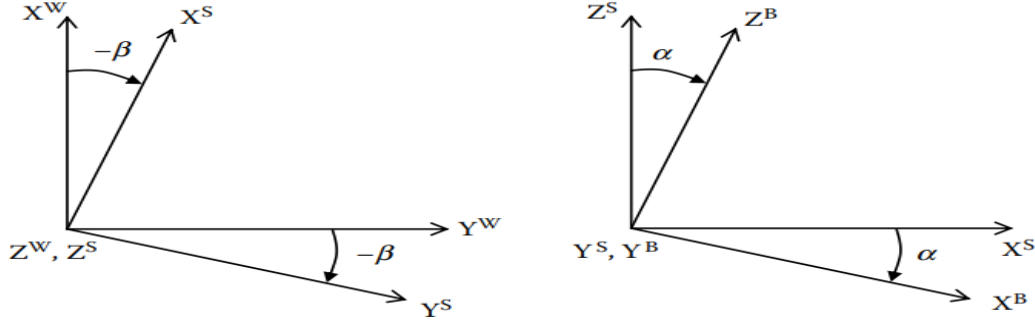


Fig. A1.7: Rotations $(-\beta)$ of wind axis system and α of the stability axis system

$$\begin{bmatrix} X^W \\ Y^W \\ Z^W \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X^S \\ Y^S \\ Z^S \end{bmatrix} = R_{-\beta} \begin{bmatrix} X^S \\ Y^S \\ Z^S \end{bmatrix} \quad (\text{A1.17})$$

Second rotation: $X^S - Y^S - Z^S \xrightarrow{\text{rotation } \alpha \text{ about } Y^S} X^B - Y^B - Z^B$

$$\begin{bmatrix} X^S \\ Y^S \\ Z^S \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} = R_{\alpha} \begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} \quad (\text{A1.18})$$

The rotation matrices $R_{-\beta}$ and R_{α} satisfy the orthogonality conditions as well. Thus, the relation between the wind-fixed and body-fixed axes can be written using the following transformation rule:

$$\begin{bmatrix} X^W \\ Y^W \\ Z^W \end{bmatrix} = R_{-\beta} R_{\alpha} \begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} \quad (\text{A1.19})$$

$$\begin{bmatrix} X^B \\ Y^B \\ Z^B \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X^W \\ Y^W \\ Z^W \end{bmatrix} = R_{\alpha}^T R_{-\beta}^T \begin{bmatrix} X^W \\ Y^W \\ Z^W \end{bmatrix} \quad (\text{A1.20})$$

This transformation rule in Eqn. (A1.20) can be used to transform a vector given in wind-fixed axis system into its components in the body-fixed axis system. For example, the aircraft relative wind velocity along X^W axis can be written in the body-fixed axis system as follows in Eqn. (A1.21):

$$\begin{aligned}
\begin{bmatrix} u \\ v \\ w \end{bmatrix} &= R_{\alpha}^T R_{-\beta}^T \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} V \cos \alpha \cos \beta \\ V \sin \beta \\ V \sin \alpha \cos \beta \end{bmatrix} \tag{A1.21}
\end{aligned}$$

A1.5 MATHEMATICAL MODELLING:

A1.5.1 Aircraft longitudinal dynamics in body-fixed frame:

The general motion of an aircraft can then be described by Newton's second law of motion in translational form in body-fixed frame:

$$F = m \left. \frac{dV_c}{dt} \right|_I = m \left. \frac{dV_c}{dt} \right|_B + m(\omega_c \times V_c) \tag{A1.22}$$

Then, the force equation in body-fixed axes system may be written as

$$F = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + m \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{A1.23}$$

Scalar expansion of Eqn. (A1.23) gives.

$$\dot{u} = rv - qw + \frac{X}{m}$$

$$\dot{v} = pw - ru + \frac{Y}{m}$$

$$\dot{w} = qu - pv + \frac{Z}{m}$$

Where $X = \frac{1}{2} \rho_{air} V^2 S C_X$ is the axial force (resultant force acting along the aircraft longitudinal axis X^B), $Y = \frac{1}{2} \rho_{air} V^2 S C_Y$ is the side force (resultant force acting along the aircraft axis Y^B) and $Z = \frac{1}{2} \rho_{air} V^2 S C_Z$ is the normal force (resultant force acting along the aircraft longitudinal axis Z^B).

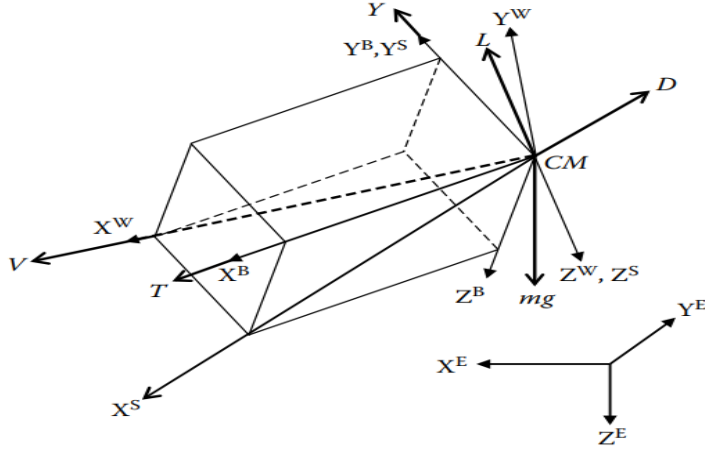


Fig. A1.8: Forces acting on the aircraft (mg along Z^E , L along $-Z^W$, D along $-X^S$, Y along Y^B , T along X^B).

Three different types of forces acting on the aircraft as shown in Fig. A1.8 constitute the resultant force as follows:

$$F = F^{\text{Gravitational}} + F^{\text{Aerodynamic}} + F^{\text{Propulsive}} \quad (\text{A1.24})$$

The gravitational force mg acts along the Z^E axis. Components of the aerodynamic force may be taken along $X^B Y^B Z^B$ axes. The propulsive force (thrust, T) may be assumed to act along X^B axis. Thus, each of these forces may be conveniently expressed in a different axis system as

$$F = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}_E + \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}_B + \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix}_B \quad (\text{A1.25})$$

where subscripts 'E' and 'B' stand for inertial (Earth-fixed) and body-fixed axis systems, respectively. X_A , Y_A , and Z_A are the components of aerodynamic forces along the aircraft-fixed body axes. We now need to transform the gravitational force to the body-fixed axis system. For that we need to know the proper relations between different axis systems, given by transformation matrices.

In terms of the force coefficients:

$$\dot{u} = rv - qw + \frac{\bar{q} S C_X^A}{m} + \frac{T}{m} - g \sin \theta \quad (\text{A1.26a})$$

$$\dot{v} = pw - ru + \frac{\bar{q}sC_Y^A}{m} + g \sin \Phi \cos \theta \quad (\text{A1.26b})$$

$$\dot{w} = qu - pv + \frac{\bar{q}sC_Z^A}{m} + g \cos \Phi \cos \theta \quad (\text{A1.26c})$$

A1.5.2 Derivation of Force Equations of Aircraft in Wind-Fixed Axis System

The equation of translational motion of an aircraft in wind-fixed axes system:

$$F = m \left. \frac{dV_c}{dt} \right|_I = m \left. \frac{dV_c}{dt} \right|_W + m(\omega_W \times V_c) \quad (\text{A1.27})$$

With

$$\omega_W \times = \begin{bmatrix} 0 & -r_W & q_W \\ r_W & 0 & -p_W \\ -q_W & p_W & 0 \end{bmatrix} \quad (\text{A1.28})$$

Expanding Eqn. (A1.27), we get

$$F = \begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix} = m \begin{bmatrix} \dot{V} \\ 0 \\ 0 \end{bmatrix} + m \begin{bmatrix} 0 \\ r_W V \\ -q_W V \end{bmatrix} \quad (\text{A1.29})$$

In scalar form,

$$\dot{V} = \frac{X_W}{m} \quad (\text{A1.30a})$$

$$r_W V = \frac{Y_W}{m} \quad (\text{A1.30b})$$

$$-q_W V = \frac{Z_W}{m} \quad (\text{A1.30c})$$

The force components in wind axis appear as in Eqn. (A1.31)

$$F = \begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix} = R_\mu^T R_\gamma^T R_\chi^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + R_{-\beta} \bar{q}s \begin{bmatrix} -C_D \\ C_Y \\ -C_L \end{bmatrix} + \begin{bmatrix} \cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\ -\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \quad (\text{A1.31})$$

So, collecting terms, we can write in full as

$$F = \begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix} = \begin{bmatrix} -mg \sin \gamma \\ mg \cos \gamma \sin \mu \\ mg \cos \gamma \cos \mu \end{bmatrix} + \bar{q}s \begin{bmatrix} -C_D \cos \beta + C_Y \sin \beta \\ C_D \sin \beta + C_Y \cos \beta \\ -C_L \end{bmatrix} + \begin{bmatrix} T \cos \alpha \cos \beta \\ -T \cos \alpha \sin \beta \\ -T \sin \alpha \end{bmatrix} \quad (\text{A1.32})$$

With Eqn. (A1.30) and (A1.32), Eqn. (A1.33) developed.

$$\dot{V} = -g \sin \gamma - \frac{\bar{q}s(C_D \cos \beta - C_Y \sin \beta)}{m} + \frac{T \cos \alpha \cos \beta}{m} \quad (\text{A1.33a})$$

$$r_W V = g \cos \gamma \sin \mu + \frac{\bar{q} s (C_D \sin \beta + C_Y \cos \beta)}{m} - \frac{T \cos \alpha \sin \beta}{m} \quad (A1.33b)$$

$$-q_W V = g \cos \gamma \cos \mu + \frac{\bar{q} s C_L}{m} - \frac{T \sin \alpha}{m} \quad (A1.33c)$$

With the help of transformation matrix, body axis rates are obtained as below:

$$\begin{bmatrix} \dot{\mu} \\ \dot{\gamma} \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} 1 & \tan \gamma \sin \mu & \tan \gamma \tan \mu \\ 0 & \cos \mu & -\sin \mu \\ 0 & \sec \gamma \sin \mu & \sec \gamma \cos \mu \end{bmatrix} \begin{bmatrix} p_W \\ q_W \\ r_W \end{bmatrix} \quad (A1.34)$$

$$\begin{bmatrix} p_W \\ q_W \\ r_W \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin \gamma \\ 0 & \cos \mu & \sin \mu \cos \gamma \\ 0 & -\sin \mu & \cos \mu \cos \gamma \end{bmatrix} \begin{bmatrix} \dot{\mu} \\ \dot{\gamma} \\ \dot{\chi} \end{bmatrix} \quad (A1.35)$$

With the help of Eqn. (A1.33) and (A1.35), Aircraft force equation in wind-fixed axes is formed:

$$\dot{V} = -g \sin \gamma - \frac{\bar{q} s (C_D \cos \beta - C_Y \sin \beta)}{m} + \frac{T \cos \alpha \cos \beta}{m} \quad (A1.36a)$$

$$\dot{\alpha} = q - \frac{1}{\cos \beta} \left\{ (p \cos \alpha + r \sin \alpha) \sin \beta - \frac{g}{V} \cos \gamma \cos \mu + \frac{\bar{q} s C_L}{mV} + \frac{T \sin \alpha}{mV} \right\} \quad (A1.36b)$$

$$\dot{\beta} = (p \sin \alpha - r \cos \alpha) + \frac{g}{V} \cos \gamma \sin \mu + \frac{\bar{q} s (C_D \sin \beta + C_Y \cos \beta)}{mV} - \frac{T \cos \alpha \sin \beta}{mV} \quad (A1.36c)$$

A1.6 METHODS OF PERFORMING NATURAL RESPONSE OF AN AIRSHIP:

As shown in the Fig A1.9 below, the short-period oscillatory response mode is of a higher frequency and heavily damped. The long period oscillatory dynamic pitch response mode or phugoid mode is of low frequency and lightly damped.

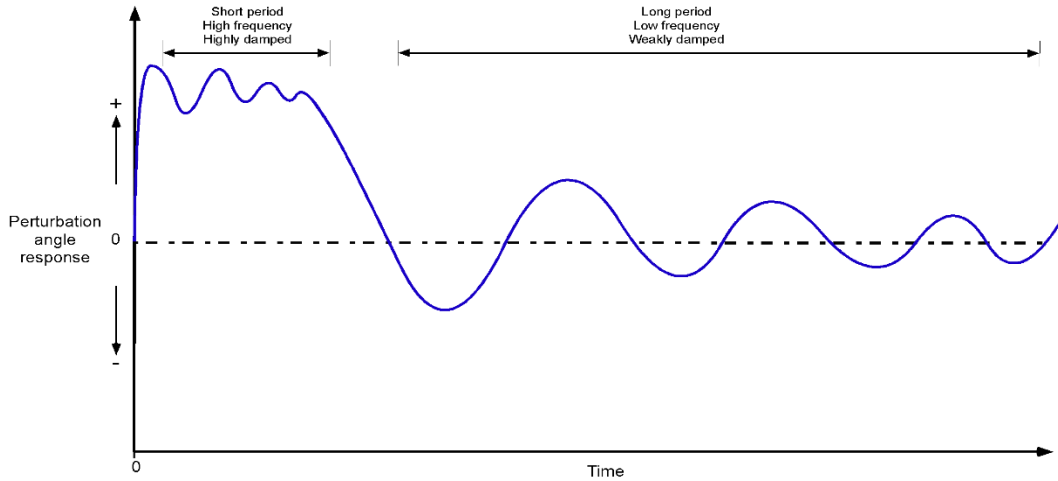


Fig. A1.9 Representative dynamic time-history response

The phugoid frequency, ω_{ph} , depends on the airspeed of the airplane, $V_\infty = U_0$, where U_0 is the initial trimmed airspeed, according to

$$\omega_{ph} = \frac{\sqrt{2}g}{U_0} \quad (A.1.37)$$

The damping in the pitch mode depends on the aircraft's lift-to-drag ratio, as given by

$$\zeta_{ph} = \frac{1}{\sqrt{2}} \left(\frac{C_L}{C_D} \right) \quad (A.1.38)$$

Let t_1 and t_2 be the times corresponding to a successive peak and valley or a successive valley and a peak in the dynamic response referred from [105]. These displacements should be measured about the trim angle and half a cycle apart, i.e., $t_2 - t_1 = \frac{T_p}{2}$, as shown in Fig. A.1.10 below.

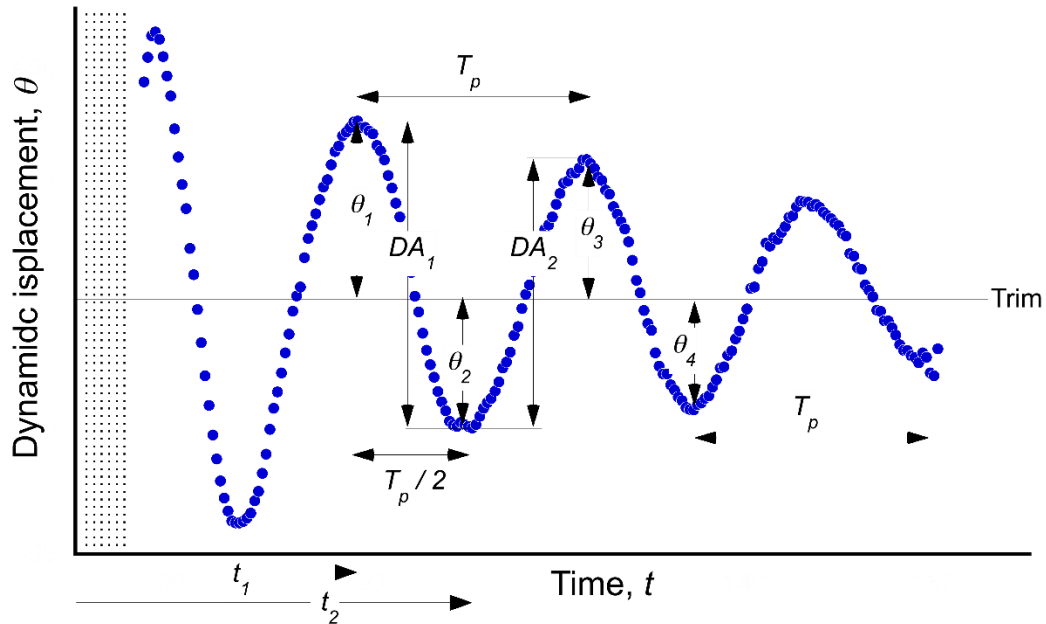


Fig. A1.10 “Peak-to-Valley Method” to determine oscillatory frequency and damping [1].

t_1 and t_2 are exactly one-half cycle apart. T_p is the cycle time or period of the dynamic response. The average is called the Transient Peak Ratio or TPR method.

$$TPR = \frac{1}{N} \sum_{i=2}^N \frac{\theta_i}{\theta_{i-1}} \quad (A1.39)$$

$$TPR = \frac{2}{N} \sum_{i=1}^N \frac{DA_i}{DA_{i-1}} \quad (A1.40)$$

where N is the number of ratios formed.

the logarithmic decrement can be written as

$$\delta = \ln(TPR) = \zeta \omega_n \left(\frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \right) = \frac{\pi \zeta}{\sqrt{1 - \zeta^2}} \quad (A1.41)$$

$$\zeta = \left| \frac{\delta}{\sqrt{\pi^2 + \delta^2}} \right| \quad (A1.42)$$

An average of the period for one cycle of the long-period dynamic response can be found using

$$T_{pav} = \frac{1}{N} \sum_{i=1}^N T_{p_i} \quad (A1.44)$$

and the average value of the damped frequency is found from

$$\omega_{d_{av}} = \frac{2\pi}{T_{pav}} \quad (A1.45)$$

Finally, the undamped frequency of the dynamic motion can be found using

$$\omega_n = \frac{\omega_{d_{av}}}{\sqrt{1 - \zeta^2}} \quad (A1.46)$$

APPENDIX A2

DERIVATION OF TWO BODY DYNAMIC MODELLING FOR HYBRID AIRSHIP

PAYLOAD SUSBSYSTEM:

Translational equations of motion of payload center of mass in payload body-fixed reference frame can be expressed as:

$$M_b \dot{V}_b + \Omega_b \times M_b V_b = F_b^A + F_b^S - F_b^C \quad (A2.1)$$

Where V_b and $\dot{V}_b$ are inertial reference frame quantities:

$$V_b = T_b V_c + \Omega_b \times R_{cb} \quad (A2.2)$$

$$\dot{V}_b = \dot{T}_b V_c + T_b \dot{V}_c + \dot{\Omega}_b \times R_{cb} + \Omega_b \dot{R}_{cb} \quad (A2.3)$$

And M_b is the payload mass matrix:

$$M_b = m_b I = \begin{bmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_b \end{bmatrix}$$

Matrix T_b is the transformation matrix from an inertial reference frame to the payload reference frame:

$$T_b =$$

$$\begin{bmatrix} \cos \theta_b \cos \psi_b & \cos \theta_b \sin \psi_b & -\sin \theta_b \\ \sin \phi_b \sin \theta_b \cos \psi_b - \cos \phi_b \sin \psi_b & \sin \phi_b \sin \theta_b \sin \psi_b + \cos \phi_b \cos \psi_b & \sin \phi_b \cos \theta_b \\ \cos \phi_b \sin \theta_b \cos \psi_b + \sin \phi_b \sin \psi_b & \cos \phi_b \sin \theta_b \sin \psi_b - \sin \phi_b \cos \psi_b & \cos \phi_b \cos \theta_b \end{bmatrix} \quad (A2.4)$$

$(\Omega_b \times)$ is cross product equivalent matrix for payload body axis angular velocity:

$$\Omega_b \times = \begin{bmatrix} 0 & -r_b & q_b \\ r_b & 0 & -p_b \\ -q_b & p_b & 0 \end{bmatrix} \quad (\text{A2.5})$$

And $(R_{cb} \times)$ is cross product equivalent matrix of components of distance of payload mass center b from joint C in payload body-fixed reference frame:

$$R_{cb} \times = \begin{bmatrix} 0 & -z_{cb} & y_{cb} \\ z_{cb} & 0 & -x_{cb} \\ -y_{cb} & x_{cb} & 0 \end{bmatrix} \quad (\text{A2.6})$$

Here $x_{cb} = 0$, $y_{cb} = 0$ and $z_{cb} = R_{cb}$ due to choice of reference frame. The distance between joint C and payload mass center is constant, i.e., $\dot{R}_{cb} = 0$, and $\dot{T}_b = -\Omega_b \times T_b$, hence

$$\dot{V}_b = (-\Omega_b \times T_b)V_c + T_b\dot{V}_c + \dot{\Omega}_b \times R_{cb} \quad (\text{A2.7})$$

Substituting V_b and $\dot{V}_b$ from Eqn. (A2.2) and (A2.7) in Eqn. (A2.1), we get

$$M_b((-\Omega_b \times T_b)V_c + T_b\dot{V}_c + \dot{\Omega}_b \times R_{cb}) + \Omega_b \times M_b(T_bV_c + \Omega_b \times R_{cb}) = F_b^A + F_b^S - F_b^C \quad (\text{A2.8})$$

$$M_bT_b\dot{V}_c + M_b\dot{\Omega}_b \times R_{cb} + \Omega_b \times M_b\Omega_b \times R_{cb} = F_b^A + F_b^S - F_b^C \quad (\text{A2.9})$$

Or,

$$M_b\dot{\Omega}_b \times R_{cb} + 0 * \dot{\Omega}_h + M_bT_b\dot{V}_c + F_b^C = F_b^A + F_b^S - \Omega_b \times M_b\Omega_b \times R_{cb}$$

$$M_b\dot{\Omega}_b \times R_{cb} + 0 * \dot{\Omega}_h + M_bT_b\dot{V}_c + T_bF_C = B1 \quad (\text{A2.9a})$$

The rotational motion of payload center of mass in payload fixed reference frame is described by:

$$I_b\dot{\Omega}_b + \Omega_b \times I_b\Omega_b = R_{cb} \times F_b^C \quad (\text{A2.10})$$

Where

$$I_b = \begin{bmatrix} I_{xb} & -I_{xyb} & -I_{xzb} \\ -I_{yxb} & I_{yb} & -I_{yzb} \\ -I_{zxb} & -I_{zyb} & I_{zb} \end{bmatrix}$$

PAYLOAD FORCES AND MOMENTS:

The external forces acting on payload sub model are:

1. Aerodynamic Force: The aerodynamic drag (C_D^b) force acting at payload mass center in payload reference frame is

$$\begin{aligned} \text{Aerodynamic force } (F_a) \text{ for payload box} &= F_b^A \\ &= \bar{q}_b S_b \begin{bmatrix} C_{\alpha_b} C_D^b \\ 0 \\ S_{\alpha_b} C_D^b \end{bmatrix} \end{aligned} \quad (\text{A2.11})$$

where $\bar{q}_b = \frac{1}{2} \rho V_b^2$ is dynamic pressure, S_b is payload box frontal area, and $\alpha_b = \tan^{-1}(\frac{w_b}{u_b})$ is the local angle of attack of payload center of mass.

2. Static Force (F_s) : It includes Buoyancy force and Gravitational force at payload mass center in payload reference frame.

$$\begin{aligned} \text{Static force } (F_s) \text{ for payload box} \\ = F_b^S = (m_b g - B) \begin{bmatrix} -S_{\theta_b} \\ S_{\phi_b} C_{\theta_b} \\ C_{\phi_b} C_{\theta_b} \end{bmatrix} \end{aligned} \quad (\text{A2.12})$$

Where g is acceleration due to gravity, B (buoyancy force) = $\rho V_h g$, $V_h = \frac{2}{3} \pi L \left(\frac{D}{2}\right)^2$, L is hull length and D is hull maximum diameter.

3. Joint Force: The inertial joint C reaction forces expressed in payload reference are

$$F_b^C = \begin{bmatrix} C_{\theta_b} C_{\psi_b} & C_{\theta_b} S_{\psi_b} & -S_{\theta_b} \\ S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b} & S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b} & S_{\phi_b} C_{\theta_b} \\ S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b} & C_{\phi_b} S_{\theta_b} S_{\psi_b} - S_{\phi_b} C_{\psi_b} & C_{\phi_b} C_{\theta_b} \end{bmatrix} \begin{bmatrix} F_{cx} \\ F_{cy} \\ F_{cz} \end{bmatrix} \quad (\text{A2.13})$$

HYBRID AIRSHIP SUSBSYSTEM:

Translational equations of motion of payload center of mass in hybrid airship body-fixed reference frame can be expressed as:

$$(M_h + M_F) \dot{V}_h + \Omega_h \times (M_h + M_F) V_h = F_h^a + F_h^s + F_h^C + F_b^T \quad (\text{A2.14})$$

Where V_h and $\dot{V}_h$ are inertial reference frame quantities:

$$V_h = T_h V_c + \Omega_h \times R_{CP} \quad (\text{A2.15})$$

$$\dot{V}_h = \dot{T}_h V_c + T_h \dot{V}_c + \dot{\Omega}_h \times R_{CP} + \Omega_h \dot{R}_{CP} \quad (\text{A2.16})$$

And M_h and M_F are hybrid airship rigid and apparent mass matrices, respectively:

$$M_h = m_h I = \begin{bmatrix} m_h & 0 & 0 \\ 0 & m_h & 0 \\ 0 & 0 & m_h \end{bmatrix} \quad (\text{A2.17})$$

$$M_F = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & 0 \\ 0 & 0 & -Z_{\dot{w}} \end{bmatrix} \quad (\text{A2.18})$$

Matrix T_h is the transformation matrix from an inertial reference frame to the hybrid airship body-fixed reference frame:

$$T_h = \begin{bmatrix} \cos \theta_h \cos \psi_h & \cos \theta_h \sin \psi_h & -\sin \theta_h \\ \sin \phi_h \sin \theta_h \cos \psi_h - \cos \phi_h \sin \psi_h & \sin \phi_h \sin \theta_h \sin \psi_h + \cos \phi_h \cos \psi_h & \sin \phi_h \cos \theta_h \\ \cos \phi_h \sin \theta_h \cos \psi_h + \sin \phi_h \sin \psi_h & \cos \phi_h \sin \theta_h \sin \psi_h - \sin \phi_h \cos \psi_h & \cos \phi_h \cos \theta_h \end{bmatrix} \quad (\text{A2.19})$$

$(\Omega_h \times)$ is cross product equivalent matrix for payload body axis angular velocity:

$$\Omega_h \times = \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \quad (\text{A2.20})$$

And $(R_{CP} \times)$ is cross- product -equivalent matrix of components of distance of hybrid airship mass center P from joint C in joint C-fixed reference frame:

$$R_{CP} \times = \begin{bmatrix} 0 & -z_{cp} & y_{cp} \\ z_{cp} & 0 & -x_{cp} \\ -y_{cp} & x_{cp} & 0 \end{bmatrix} \quad (\text{A2.21})$$

The distance between joint C and hybrid airship mass center is constant, i.e. ,

$\dot{R}_{Cb} = 0$, and $\dot{T}_h = -\Omega_h \times T_h$, hence

$$\dot{V}_h = (-\Omega_h \times T_h)V_c + T_h\dot{V}_c + \dot{\Omega}_h \times R_{CP} \quad (\text{A2.22})$$

Substituting V_h and $\dot{V}_h$ from Eqn. (A2.15) and (A2.16) in Eqn. (A2.14), we get

$$(M_h + M_F) \left((-\Omega_h \times T_h)V_c + T_h\dot{V}_c + \dot{\Omega}_h \times R_{CP} \right) + \Omega_h \times ((M_h + M_F))(T_hV_c + \Omega_h \times R_{CP}) = F_h^A + F_h^S + F_h^C + F_h^T \quad (\text{A2.23})$$

$$(M_h + M_F)T_h\dot{V}_c + (M_h + M_F)\dot{\Omega}_h \times R_{CP} - (M_h + M_F)\Omega_h \times T_hV_c + \Omega_h \times (M_h + M_F)T_hV_c + \Omega_h \times (M_h + M_F)\Omega_h \times R_{CP} = F_h^A + F_h^S + F_h^C + F_h^T \quad (\text{A2.24})$$

The rotational motion of hybrid airship center of mass in hybrid airship body-fixed reference frame is described by:

$$(I_h + I_M)\dot{\Omega}_h + \Omega_h \times (I_h + I_M)\Omega_h = M_h^A - R_{CP} \times F_h^C + M_h^S + M_h^T \quad (\text{A2.25})$$

$$\text{where, } I_h = \begin{bmatrix} I_{xx} & 0 & -I_{zx} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}_h \quad \text{and} \quad I_M = I'_0 = \begin{bmatrix} -L_{\dot{p}} & 0 & -N_{\dot{p}} \\ 0 & -M_{\dot{q}} & 0 \\ -L_{\dot{r}} & 0 & -N_{\dot{r}} \end{bmatrix}$$

$$I_h + I_M = \begin{bmatrix} I_{xx} - L\dot{p} & 0 & -(I_{zx} - N\dot{p}) \\ 0 & I_{yy} - M\dot{q} & 0 \\ -(I_{xz} - L\dot{r}) & 0 & I_{zz} - N\dot{r} \end{bmatrix}$$

HYBRID AIRSHIP FORCES AND MOMENTS:

The external forces and moments acting at hybrid airship sub model mass center are:

1. Aerodynamic Force: The aerodynamic forces acting at hybrid airship mass center in hybrid airship body-fixed reference frame are:

$$\text{Aerodynamic force } (F_a) \text{ for hybrid airship} = F_h^A = \bar{q}_p S_p \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} \quad (\text{A2.26})$$

where $\bar{q}_h = \frac{1}{2} \rho V_h^2$ is dynamic pressure, S_h is hybrid airship (hull with wing) reference area, and $\alpha_h = \tan^{-1}(\frac{w_h}{u_h})$ is hybrid airship angle of attack and C_X , C_Y , C_Z are components of aerodynamic force in hybrid airship fixed reference frame.

2. Aerodynamic Moments: The aerodynamic moments acting at hybrid airship mass center in hybrid airship frame are:

$$\text{Aerodynamic moment } (M_a) \text{ for hybrid airship} = M_h^A = \bar{q}_h S_h \begin{bmatrix} bC_l \\ cC_m \\ bC_n \end{bmatrix} \quad (\text{A2.27})$$

3. Static Force (F_s) : It includes Buoyancy force and Gravitational force at hybrid airship mass center as expressed in hybrid airship reference frame.

$$\text{Static force } (F_s) \text{ for hybrid airship} = F_h^S = \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}_h = (m_h g - B) \begin{bmatrix} -S_{\theta_h} \\ S_{\phi_h} C_{\theta_h} \\ C_{\phi_h} C_{\theta_h} \end{bmatrix}$$

(A2.28)

4. Static Moments: Static moment acting on center of buoyancy of hybrid airship.

Let $r_G = (a_x, a_y, a_z)$ represents the coordinate location of the center of gravity with respect to origin, and $r_B = (b_x, b_y, b_z)$ represents the coordinate location of the center of buoyancy with respect to the origin.

$$\text{Static moments for hybrid airship} = \begin{bmatrix} L_s \\ M_s \\ N_s \end{bmatrix}_h \quad (\text{A2.29})$$

Where,

$$L_s = (mga_z + Bb_z) \sin \phi_h \cos \theta_h$$

$$M_s = -(mga_z + Bb_z) \sin \theta_h - (mga_x + Bb_x) \cos \phi_h \cos \theta_h$$

$$N_s = (mga_x + Bb_x) \sin \phi_h \cos \theta_h$$

5. Joint Force: The components of inertial joint C reaction forces expressed in hybrid airship reference frame are:

$$F_h^C = \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix} \begin{bmatrix} F_{cx} \\ F_{cy} \\ F_{cz} \end{bmatrix} \quad (\text{A2.30})$$

6. Propulsion/Thrust force and moment: The thrust is parallel to the airship's reference line.

The simplified version of the equation for propulsion system of hybrid airship is given in Eqn. (A2.31).

$$M_P = M_T = T d_z \quad (\text{A2.31a})$$

$$Y_p = Z_p = L_p = N_p = 0 \quad (\text{A2.31b})$$

$$X_p = X_T = T \quad (\text{A2.31c})$$

Where M_T is the moment due to propulsion system or thrust, $X_p = X_T = T$ is the thrust in axial direction, $Z_T=0$ is the thrust in normal direction. T is total thrust generated by the engine and d_z is the distance of propulsion system from CV.

9DOF MODEL OF HYBRID AIRSHIP:

$$\begin{bmatrix} -M_b R_{Cb} & 0 & M_b T_b & T_b \\ 0 & -(M_h + M_F) R_{ch} (M_h + M_F) T_h & -T_h & \\ I_b & 0 & 0 & -R_{Cb} T_b \\ 0 & I_h + I_M & 0 & R_{ch} T_h \end{bmatrix} \begin{bmatrix} \dot{\Omega}_b \\ \dot{\Omega}_h \\ \dot{V}_c \\ F_c \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \quad (\text{A2.32})$$

Where

$$B_1 = F_b^A + F_b^G - \Omega_b \times M_b \Omega_b \times R_{Cb} \quad (\text{A2.33a})$$

$$B_2 = F_h^A + F_h^B + F_h^G - \Omega_h \times (M_h + M_F) \Omega_h \times R_{ch} + M_F \Omega_h \times T_h V_c - \Omega_h \times M_F T_h V_c \quad (\text{A2.33b})$$

$$B_3 = -\Omega_b \times I_b \Omega_b \quad (\text{A2.33c})$$

$$B_4 = M_h^A - \Omega_h \times (I_h + I_M) \Omega_h \quad (\text{A2.33d})$$

Eqn. (A2.9) is written here for simplification purpose.

$$M_b T_b \dot{V}_c + M_b \dot{\Omega}_b \times R_{Cb} + \Omega_b \times M_b \Omega_b \times R_{Cb} = F_b^A + F_b^S - F_b^C \quad (\text{A2.9})$$

Eqn. (A2.9) is rearranged according to sequence $(\dot{\Omega}_b, \dot{\Omega}_h, \dot{V}_c \text{ and } F_c)$

$$M_b \dot{\Omega}_b \times R_{Cb} + 0 * \dot{\Omega}_h + M_b T_b \dot{V}_c + F_b^C = F_b^A + F_b^S - \Omega_b \times M_b \Omega_b \times R_{Cb} \quad (\text{A2.32})$$

$$B_1 = F_b^A + F_b^S - \Omega_b \times M_b \Omega_b \times R_{Cb} \quad (\text{A2.33})$$

So,

$$M_b \dot{\Omega}_b \times R_{Cb} + 0 * \dot{\Omega}_h + M_b T_b \dot{V}_c + F_b^C = B1$$

Or

$$M_b \dot{\Omega}_b \times R_{cb} + 0 * \dot{\Omega}_h + M_b T_b \dot{V}_c + T_b F_C = B_1 \quad (\text{A2.34})$$

Similarly, Eqn. (A2.24) is written for simplification.

$$(M_h + M_F) T_h \dot{V}_c + (M_h + M_F) \dot{\Omega}_h \times R_{CP} - (M_h + M_F) \Omega_h \times T_h V_c + \Omega_h \times (M_h + M_F) T_h V_c + \Omega_h \times (M_h + M_F) \Omega_h \times R_{CP} = F_h^A + F_h^S + F_h^C + F_h^T$$

Eqn. (A2.24) is rearranged according to sequence $(\dot{\Omega}_b, \dot{\Omega}_h, \dot{V}_c \text{ and } F_C)$

$$0 * \dot{\Omega}_b + (M_h + M_F) \dot{\Omega}_h \times R_{CP} + (M_h + M_F) T_h \dot{V}_c - T_h F_C = F_h^A + F_h^S + F_h^T + (\textcolor{red}{M_h + M_F}) \Omega_h \times T_h V_c - \Omega_h \times (\textcolor{red}{M_h + M_F}) T_h V_c - \Omega_h \times (M_h + M_F) \Omega_h \times R_{CP} \quad (\text{A2.35})$$

$$B_2 = F_h^A + F_h^S + F_h^T - \Omega_h \times (M_h + M_F) \Omega_h \times R_{CP} + \textcolor{red}{M_F} \Omega_h \times T_h V_c - \Omega_h \times \textcolor{red}{M_F} T_h V_c \quad (\text{A2.36})$$

$$0 * \dot{\Omega}_b + (M_h + M_F) \dot{\Omega}_h \times R_{CP} + (M_h + M_F) T_h \dot{V}_c - T_h F_C = B_2 \quad (\text{A2.37})$$

Eqn. (A2.10) is written for simplification.

$$I_b \dot{\Omega}_b + \Omega_b \times I_b \Omega_b = R_{cb} \times F_b^C$$

Eqn. (A2.10) is rearranged according to sequence $(\dot{\Omega}_b, \dot{\Omega}_h, \dot{V}_c \text{ and } F_C)$

$$I_b \dot{\Omega}_b + 0 * \dot{\Omega}_h + 0 * \dot{V}_c - R_{cb} \times T_b F_C = -\Omega_b \times I_b \Omega_b \quad (\text{A2.38})$$

$$B_3 = -\Omega_b \times I_b \Omega_b \quad (\text{A2.39})$$

$$I_b \dot{\Omega}_b + 0 * \dot{\Omega}_h + 0 * \dot{V}_c - R_{cb} \times T_b F_C = B_3 \quad (\text{A2.40})$$

Eqn. (A2.25) is written for simplification.

$$(I_h + I_M) \dot{\Omega}_h + \Omega_h \times (I_h + I_M) \Omega_h = M_h^A - R_{CP} \times F_h^C + M_h^S + M_h^T$$

Eqn. (A2.25) is rearranged according to sequence $(\dot{\Omega}_b, \dot{\Omega}_h, \dot{V}_c \text{ and } F_C)$

$$0 * \dot{\Omega}_b + (I_h + I_M) \dot{\Omega}_h + 0 * \dot{V}_c + R_{CP} \times T_h F_C = M_h^A + M_h^S + M_h^T - \Omega_h \times (I_h + I_M) \Omega_h \quad (\text{A2.41})$$

$$B_4 = M_h^A + M_h^S + M_h^T - \Omega_h \times (I_h + I_M)\Omega_h \quad (A2.42)$$

$$0 * \dot{\Omega}_b + (I_h + I_M)\dot{\Omega}_h + 0 * \dot{V}_c + R_{CP} \times T_h F_C = B_4 \quad (A2.43)$$

Eqn. (A2.34), (A2.37), (A2.40) and (A2.43) is rewritten here for forming matrix form.

$$M_b \dot{\Omega}_b \times R_{Cb} + 0 * \dot{\Omega}_h + M_b T_b \dot{V}_c + T_b F_C = B_1$$

$$0 * \dot{\Omega}_b + (M_h + M_F)\dot{\Omega}_h \times R_{CP} + (M_h + M_F)T_h \dot{V}_c - T_h F_C = B_2$$

$$I_b \dot{\Omega}_b + 0 * \dot{\Omega}_h + 0 * \dot{V}_c - R_{Cb} \times T_b F_C = B_3$$

$$0 * \dot{\Omega}_b + (I_h + I_M)\dot{\Omega}_h + 0 * \dot{V}_c + R_{CP} \times T_h F_C = B_4$$

Where

$$B_1 = F_b^A + F_b^S - \Omega_b \times M_b \Omega_b \times R_{Cb}$$

$$B_2 = F_h^A + F_h^S + F_h^T - \Omega_h \times (M_h + M_F)\Omega_h \times R_{CP} + M_F \Omega_h \times T_h V_c - \Omega_h \times M_F T_h V_c$$

$$B_3 = -\Omega_b \times I_b \Omega_b$$

$$B_4 = M_h^A + M_h^S + M_h^T - \Omega_h \times (I_h + I_M)\Omega_h$$

9DOF MODEL OF HYBRID AIRSHIP:

$$\begin{bmatrix} -M_b R_{Cb} \times & 0 & M_b T_b & T_b \\ 0 & -(M_h + M_F) R_{CP} \times & (M_h + M_F) T_h & -T_h \\ I_b & 0 & 0 & -R_{Cb} \times T_b \\ 0 & I_h + I_M & 0 & R_{CP} \times T_h \end{bmatrix} \begin{bmatrix} \dot{\Omega}_b \\ \dot{\Omega}_h \\ \dot{V}_c \\ F_C \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$$

(A2.44)

To obtain the nonlinear scalar form of 9DOF model:

ROUGH CALCULATION (Cross product matrix multiplication)

$$I_b = \begin{bmatrix} I_{xb} & -I_{xyb} & -I_{xzb} \\ -I_{yxb} & I_{y b} & -I_{y z b} \\ -I_{zxb} & -I_{zyb} & I_{z b} \end{bmatrix}$$

$$\begin{aligned}
-\mathbf{M}_b \mathbf{R}_{Cb} \times &= - \begin{bmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_b \end{bmatrix} \begin{bmatrix} 0 & -z_{cb} & y_{cb} \\ z_{cb} & 0 & -x_{cb} \\ -y_{cb} & x_{cb} & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & m_b z_{cb} & -m_b y_{cb} \\ -m_b z_{cb} & 0 & m_b x_{cb} \\ m_b y_{cb} & -m_b x_{cb} & 0 \end{bmatrix}
\end{aligned}$$

$$\mathbf{M}_b \mathbf{T}_b$$

$$\begin{aligned}
&= \begin{bmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_b \end{bmatrix} \begin{bmatrix} C_{\theta_b} C_{\psi_b} & C_{\theta_b} S_{\psi_b} & -S_{\theta_b} \\ S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b} & S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b} & S_{\phi_b} C_{\theta_b} \\ S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b} & C_{\phi_b} S_{\theta_b} S_{\psi_b} - S_{\phi_b} C_{\psi_b} & C_{\phi_b} C_{\theta_b} \end{bmatrix} \\
&= \begin{bmatrix} m_b C_{\theta_b} C_{\psi_b} & m_b C_{\theta_b} S_{\psi_b} & -m_b S_{\theta_b} \\ m_b S_{\phi_b} S_{\theta_b} C_{\psi_b} - m_b C_{\phi_b} S_{\psi_b} & m_b S_{\phi_b} S_{\theta_b} S_{\psi_b} + m_b C_{\phi_b} C_{\psi_b} & m_b S_{\phi_b} C_{\theta_b} \\ m_b S_{\phi_b} S_{\theta_b} C_{\psi_b} - m_b C_{\phi_b} S_{\psi_b} & m_b C_{\phi_b} S_{\theta_b} S_{\psi_b} - m_b S_{\phi_b} C_{\psi_b} & m_b C_{\phi_b} C_{\theta_b} \end{bmatrix}
\end{aligned}$$

$$\boldsymbol{\Omega}_b \times \mathbf{M}_b \boldsymbol{\Omega}_b \times \mathbf{R}_{Cb}$$

$$\begin{aligned}
&= \begin{bmatrix} 0 & -r_b & q_b \\ r_b & 0 & -p_b \\ -q_b & p_b & 0 \end{bmatrix} \begin{bmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_b \end{bmatrix} \begin{bmatrix} 0 & -r_b & q_b \\ r_b & 0 & -p_b \\ -q_b & p_b & 0 \end{bmatrix} \begin{bmatrix} x_{Cb} \\ y_{Cb} \\ z_{Cb} \end{bmatrix} \\
&= \begin{bmatrix} 0 & -r_b m_b & q_b m_b \\ r_b m_b & 0 & -p_b m_b \\ -q_b m_b & p_b m_b & 0 \end{bmatrix} \begin{bmatrix} 0 & -r_b & q_b \\ r_b & 0 & -p_b \\ -q_b & p_b & 0 \end{bmatrix} \begin{bmatrix} x_{Cb} \\ y_{Cb} \\ z_{Cb} \end{bmatrix} \\
&= \begin{bmatrix} -r_b^2 m_b - q_b^2 m_b & p_b q_b m_b & p_b r_b m_b \\ p_b q_b m_b & -r_b^2 m_b - p_b^2 m_b & q_b r_b m_b \\ p_b r_b m_b & q_b r_b m_b & -q_b^2 m_b - p_b^2 m_b \end{bmatrix} \begin{bmatrix} x_{Cb} \\ y_{Cb} \\ z_{Cb} \end{bmatrix} \\
&= \begin{bmatrix} -r_b^2 m_b x_{Cb} - q_b^2 m_b x_{Cb} + p_b q_b m_b y_{Cb} + p_b r_b m_b z_{Cb} \\ p_b q_b m_b x_{Cb} - r_b^2 m_b y_{Cb} - p_b^2 m_b y_{Cb} + q_b r_b m_b z_{Cb} \\ p_b r_b m_b x_{Cb} + q_b r_b m_b y_{Cb} - q_b^2 m_b z_{Cb} - p_b^2 m_b z_{Cb} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
-(\mathbf{M}_h + \mathbf{M}_F) \mathbf{R}_{CP} \times &= \begin{bmatrix} m_h - X_{\dot{u}} & 0 & 0 \\ 0 & m_h - Y_{\dot{v}} & 0 \\ 0 & 0 & m_h - Z_{\dot{w}} \end{bmatrix} \begin{bmatrix} 0 & -z_{cp} & y_{cp} \\ z_{cp} & 0 & -x_{cp} \\ -y_{cp} & x_{cp} & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & -z_{cp}(m_h - X_{\dot{u}}) & y_{cp}(m_h - X_{\dot{u}}) \\ z_{cp}(m_h - Y_{\dot{v}}) & 0 & -x_{cp}(m_h - Y_{\dot{v}}) \\ -y_{cp}(m_h - Z_{\dot{w}}) & x_{cp}(m_h - Z_{\dot{w}}) & 0 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& (\mathbf{M}_h + \mathbf{M}_F) \mathbf{T}_h \\
&= \begin{bmatrix} m_h - X_{\dot{u}} & 0 & 0 \\ 0 & m_h - Y_{\dot{v}} & 0 \\ 0 & 0 & m_h - Z_{\dot{w}} \end{bmatrix} \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix} \\
&= \begin{bmatrix} (m_h - X_{\dot{u}}) C_{\theta_h} C_{\psi_h} & (m_h - X_{\dot{u}}) C_{\theta_h} S_{\psi_h} & -(m_h - X_{\dot{u}}) S_{\theta_h} \\ (m_h - Y_{\dot{v}}) (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) & (m_h - Y_{\dot{v}}) (S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h}) & (m_h - Y_{\dot{v}}) S_{\phi_h} C_{\theta_h} \\ (m_h - Z_{\dot{w}}) (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) & (m_h - Z_{\dot{w}}) (C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h}) & (m_h - Z_{\dot{w}}) C_{\phi_h} C_{\theta_h} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& \boldsymbol{\Omega}_h \times (\mathbf{M}_h + \mathbf{M}_F) \boldsymbol{\Omega}_h \times \mathbf{R}_{ch} \\
&= \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \begin{bmatrix} m_h - X_{\dot{u}} & 0 & 0 \\ 0 & m_h - Y_{\dot{v}} & 0 \\ 0 & 0 & m_h - Z_{\dot{w}} \end{bmatrix} \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \begin{bmatrix} x_{CP} \\ y_{CP} \\ z_{CP} \end{bmatrix} \\
&= \begin{bmatrix} 0 & -r_h(m_h - Y_{\dot{v}}) & q_h(m_h - Z_{\dot{w}}) \\ r_h(m_h - X_{\dot{u}}) & 0 & -p_h(m_h - Z_{\dot{w}}) \\ -q_h(m_h - X_{\dot{u}}) & p_h(m_h - Y_{\dot{v}}) & 0 \end{bmatrix} \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \begin{bmatrix} x_{CP} \\ y_{CP} \\ z_{CP} \end{bmatrix} \\
&= \\
&\begin{bmatrix} -r_h^2(m_h - Y_{\dot{v}}) - q_h^2(m_h - Z_{\dot{w}}) & p_h q_h(m_h - Z_{\dot{w}}) & p_h r_h(m_h - Y_{\dot{v}}) \\ p_h q_h(m_h - Z_{\dot{w}}) & -r_h^2(m_h - X_{\dot{u}}) - p_h^2(m_h - Z_{\dot{w}}) & q_h r_h(m_h - X_{\dot{u}}) \\ p_h r_h(m_h - Y_{\dot{v}}) & q_h r_h(m_h - X_{\dot{u}}) & -q_h^2(m_h - X_{\dot{u}}) - p_h^2(m_h - Y_{\dot{v}}) \end{bmatrix} \begin{bmatrix} x_{CP} \\ y_{CP} \\ z_{CP} \end{bmatrix} \\
&= \begin{bmatrix} -r_h^2(m_h - Y_{\dot{v}}) x_{CP} - q_h^2(m_h - Z_{\dot{w}}) x_{CP} + p_h q_h(m_h - Z_{\dot{w}}) y_{CP} + p_h r_h(m_h - Y_{\dot{v}}) z_{CP} \\ p_h(m_h - Z_{\dot{w}}) x_{CP} - r_h^2(m_h - X_{\dot{u}}) y_{CP} - p_h^2(m_h - Z_{\dot{w}}) y_{CP} + q_h r_h(m_h - X_{\dot{u}}) z_{CP} \\ p_h r_h(m_h - Y_{\dot{v}}) x_{CP} + q_h r_h(m_h - X_{\dot{u}}) y_{CP} - q_h^2(m_h - X_{\dot{u}}) z_{CP} - p_h^2(m_h - Y_{\dot{v}}) z_{CP} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& \mathbf{M}_F \boldsymbol{\Omega}_h \times \mathbf{T}_h \mathbf{V}_c \\
&= \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & 0 \\ 0 & 0 & -Z_{\dot{w}} \end{bmatrix} \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix} \begin{bmatrix} u_C \\ v_C \\ w_C \end{bmatrix} \\
&= \\
&\begin{bmatrix} 0 & r_h X_{\dot{u}} & -q_h X_{\dot{u}} \\ -r_h Y_{\dot{v}} & 0 & p_h Y_{\dot{v}} \\ q_h Z_{\dot{w}} & -p_h Z_{\dot{w}} & 0 \end{bmatrix} \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix} \begin{bmatrix} u_C \\ v_C \\ w_C \end{bmatrix} \\
&= \\
&[-r_h Y_{\dot{v}} C_{\theta_h} C_{\psi_h} + p_h Y_{\dot{v}} (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})] u_C + [-r_h Y_{\dot{v}} C_{\theta_h} S_{\psi_h} + \\
&p_h Y_{\dot{v}} (C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h})] v_C + [r_h Y_{\dot{v}} S_{\theta_h} + p_h Y_{\dot{v}} C_{\phi_h} C_{\theta_h}] w_C
\end{aligned}$$

$$\mathbf{\Omega}_h \times \mathbf{M}_F \mathbf{T}_h \mathbf{V}_c$$

$$= \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \begin{bmatrix} -\dot{X}_u & 0 & 0 \\ 0 & -\dot{Y}_v & 0 \\ 0 & 0 & -\dot{Z}_w \end{bmatrix} \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 0 & r_h \dot{Y}_v & -q_h \dot{Z}_w \\ -r_h \dot{X}_u & 0 & p_h \dot{Z}_w \\ q_h \dot{X}_u & -p_h \dot{Y}_v & 0 \end{bmatrix} \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix}$$

$$= [-r_h \dot{X}_u C_{\theta_h} C_{\psi_h} + p_h \dot{Z}_w (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})] u_c + [-r_h \dot{X}_u C_{\theta_h} S_{\psi_h} + p_h \dot{Z}_w (C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h})] v_c + [r_h \dot{X}_u S_{\theta_h} + p_h \dot{Z}_w C_{\phi_h} C_{\theta_h}] w_c$$

$$\mathbf{R}_{Cb} \times \mathbf{T}_b$$

$$= \begin{bmatrix} 0 & -z_{cb} & y_{cb} \\ z_{cb} & 0 & -x_{cb} \\ -y_{cb} & x_{cb} & 0 \end{bmatrix} \begin{bmatrix} C_{\theta_b} C_{\psi_b} & C_{\theta_b} S_{\psi_b} & -S_{\theta_b} \\ S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b} & S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b} & S_{\phi_b} C_{\theta_b} \\ S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b} & C_{\phi_b} S_{\theta_b} S_{\psi_b} - S_{\phi_b} C_{\psi_b} & C_{\phi_b} C_{\theta_b} \end{bmatrix}$$

$$[-(S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) z_{cb} + (S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) y_{cb}] \dot{u}_c +$$

$$[-(S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b}) z_{cb} + (C_{\phi_b} S_{\theta_b} S_{\psi_b} - S_{\phi_b} C_{\psi_b}) y_{cb}] \dot{v}_c + [-S_{\phi_b} C_{\theta_b} z_{cb} + C_{\phi_b} C_{\theta_b} y_{cb}] \dot{w}_c$$

$$[-y_{cb} C_{\theta_b} C_{\psi_b} + (S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) x_{cb}] \dot{u}_c + [-y_{cb} C_{\theta_b} S_{\psi_b} + x_{cb} (S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b})] \dot{v}_c + [y_{cb} S_{\theta_b} + x_{cb} S_{\phi_b} C_{\theta_b}] \dot{w}_c$$

$$\mathbf{\Omega}_b \times \mathbf{I}_b \mathbf{\Omega}_b = \begin{bmatrix} 0 & -r_b & q_b \\ r_b & 0 & -p_b \\ -q_b & p_b & 0 \end{bmatrix} \begin{bmatrix} I_{xb} & -I_{xyb} & -I_{xzb} \\ -I_{yxb} & I_{y b} & -I_{yzb} \\ -I_{zxb} & -I_{zyb} & I_{zb} \end{bmatrix} \begin{bmatrix} p_b \\ q_b \\ r_b \end{bmatrix}$$

$$[I_{yxb} r_b - I_{zxb} q_b] p_b + [-I_{y b} r_b - I_{zyb} q_b] q_b + [I_{yzb} r_b + I_{zb} q_b] r_b$$

$$[-I_{xb} q_b - I_{yxb} p_b] p_b + [I_{xyb} q_b + I_{y b} p_b] q_b + [I_{xzb} q_b + I_{yzb} p_b] r_b$$

$$\mathbf{R}_{CP} \times \mathbf{T}_h$$

$$= \begin{bmatrix} 0 & -z_{cp} & y_{cp} \\ z_{cp} & 0 & -x_{cp} \\ -y_{cp} & x_{cp} & 0 \end{bmatrix} \begin{bmatrix} C_{\theta_h} C_{\psi_h} & C_{\theta_h} S_{\psi_h} & -S_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h} & S_{\phi_h} C_{\theta_h} \\ S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h} & C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h} & C_{\phi_h} C_{\theta_h} \end{bmatrix}$$

$$[-(S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})z_{CP} + (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})y_{CP}]F_{Cx} +$$

$$[-(S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h})z_{CP} + (C_{\phi_h} S_{\theta_h} S_{\psi_h} -$$

$$S_{\phi_h} C_{\psi_h})y_{CP}]F_{Cy} + [-S_{\phi_h} C_{\theta_h} z_{CP} + C_{\phi_h} C_{\theta_h} y_{CP}]F_{Cz}$$

$$[-y_{CP} C_{\theta_h} C_{\psi_h} + (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})x_{CP}]F_{Cx} + [-y_{CP} C_{\theta_h} S_{\psi_h} +$$

$$x_{CP} (S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h})]F_{Cy} + [y_{CP} S_{\theta_h} + x_{CP} S_{\phi_h} C_{\theta_h}]F_{Cz}$$

$$\mathbf{\Omega}_h \times (\mathbf{I}_h + \mathbf{I}_M) \mathbf{\Omega}_h$$

$$= \begin{bmatrix} 0 & -r_h & q_h \\ r_h & 0 & -p_h \\ -q_h & p_h & 0 \end{bmatrix} \begin{bmatrix} I_{xx} - L_{\dot{p}} & 0 & -(I_{zx} - N_{\dot{p}}) \\ 0 & I_{yy} - M_{\dot{q}} & 0 \\ -(I_{xz} - L_{\dot{r}}) & 0 & I_{zz} - N_{\dot{r}} \end{bmatrix} \begin{bmatrix} p_h \\ q_h \\ r_h \end{bmatrix}$$

$$-(I_{xz} - L_{\dot{r}}) q_h p_h - (I_{yy} - M_{\dot{q}}) r_h q_h + (I_{zz} - N_{\dot{r}}) q_h r_h$$

$$-(I_{xx} - L_{\dot{p}}) q_h p_h - (I_{yy} - M_{\dot{q}}) p_h q_h + (I_{zx} - N_{\dot{p}}) q_h r_h$$

To derive 4DOF longitudinal model ,the required scalar form of 9 DOF hybrid Airship:

Sub model Payload box: Force equations: Side force ($\dot{q}_b$):

$$-m_b z_{cb} \dot{p}_b + m_b x_{cb} \dot{r}_b + m_b S_{\phi_b} S_{\theta_b} C_{\psi_b} \dot{u}_c - m_b C_{\phi_b} S_{\psi_b} \dot{u}_c +$$

$$m_b S_{\phi_b} S_{\theta_b} S_{\psi_b} \dot{v}_c + m_b C_{\phi_b} C_{\psi_b} \dot{v}_c + m_b S_{\phi_b} C_{\theta_b} \dot{w}_c + S_{\phi_b} S_{\theta_b} C_{\psi_b} F_{Cx} -$$

$$C_{\phi_b} S_{\psi_b} F_{Cx} + S_{\phi_b} S_{\theta_b} S_{\psi_b} F_{Cy} + C_{\phi_b} C_{\psi_b} F_{Cy} + S_{\phi_b} C_{\theta_b} F_{Cz} = 0 + (m_b g -$$

$$B) S_{\phi_b} C_{\theta_b} + p_b q_b m_b x_{cb} - r_b^2 m_b y_{cb} - p_b^2 m_b y_{cb} + q_b r_b m_b z_{cb}$$

Sub model hybrid Airship: Force equations: Side force ($\dot{q}_h$):

$$z_{cp} (m_h - Y_{\dot{v}}) \dot{p}_h - x_{cp} (m_h - Y_{\dot{v}}) \dot{r}_h + (m_h - Y_{\dot{v}}) (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) \dot{u}_c +$$

$$(m_h - Y_{\dot{v}}) (S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h}) \dot{v}_c + (m_h - Y_{\dot{v}}) S_{\phi_h} C_{\theta_h} \dot{w}_c - (S_{\phi_h} S_{\theta_h} C_{\psi_h} -$$

$$C_{\phi_h} S_{\psi_h}) F_{Cx} - (S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h}) F_{Cy} - S_{\phi_h} C_{\theta_h} F_{Cz} = \bar{q}_p S_p C_y + (m_h g -$$

$$B) S_{\phi_h} C_{\theta_h} - p_h (m_h - Z_{\dot{w}}) x_{cp} r_h^2 + (m_h - X_{\dot{u}}) y_{cp} + p_h^2 (m_h - Z_{\dot{w}}) y_{cp} -$$

$$q_h r_h (m_h - X_{\dot{u}}) z_{cp} + [-r_h Y_{\dot{v}} C_{\theta_h} C_{\psi_h} + p_h Y_{\dot{v}} (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})] u_c +$$

$$\begin{aligned}
& [-r_h Y_{\dot{v}} C_{\theta_h} S_{\psi_h} + p_h Y_{\dot{v}} (C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h})] v_c + [r_h Y_{\dot{v}} S_{\theta_h} + \\
& p_h Y_{\dot{v}} C_{\phi_h} C_{\theta_h}] w_c + [-r_h X_{\dot{u}} C_{\theta_h} C_{\psi_h} + p_h Z_{\dot{w}} (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h})] u_c + \\
& [-r_h X_{\dot{u}} C_{\theta_h} S_{\psi_h} + p_h Z_{\dot{w}} (C_{\phi_h} S_{\theta_h} S_{\psi_h} - S_{\phi_h} C_{\psi_h})] v_c + [r_h X_{\dot{u}} S_{\theta_h} + \\
& p_h Z_{\dot{w}} C_{\phi_h} C_{\theta_h}] w_c
\end{aligned}$$

Sub model Payload box: Moment equations: Axial ($\dot{u}_c$):

$$\begin{aligned}
& I_{xb} \dot{p}_b - I_{xyb} \dot{q}_b - I_{xzb} \dot{r}_b + [(S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) z_{cb} - (S_{\phi_b} S_{\theta_b} C_{\psi_b} - \\
& C_{\phi_b} S_{\psi_b}) y_{cb}] \dot{u}_c - [-(S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b}) z_{cb} + (C_{\phi_b} S_{\theta_b} S_{\psi_b} - \\
& S_{\phi_b} C_{\psi_b}) y_{cb}] \dot{v}_c - [-S_{\phi_b} C_{\theta_b} z_{cb} + C_{\phi_b} C_{\theta_b} y_{cb}] \dot{w}_c = -[I_{yxb} r_b - I_{zxb} q_b] p_b - \\
& [-I_{ybr} - I_{zyb} q_b] q_b - [I_{yzb} r_b + I_{zbr} q_b] r_b
\end{aligned}$$

Normal ($\dot{w}_c$):

$$\begin{aligned}
& -I_{zxb} \dot{p}_b - I_{zyb} \dot{q}_b + I_{zb} \dot{r}_b + [y_{cb} C_{\theta_b} C_{\psi_b} - (S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) x_{cb}] \dot{u}_c - \\
& [-y_{cb} C_{\theta_b} S_{\psi_b} + x_{cb} (S_{\phi_b} S_{\theta_b} S_{\psi_b} + C_{\phi_b} C_{\psi_b})] \dot{v}_c - [y_{cb} S_{\theta_b} + x_{cb} S_{\phi_b} C_{\theta_b}] \dot{w}_c = - \\
& [-I_{xb} q_b - I_{yxb} p_b] p_b - [I_{xyb} q_b + I_{ybr} p_b] q_b - [I_{xzb} q_b + I_{yzb} p_b] r_b
\end{aligned}$$

Sub model hybrid airship: Moment equations:

Axial (F_{Cx})

$$\begin{aligned}
& (I_{xx} - L_{\dot{p}}) \dot{p}_h - (I_{zx} - N_{\dot{p}}) \dot{r}_h + [-(S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) z_{CP} + (S_{\phi_h} S_{\theta_h} C_{\psi_h} - \\
& C_{\phi_h} S_{\psi_h}) y_{CP}] F_{Cx} + [-(S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h}) z_{CP} + (C_{\phi_h} S_{\theta_h} S_{\psi_h} - \\
& S_{\phi_h} C_{\psi_h}) y_{CP}] F_{Cy} + [-S_{\phi_h} C_{\theta_h} z_{CP} + C_{\phi_h} C_{\theta_h} y_{CP}] F_{Cz} = \bar{q}_h S_h b C_l + (m g a_z + \\
& B b_z) + (I_{xz} - L_{\dot{r}}) q_h p_h + (I_{yy} - M_{\dot{q}}) r_h q_h - (I_{zz} - N_{\dot{r}}) q_h r_h
\end{aligned}$$

Normal (F_{Cz})

$$\begin{aligned}
& -(I_{xz} - L_{\dot{r}}) \dot{p}_h + (I_{zz} - N_{\dot{r}}) \dot{r}_h + [-y_{CP} C_{\theta_h} C_{\psi_h} + (S_{\phi_h} S_{\theta_h} C_{\psi_h} - \\
& C_{\phi_h} S_{\psi_h}) x_{CP}] F_{Cx} + [-y_{CP} C_{\theta_h} S_{\psi_h} + x_{CP} (S_{\phi_h} S_{\theta_h} S_{\psi_h} + C_{\phi_h} C_{\psi_h})] F_{Cy} +
\end{aligned}$$

$$[y_{CP}S_{\theta_h} + x_{CP}S_{\phi_h}C_{\theta_h}]F_{Cz} = \bar{q}_h S_h b C_n + (mga_x + Bb_x) \sin \phi_h \cos \theta_h + (I_{xx} - L_{\dot{p}}) q_h p_h + (I_{yy} - M_{\dot{q}}) p_h q_h - (I_{zz} - N_{\dot{r}}) q_h r_h$$

$$I_h + I_M = \begin{bmatrix} I_{xx} - L_{\dot{p}} & 0 & -(I_{zx} - N_{\dot{p}}) \\ 0 & I_{yy} - M_{\dot{q}} & 0 \\ -(I_{xz} - L_{\dot{r}}) & 0 & I_{zz} - N_{\dot{r}} \end{bmatrix}$$

Longitudinal model 4DOF:

The two body longitudinal dynamics 4DOF model consists of axial force equation ($\dot{u}_c$), normal force equation ($\dot{w}_c$), pitching moment equation for suspended payload box ($\dot{q}_b$), pitching moment equation for hybrid airship ($\dot{q}_h$), Axial (F_{Cx}) and normal (F_{Cz}). All the 9 DOF non-linear equation of motion is given in body axis frame of reference. The longitudinal 4 DOF non-linear equation of motion for two body dynamics of hybrid airship is proposed here by considering the associated terms used for longitudinal dynamics and dropping out the remaining terms in the given 9 DOF nonlinear equation of motion for hybrid airship expressed in scalar form above.

$$\text{Side force } (\dot{q}_b): \quad m_b S_{\phi_b} S_{\theta_b} C_{\psi_b} \dot{u}_c - m_b C_{\phi_b} S_{\psi_b} \dot{u}_c + m_b S_{\phi_b} C_{\theta_b} \dot{w}_c + S_{\phi_b} S_{\theta_b} C_{\psi_b} F_{Cx} - C_{\phi_b} S_{\psi_b} F_{Cx} + S_{\phi_b} C_{\theta_b} F_{Cz} = 0 + (m_b g - B) S_{\phi_b} C_{\theta_b} - p_b^2 m_b y_{cb} \quad (\text{A2.45})$$

Side force ($\dot{q}_h$):

$$(m_h - Y_{\dot{v}})(S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) \dot{u}_c + (m_h - Y_{\dot{v}}) S_{\phi_h} C_{\theta_h} \dot{w}_c - (S_{\phi_h} S_{\theta_h} C_{\psi_h} - C_{\phi_h} S_{\psi_h}) F_{Cx} - S_{\phi_h} C_{\theta_h} F_{Cz} = \bar{q}_p S_p C_y + (m_h g - B) S_{\phi_h} C_{\theta_h} - q_h r_h (m_h - X_{\dot{u}}) z_{CP} \quad (\text{A2.46})$$

Axial ($\dot{u}_c$):

$$-I_{xyb} \dot{q}_b + [(S_{\phi_b} S_{\theta_b} C_{\psi_b} - C_{\phi_b} S_{\psi_b}) z_{cb}] \dot{u}_c - [-S_{\phi_b} C_{\theta_b} z_{cb} + C_{\phi_b} C_{\theta_b} y_{cb}] \dot{w}_c = -[-I_{ybr} r_b - I_{zyb} q_b] q_b \quad (\text{A2.47})$$

Normal ($\dot{w}_c$):

$$\begin{aligned}
& -I_{zyb}\dot{q}_b + [y_{cb}C_{\theta_b}C_{\psi_b} - (S_{\phi_b}S_{\theta_b}C_{\psi_b} - C_{\phi_b}S_{\psi_b})x_{cb}]\dot{u}_c - \\
& [x_{cb}S_{\phi_b}C_{\theta_b}]w_c = [I_{xyb}q_b + I_{ybp_b}]q_b
\end{aligned} \tag{A2.48}$$

Sub model hybrid airship:

Moment equations:

Axial (F_{Cx})

$$\begin{aligned}
& [-(S_{\phi_h}S_{\theta_h}C_{\psi_h} - C_{\phi_h}S_{\psi_h})z_{CP}]F_{Cx} + [-(S_{\phi_h}S_{\theta_h}S_{\psi_h} + \\
& C_{\phi_h}C_{\psi_h})z_{CP} + [-S_{\phi_h}C_{\theta_h}z_{CP} + C_{\phi_h}C_{\theta_h}y_{CP}]F_{Cz} = \bar{q}_h S_h b C_l + (mga_z + Bb_z)
\end{aligned} \tag{A2.49}$$

Normal (F_{Cz})

$$\begin{aligned}
& +[-y_{CP}C_{\theta_h}C_{\psi_h} + (S_{\phi_h}S_{\theta_h}C_{\psi_h} - C_{\phi_h}S_{\psi_h})x_{CP}]F_{Cx} + [y_{CP}S_{\theta_h} + \\
& x_{CP}S_{\phi_h}C_{\theta_h}]F_{Cz} = \bar{q}_h S_h b C_n + (mga_x + Bb_x) \sin \phi_h \cos \theta_h
\end{aligned} \tag{A2.50}$$

After replacing the required terms in Eqns. (A2.45 to A2.50) We get the 4DOF model in matrix form as:

$$\begin{bmatrix}
m_b \cos \theta_b & -m_b \sin \theta_b & m_b R_b & 0 & \cos \theta_b & -\sin \theta_b \\
m_b \sin \theta_b & m_b \cos \theta_b & 0 & 0 & \sin \theta_b & \cos \theta_b \\
0 & 0 & I_b & 0 & -R_b \cos \theta_b & R_b \sin \theta_b \\
m_{pA} \cos \theta_p & -m_{pA} \sin \theta_p & 0 & m_{pA} R_p \cos \mu & -\cos \theta_p & \sin \theta_p \\
m_{pC} \sin \theta_p & m_{pC} \cos \theta_p & 0 & -m_{pC} R_p \sin \mu & -\sin \theta_p & -\cos \theta_p \\
0 & 0 & 0 & I_{PF} & z_{cp} & -x_{cp}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_c \\
\dot{w}_c \\
\dot{q}_b \\
\dot{q}_p \\
F_x \\
F_z
\end{bmatrix} =
\begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4 \\
b_5 \\
b_6
\end{bmatrix} \tag{A2.51}$$

$$b_1 = -m_b g \sin \theta_b - Q_b S_b C_{D_b} \cos \alpha_b \tag{A2.52a}$$

$$b_2 = m_b g \cos \theta_b - Q_b S_b C_{D_b} \sin \alpha_b + m_b R_b q_b^2 \tag{A2.52b}$$

$$b_3 = 0 \quad (\text{A2.52c})$$

$$b_4 = -m_p g \sin \theta_p + Q_p S_p C_x + m_{pc} R_p \sin \mu q_p^2 - (C - A)(u_C \sin \theta_p + w_C \cos \theta_p) q_p \quad (\text{A2.52d})$$

$$b_5 = m_p g \cos \theta_p + Q_p S_p C_z + m_{pA} R_p \cos^\mu q_p^2 - (C - A)(U_C \cos \theta_p - w_C \sin \theta_p) q_p \quad (\text{A2.52e})$$

$$b_6 = -x_{pa} Q_p S_p C_z + Q_p S_{pC} C_M \quad (\text{A2.52f})$$

APPENDIX A3

AERODYNAMIC AND FLIGHT GEOMETRY DATA

Table A3.1. Airship Geometry [28]

Parameter of a HULL	Value
Mass of a Hull, m_h	10 kg
Overall Airship length, l	3.75 m
Maximum Airship Diameter, D	1.6 m
Hull Volume, V	5.6 m^3
Reference Area of a Hull, S_k	3.16 m^2
Reference length of a Hull, c	1.78 m
Ellipsoid shaped semi-minor axis, b	0.8 m
a_1	1.55 m
a_2	2.75 m
Airship Wingspan, b_w	3.06 m
Airship Wing area, S_w	1.72 m^2
Airship tail span, b_t	3.06 m
Airship tail area, S_t	0.916 m^2
R_{ck}	0.8 m
Wing MAC $\bar{c}$ (m)	0.47
Tail MAC $\bar{c}$ (m)	0.51

TABLE A3.2 Payload Geometry

Payload Parameter	Value
mb	2 kg
Sb	0.1x0.2 m ²
Rcb	0.25 m
C _D ^B	1.05

Table A3.3 Aerodynamic data FOR NDI AND 4DOF model

Parameter	Value
C_{l_q}	$-0.0104 (q_h c / 2V_h)^{-1}$
C_{l_p}	$-0.0359 (p_h b / 2V_h)^{-1}$
C_{Y_β}	0.000019 /deg
C_{Y_r}	$-0.000013 (r_h b / 2V_h)^{-1}$
C_{l_β}	-0.0004 /deg
C_{l_p}	$-0.0062 (p_h b / 2V_h)^{-1}$
C_{l_r}	$-0.0000016 (r_h b / 2V_h)^{-1}$
C_{n_β}	0.0047 /deg
C_{n_p}	$-0.0083 (p_h b / 2V_h)^{-1}$
C_{n_r}	$0.0000042 (r_h b / 2V_h)^{-1}$
C_{n_p}	$-0.0083 (p_h b / 2V_h)^{-1}$

Aerodynamic Control derivatives]

Parameter	Value
$C_{L_{\delta_a}}$	0.2350
$C_{Y_{\delta_a}}$	0.1368
$C_{D_{\delta_a}}$	0.0957
$C_{l_{\delta_a}}$	-0.0063
$C_{m_{\delta_a}}$	0.2940
$C_{n_{\delta_a}}$	0.0155

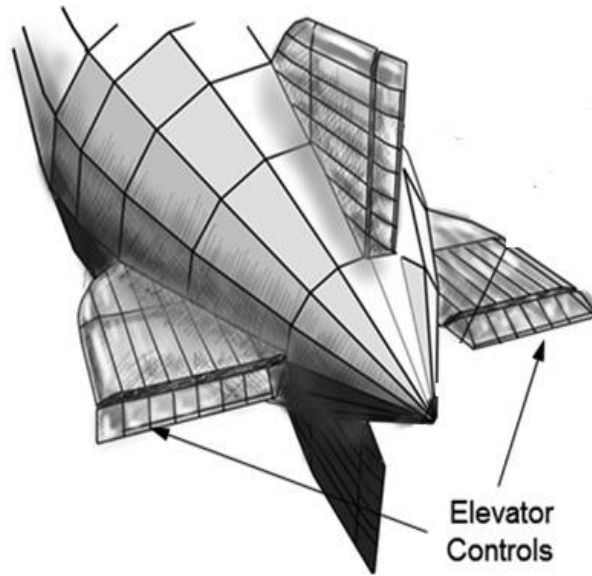


Fig. A3.1 Elevator control on Airship [64]

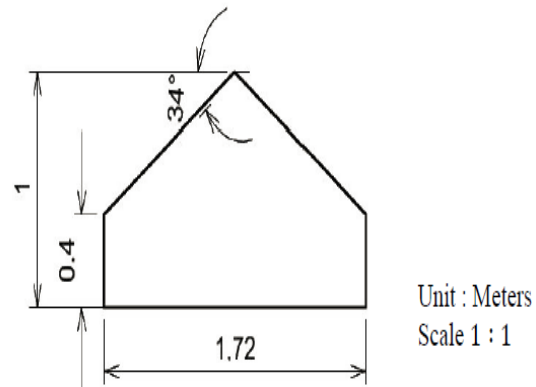
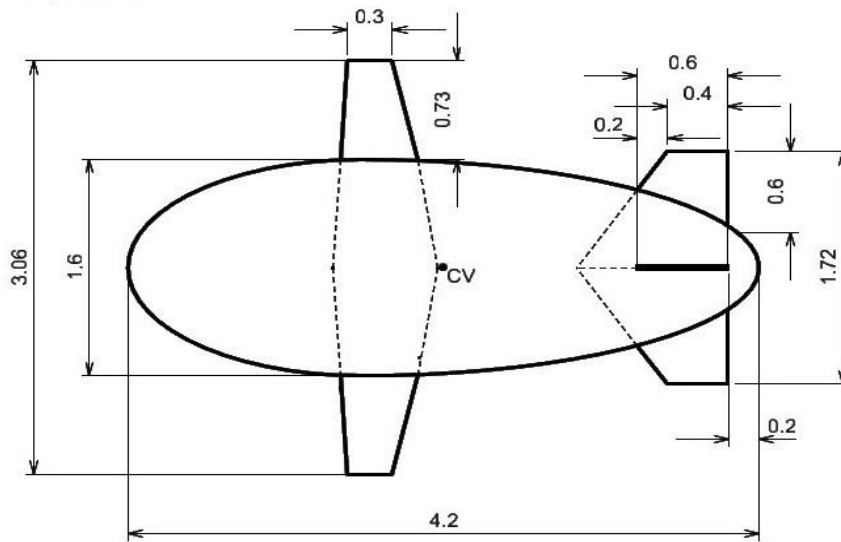
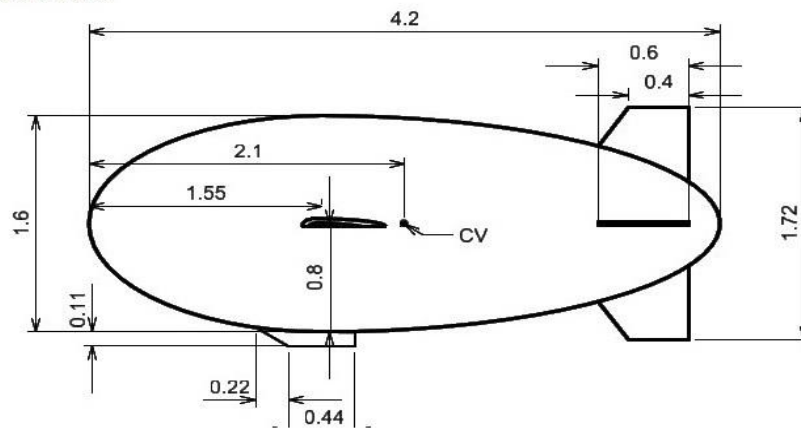


Fig. A3.2 Finite tail geometry of airship.[28]

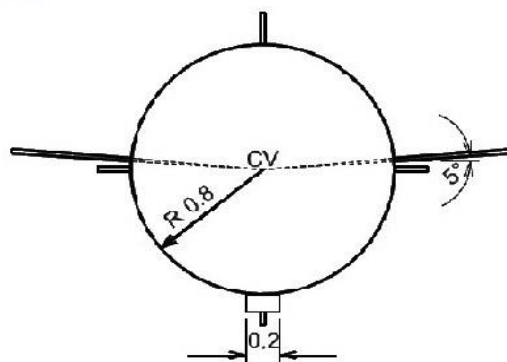
Top View



Side View



Front View



Unit: Meters
Scale 1:1

Fig. A3.3 Airship Complete sketch [28]

AERODYNAMIC DATA

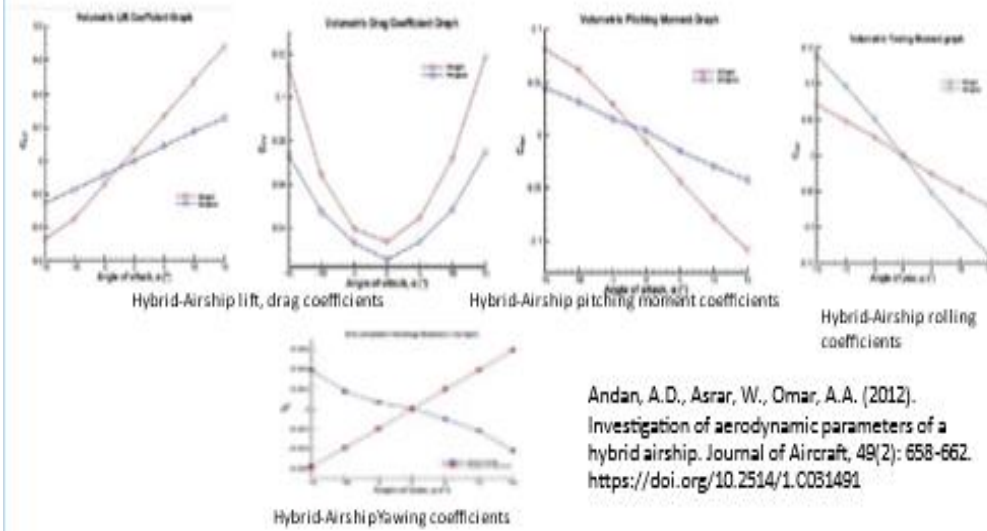


Fig. A3.4 Wind tunnel data (Real flight data) [96]

Table A3.4: Winged hybrid airship operating condition [28]

Cruising Speed, V_0	20 m/s
Mach Number, Ma	0.04
Cruising altitude, h_0	457.2 m
Air density, ρ	1.14 kg/m ³
Static Temperature, T	300 K
Reynolds number, Re	2.16×10^6

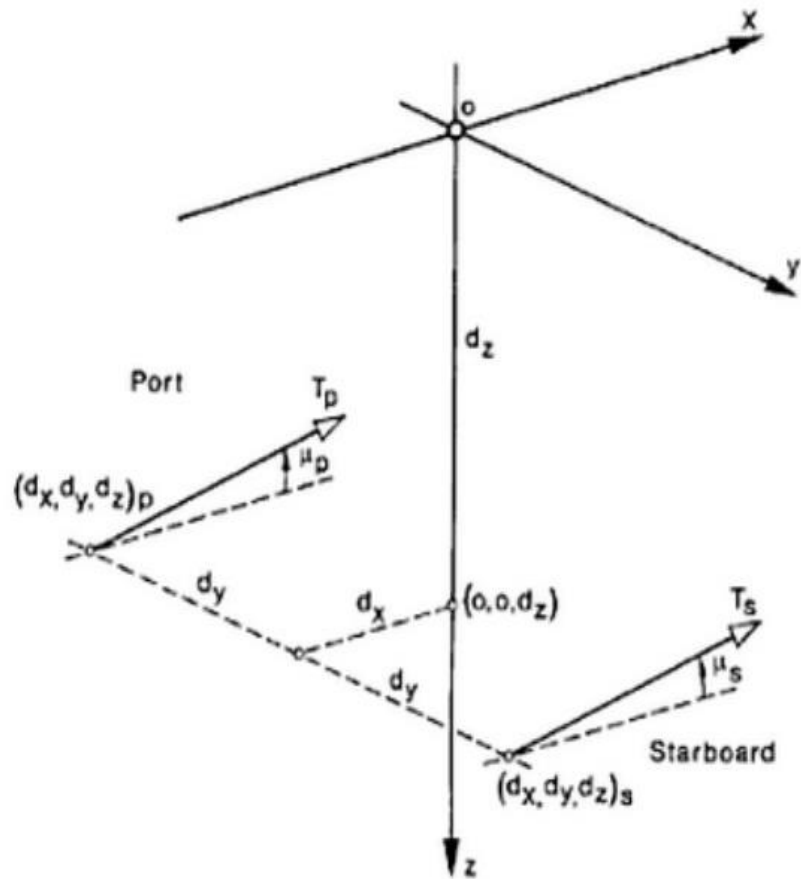


Fig. A3.5 Typical propulsion system geometry [36]

APPENDIX A4

ADDITIONAL SIMULATION RESULTS

A4.1 SIMULATION RESULTS OF SINGLE BODY LONGITUDINAL (3DOF) MODEL:

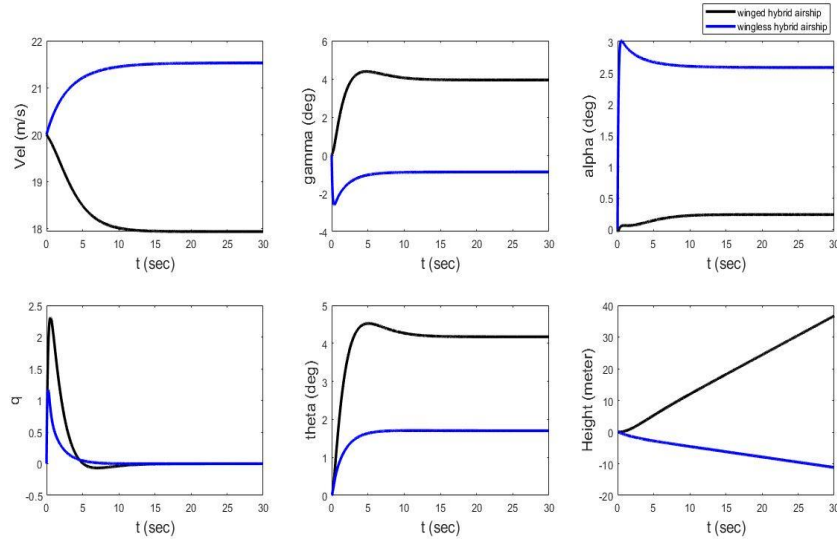
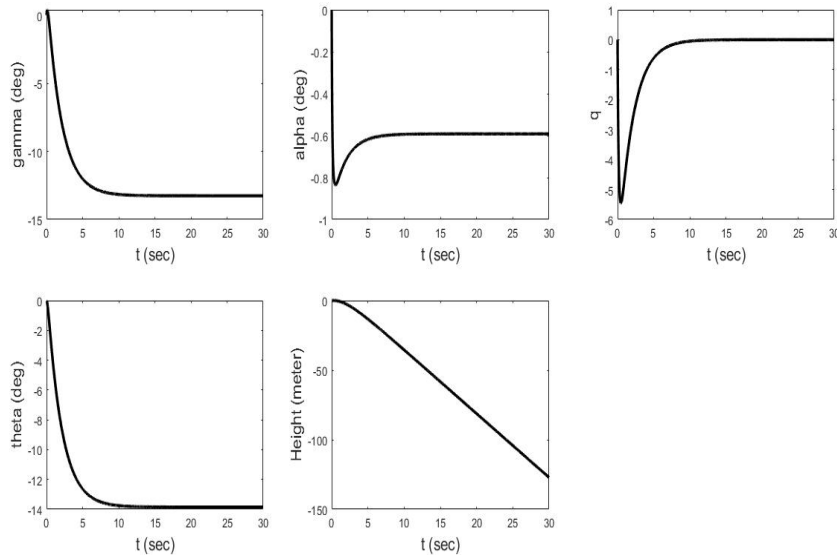


Fig. A4.1 ASIMULATION RESULT of 3DOF MODEL OF HYBRID AIRSHIP WITH WING AND WITHOUT WING



Fig, A4.2 SIMULATION RESULT of 3DOF MODEL OF HYBRID AIRSHIP LEVEL WITHOUT WING

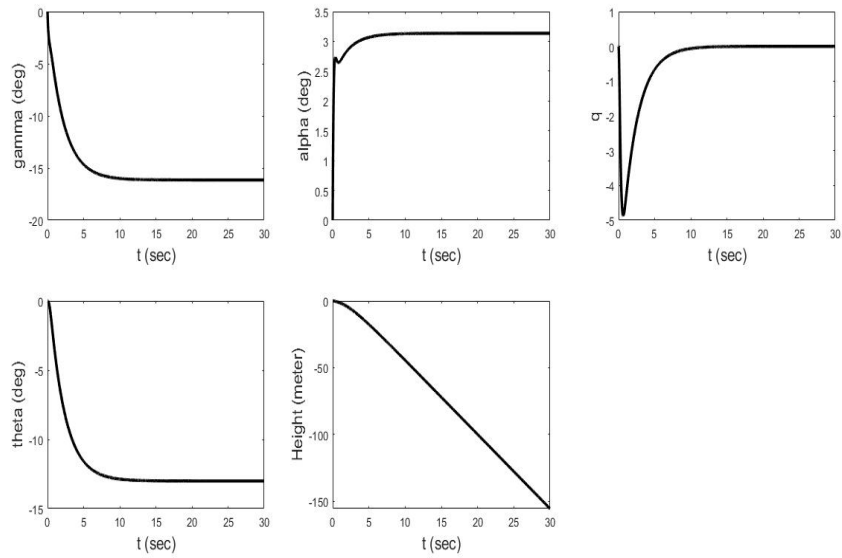


Fig. A4.3 SIMULATION RESULT of 3DOF MODEL OF HYBRID AIRSHIP LEVEL WITH WING

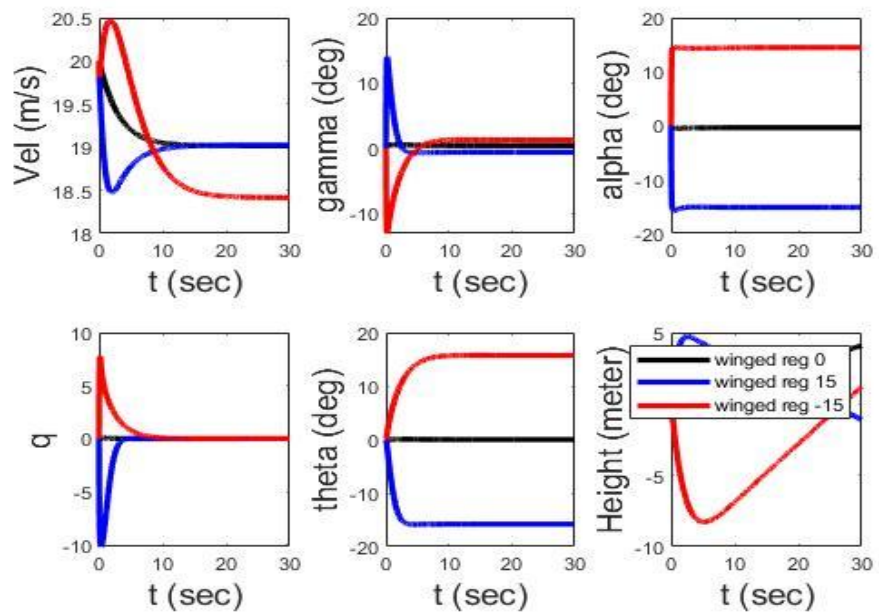


Fig. A4.4 SIMULATION RESULT of 3DOF MODEL OF HYBRID AIRSHIP WITH WING

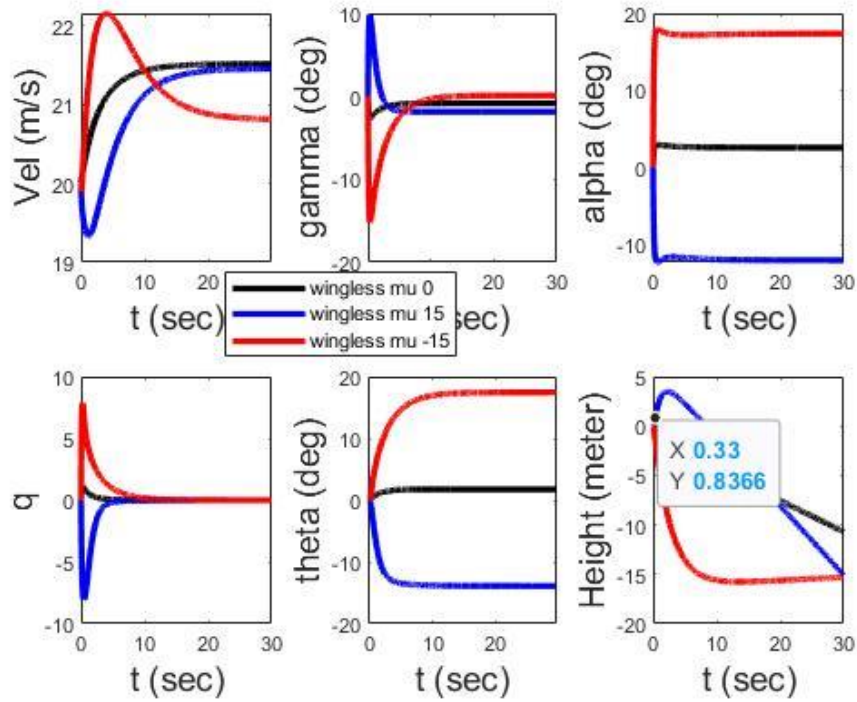


Fig. A4.5 SIMULATION RESULT of 3DOF MODEL OF HYBRID AIRSHIP WITHOUT WING

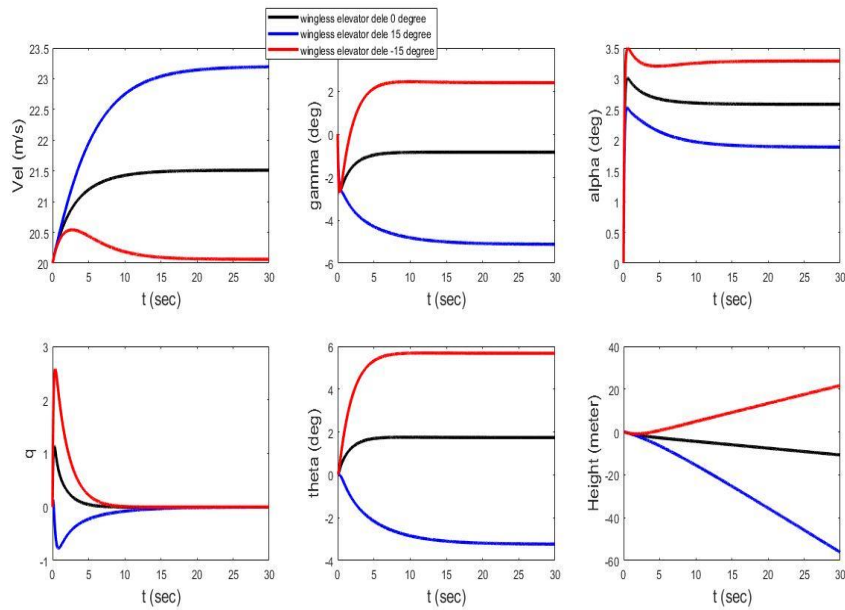


Fig. A4.6 SIMULATION RESULT of 3DOF MODEL OF HYBRID AIRSHIP WITHOUT WING WITH THREE DIFFERENT LEVEL OF ELEVATOR

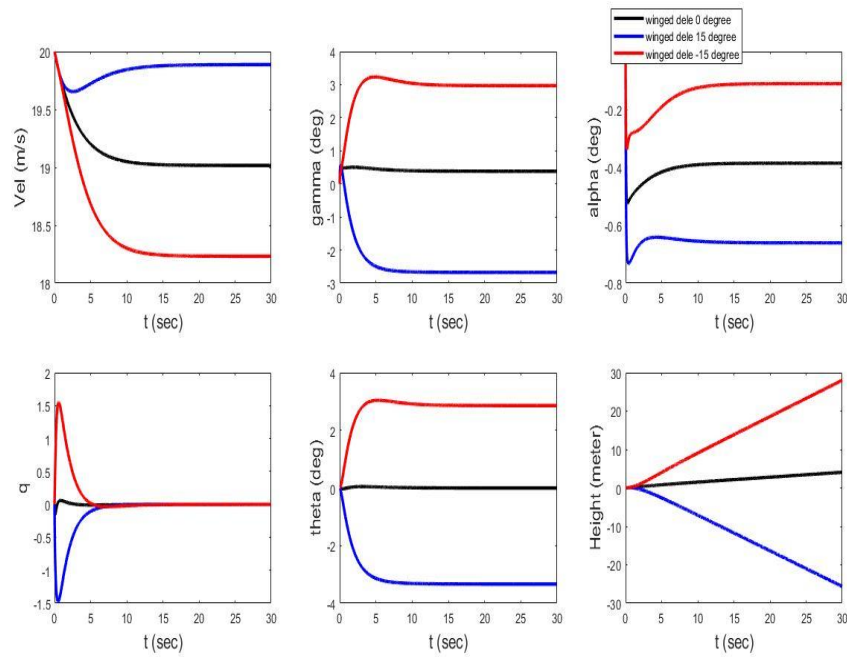


Fig. A4.7 Simulation Result of 3DOF Model of Hybrid Airship with Wing with Three Different Level of Elevator

A4.2 BIFURCATION RESULTS OF SINGLE BODY LONGITUDINAL (3DOF) MODEL:

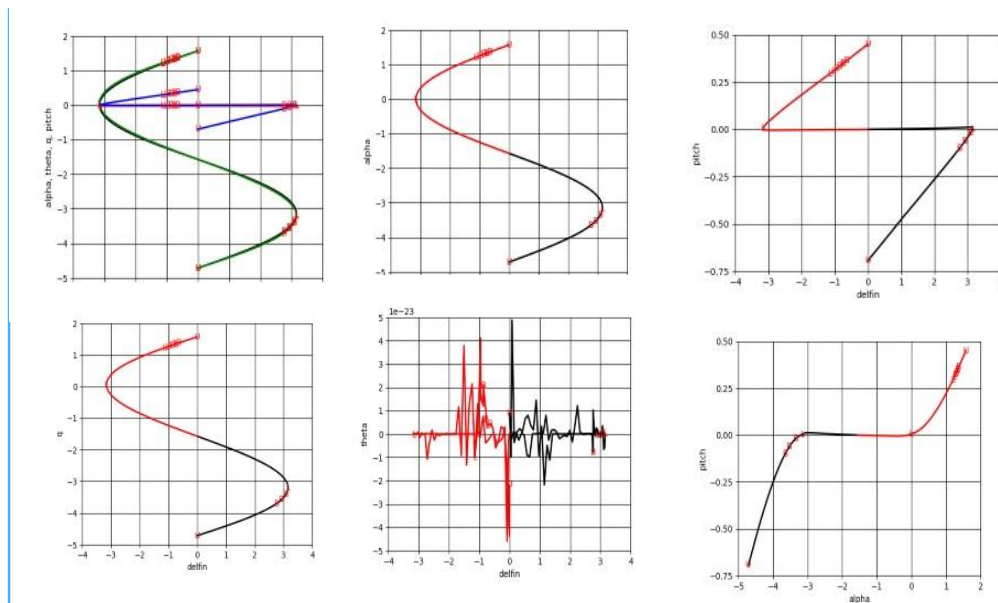


Fig. A4.8 BIFURCATION RESULT HA WINGED WITH ELEVATOR

DEFLECTION

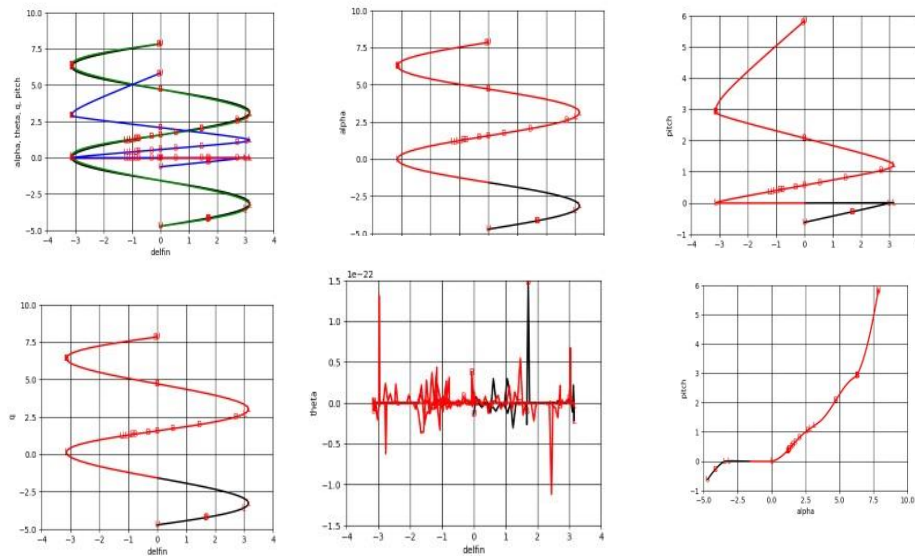


Fig. A4.9 BIFURCATION RESULT HA WINGLESS WITH ELEVATOR DEFLECTION

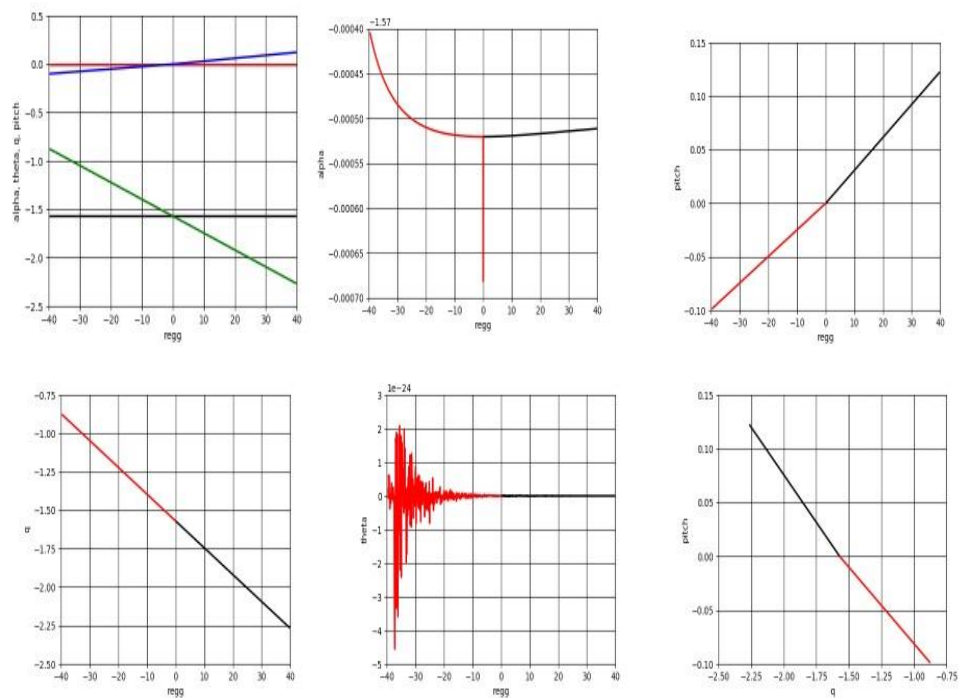


Fig. A4.10 BIFURCATION RESULT HA WINGED WITH DIFFERENT REGGING ANGLE

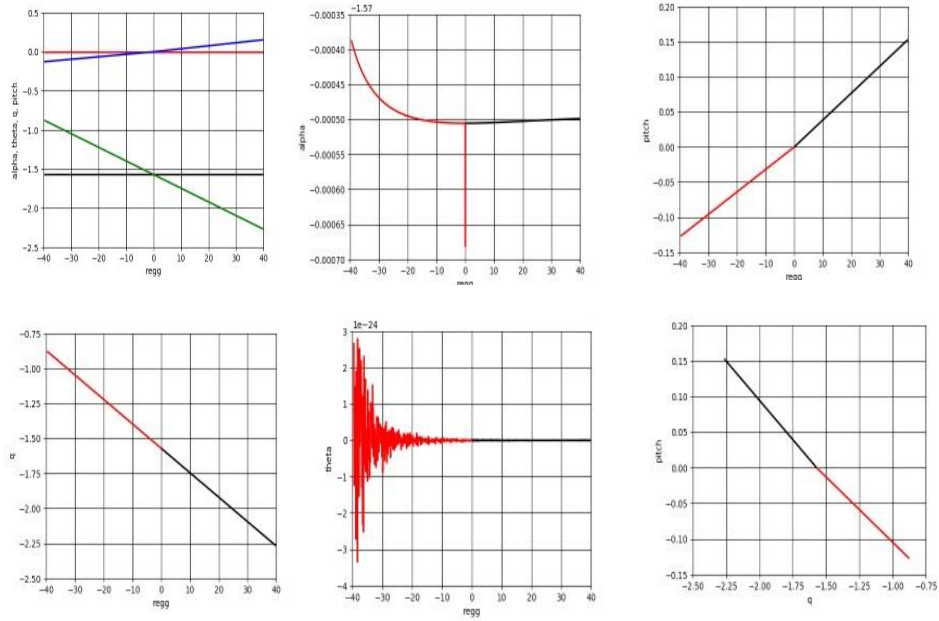


Fig. A4.11 BIFURCATION RESULT HA WINGLESS WITH DIFFERENT REGGING ANGLE

A4.3 ADDITIONAL SIMULATION RESULTS OF TWO BODY LONGITUDINAL (4DOF) MODEL:

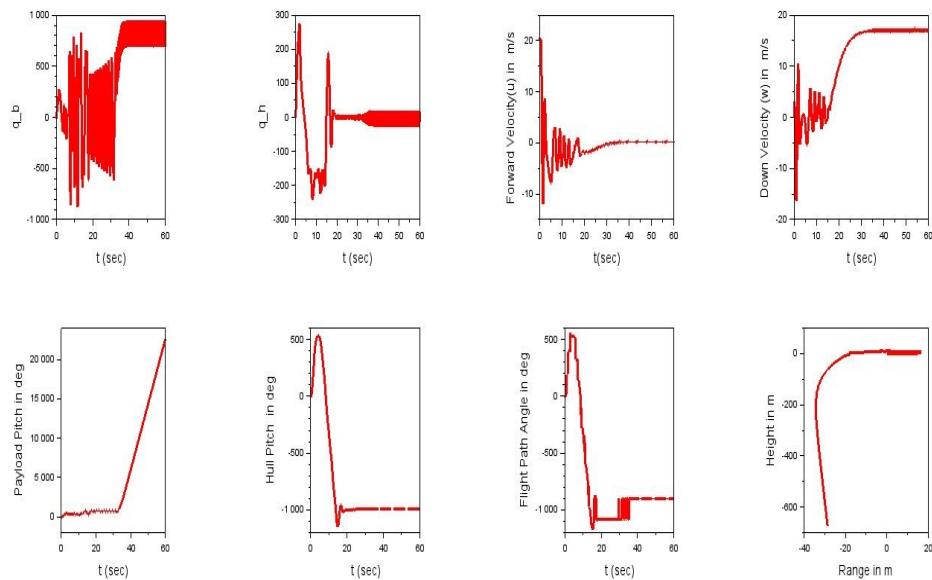


Fig. A4.12 Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload dele 0 reg -20 deg.

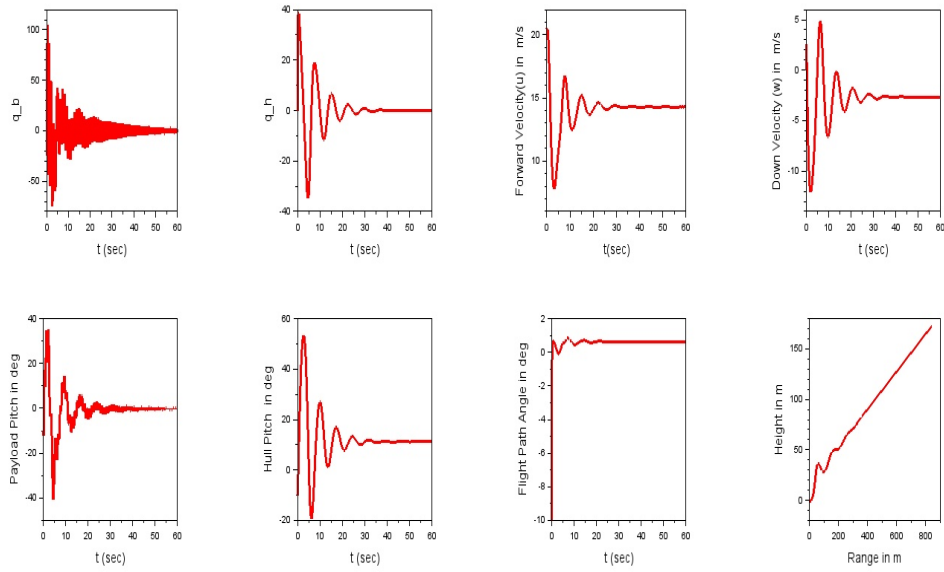


Fig. A4.13 Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload dele 0 reg +20 deg.

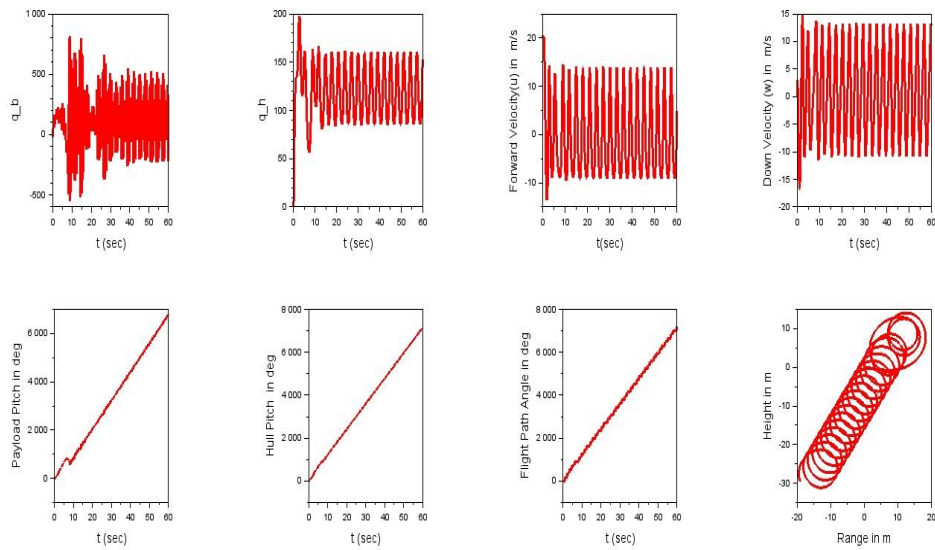


Fig. A4.14 Simulation result of 4DOF longitudinal dynamics of hybrid airship with suspended payload dele 0 reg -10 deg.

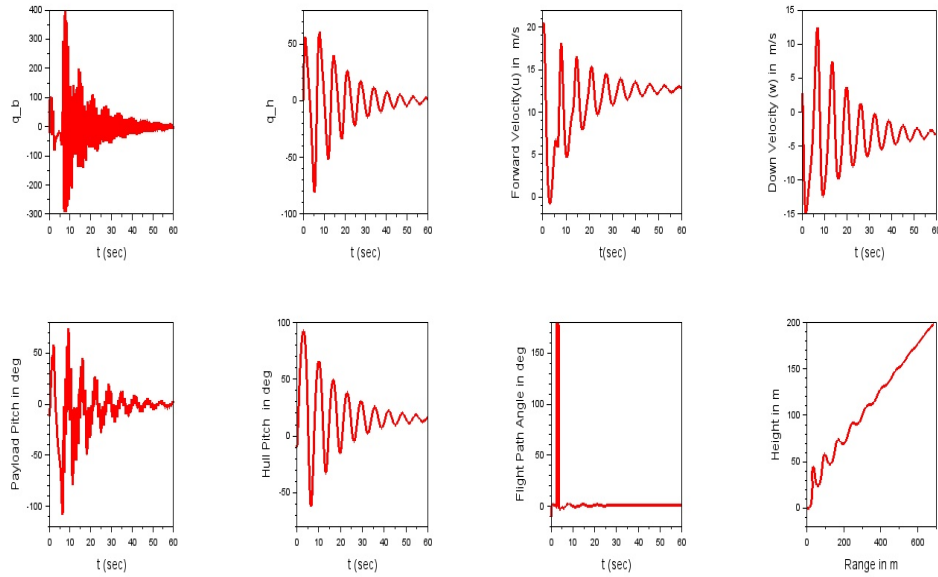


Fig. A4.15 Simulation result of 4DOF longitudinal dynamics of hybrid airship
with suspended payload dele 0 reg +10 deg

A4.4 ADDITIONAL CLOSED LOOP RESPONSE OF 3DOF MODEL WITH CONTROLLERS:

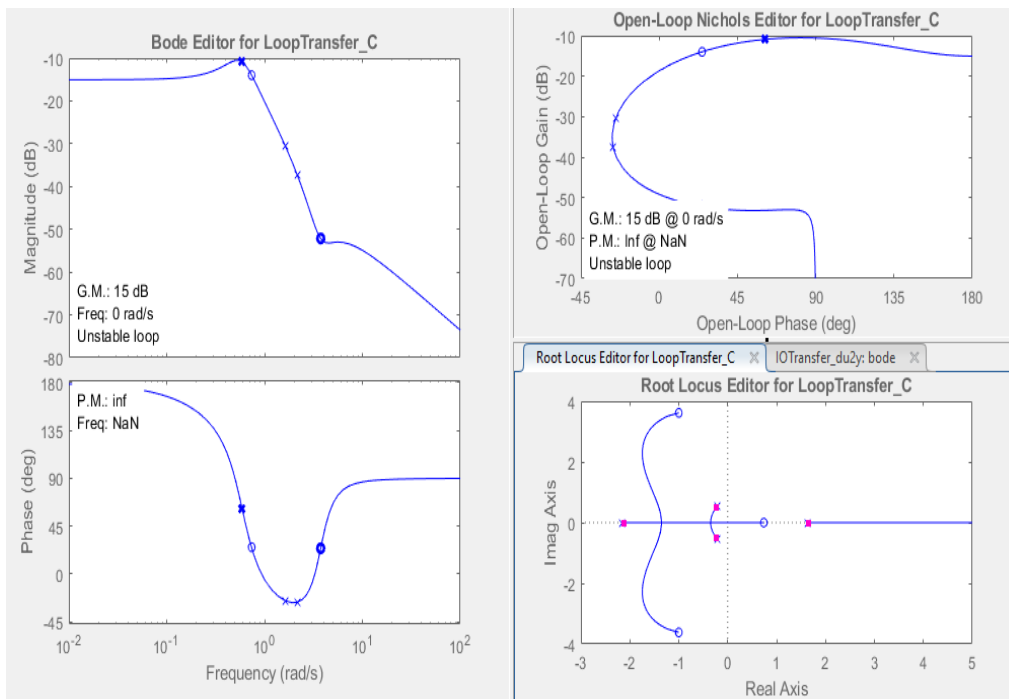


Fig. A4.16 CLOSED LOOP RESPONSE OF G11

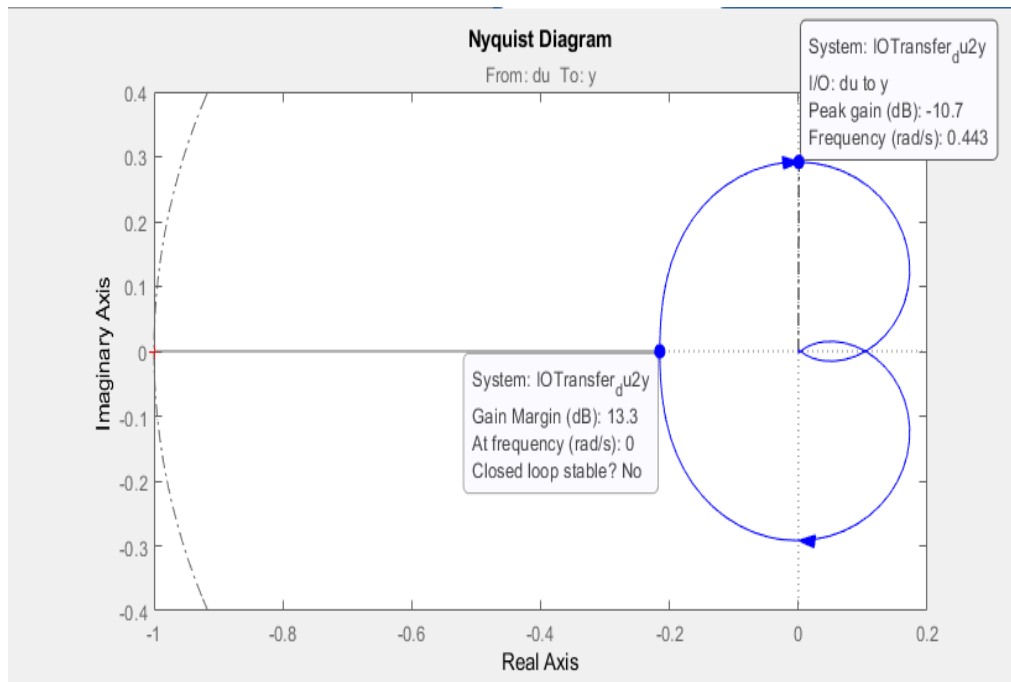


Fig. A4.17 CLOSED LOOP RESPONSE (NYQUIST PLOT) OF G11

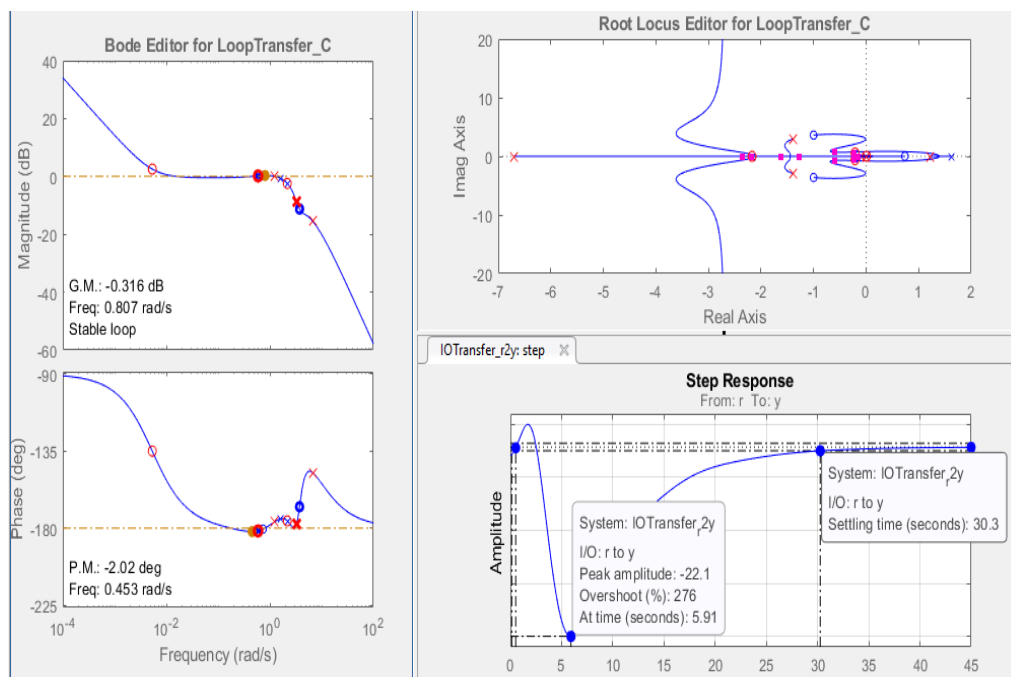


Fig. A4.18 CLOSED LOOP AND OPEN RESPONSE OF G11 WITH LQG COMPENSATOR DESIGN

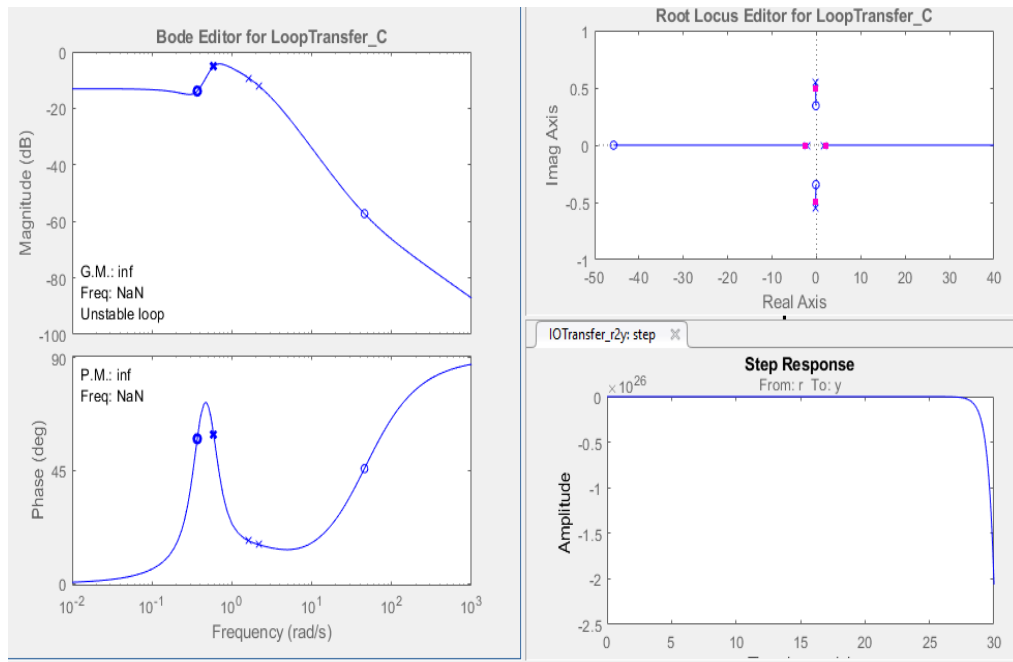


Fig. A4.19 CLOSED LOOP AND OPEN RESPONSE OF G12

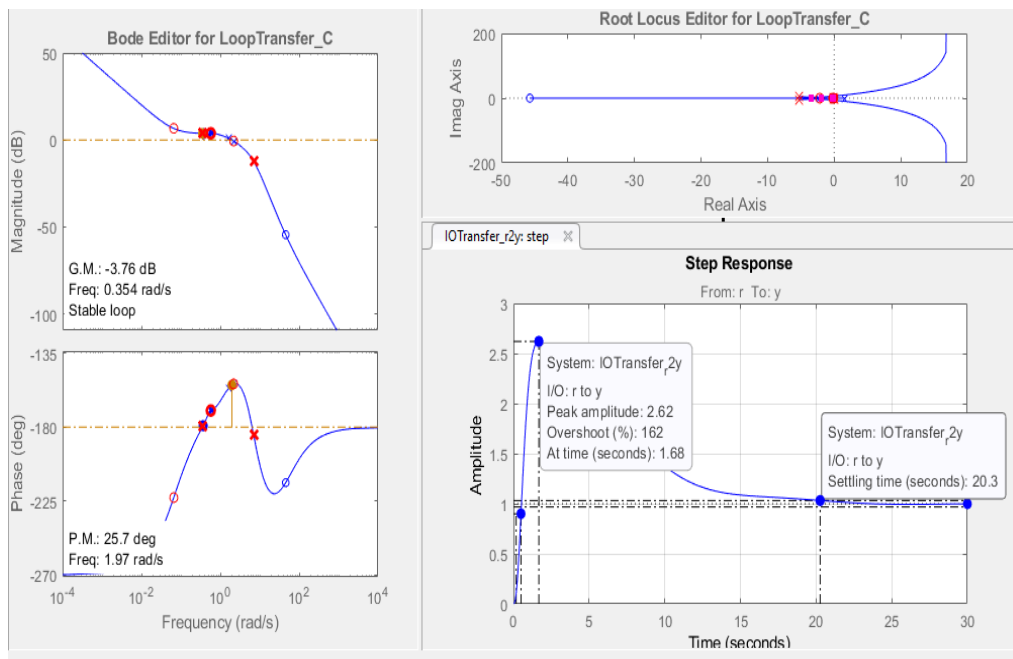


Fig. A4.20 CLOSED LOOP AND OPEN RESPONSE OF G12 WITH LQG COMPENSATOR DESIGN

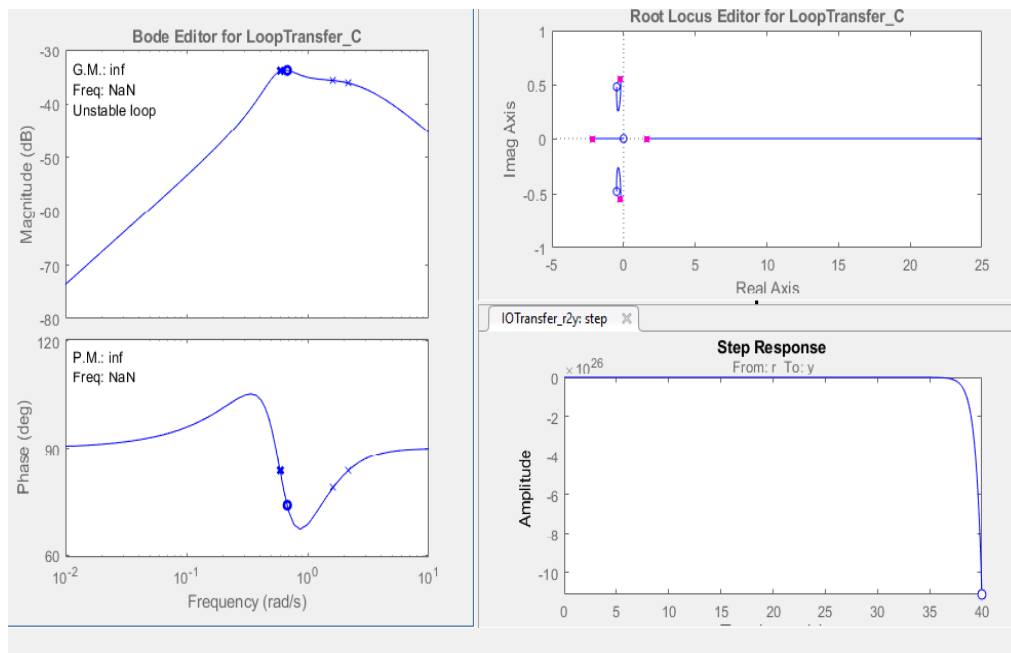


Fig. A4.21 CLOSED LOOP AND OPEN RESPONSE OF G13

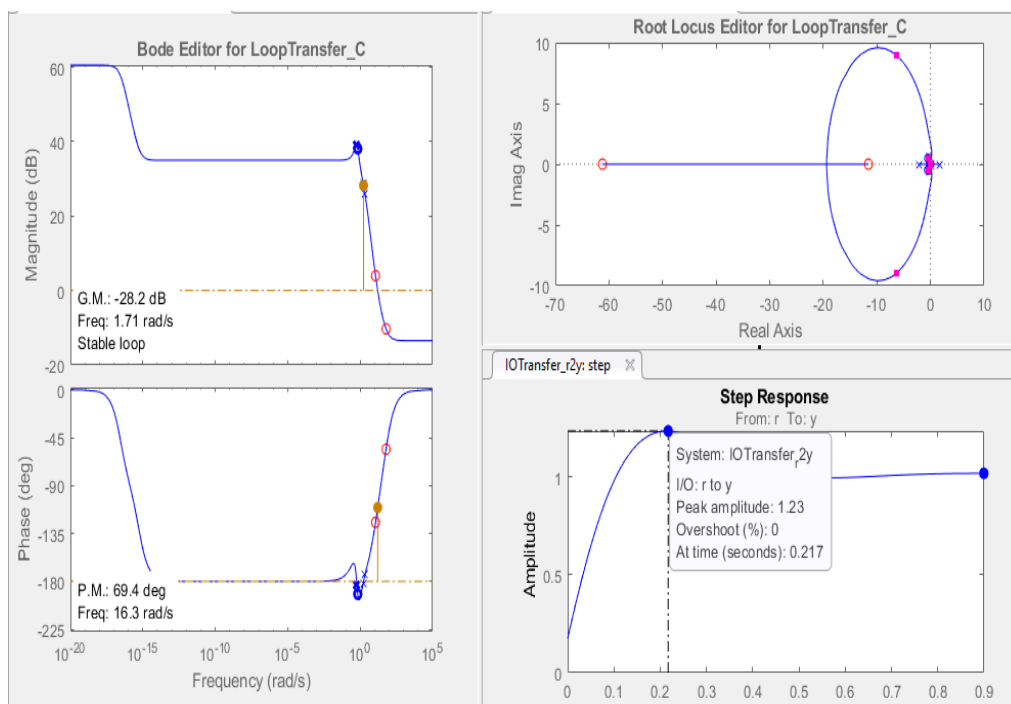


Fig. A4.22 CLOSED LOOP AND OPEN RESPONSE OF G13 WITH PID DESIGN

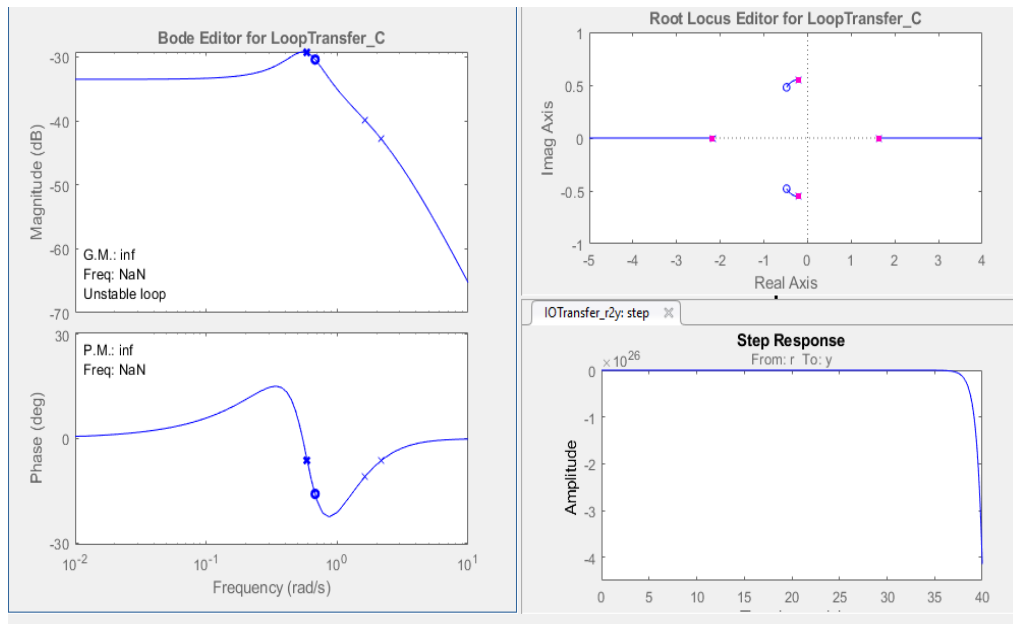


Fig. A4.23 CLOSED LOOP AND OPEN RESPONSE OF G14

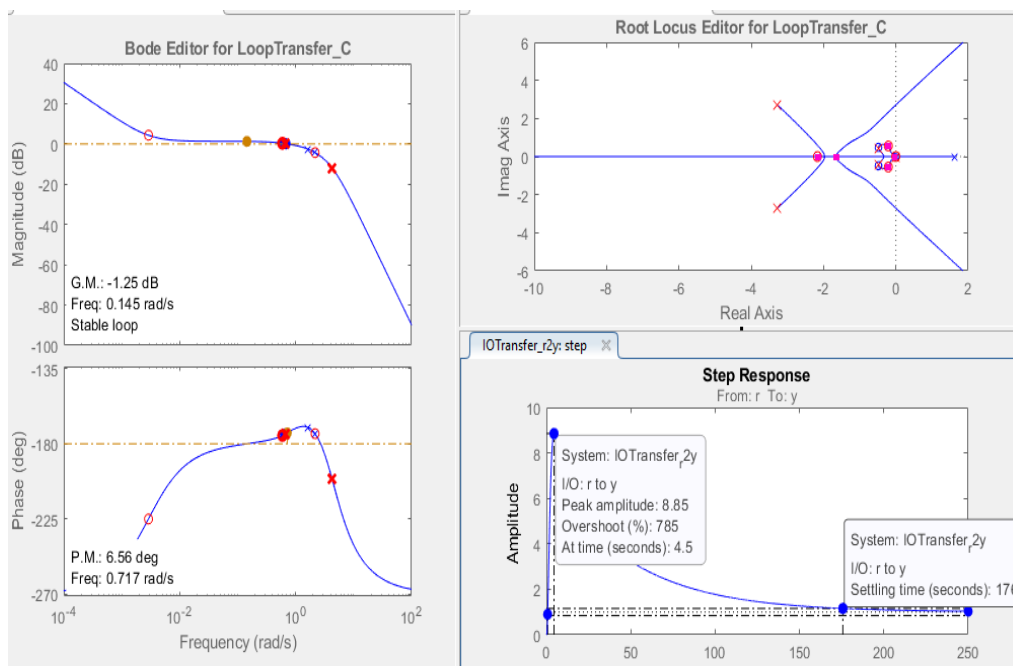


Fig. A4.24 CLOSED LOOP AND OPEN RESPONSE OF G14 WITH LQG COMPENSATOR DESIGN

TRACKING CAPABILITIES OF HYBRID AIRSHIP:

TABLE A4.1: Lateral dynamics mode analysis of linearized hybrid airship for validation [28]:

System	Pole/longitudinal eigen values	damping	Frequency (rad/s)	Time constant (s)	Stability	mode
System matrix A'	-0.2659+ 3.099i	0.085	3.111	3.761	yes	Pendulum mode
	-0.2659 – 3.0994i	0.085	3.111	3.761	yes	Pendulum mode
	0.8933			-1.119	no	Heave mode
	-1.3055			0.766	yes	Surge mode

The lateral state matrix computed for hybrid winged airship and operating conditions taken from [28].

Modes of motion of a conventional aircraft has two longitudinal oscillatory dynamic modes as short period oscillation consists of pitch rate (q) and pitch angle (θ) and long period oscillation or phugoid mode consists of linear velocity in x body axis (u) and linear velocity in z body axis (w). Modes of motion of a conventional airship has two real roots of u and w and one complex pair of roots of q and θ . The longitudinal motion characteristic roots of a winged hybrid airship consist of a stable first order root (u), a pair of stable short period oscillatory roots (q and θ) and an unstable first order root (w). The first longitudinal mode is the surge mode, and it is characterized by the drag variation due to speed. It is stable for conventional airship with long time constant since the drag variation due to speed is small and negative. There is increase in drag for winged hybrid airship in comparison to a conventional wingless airship. Since the drag variation due to speed is increased, the surge mode for the winged hybrid airship is stable with small time constant of 0.442 sec. The second longitudinal mode of motion is longitudinal pendulum mode in nature. This is

stable for conventional wingless airship with low frequency oscillation in pitch and also stable for winged hybrid airship with moderate to low values of damping ratio and natural frequency. The third longitudinal mode is the heave mode and is stable with short time constant at hovering and at low speed for both conventional wingless airship and for winged hybrid airship.

As speed increases, the additional aerodynamic effects from the lift contributed by hull due to normal velocity and pitch damping begin to dominate. For a conventional wingless airship, the aerodynamic lift due to normal velocity may not be very dominant. Therefore, the heave mode is stable for wingless airship. Due to strong dynamic effects from the hull plus wing configuration in winged hybrid airship, the heave mode is unstable here. The additional lift from the wing that tripled the overall aerodynamic lift can cause a strong upward force that is not recoverable by the pitch damping. The winged hybrid airship is statically stable and dynamically unstable. When an unstable mode of motion is excited, it will never return to equilibrium position if no correction is made through a control system. Although unstable mode is an undesirable characteristic, some aerial vehicles are designed to be dynamically unstable for better manoeuvrability, while instability is continually corrected by on board “fly-by-wire” system.

The lateral-directional eigenvalues and modes characteristic are given in Table A4.1. The winged hybrid airship’s lateral directional motion include a stable first order root due to linear velocity along y body axis (v), an unstable first order root due to roll angle (ϕ) and a pair of stable oscillatory roots due to roll and yaw rates in body axis (p and r). The first lateral directional mode is the yaw subsidence mode and it is stable and considered as fast mode. This mode characteristic is significantly dependent on the yaw damping derivative, and it is more stable as speed increases because the yaw damping increases with speed. The second mode is oscillatory roll mode and is stable. The roll mode is stabilizing when the airship is buoyant. This mode is similar to aircraft’s second order response dutch roll mode. The aerodynamic roll damping of the winged hybrid airship is small, with $C_{l_\beta} = -0.0004$, thus the roll coupling is not too significant. The yaw damping is higher than the roll damping. Therefore, this

mode is overall dynamically stable. Fig. A4.25 is the hybrid airship response with two body dynamic approach with 20, 10 and 5 degree elevator deflection by changing the elevator are 10, 20 and 30 percent of tail areas.

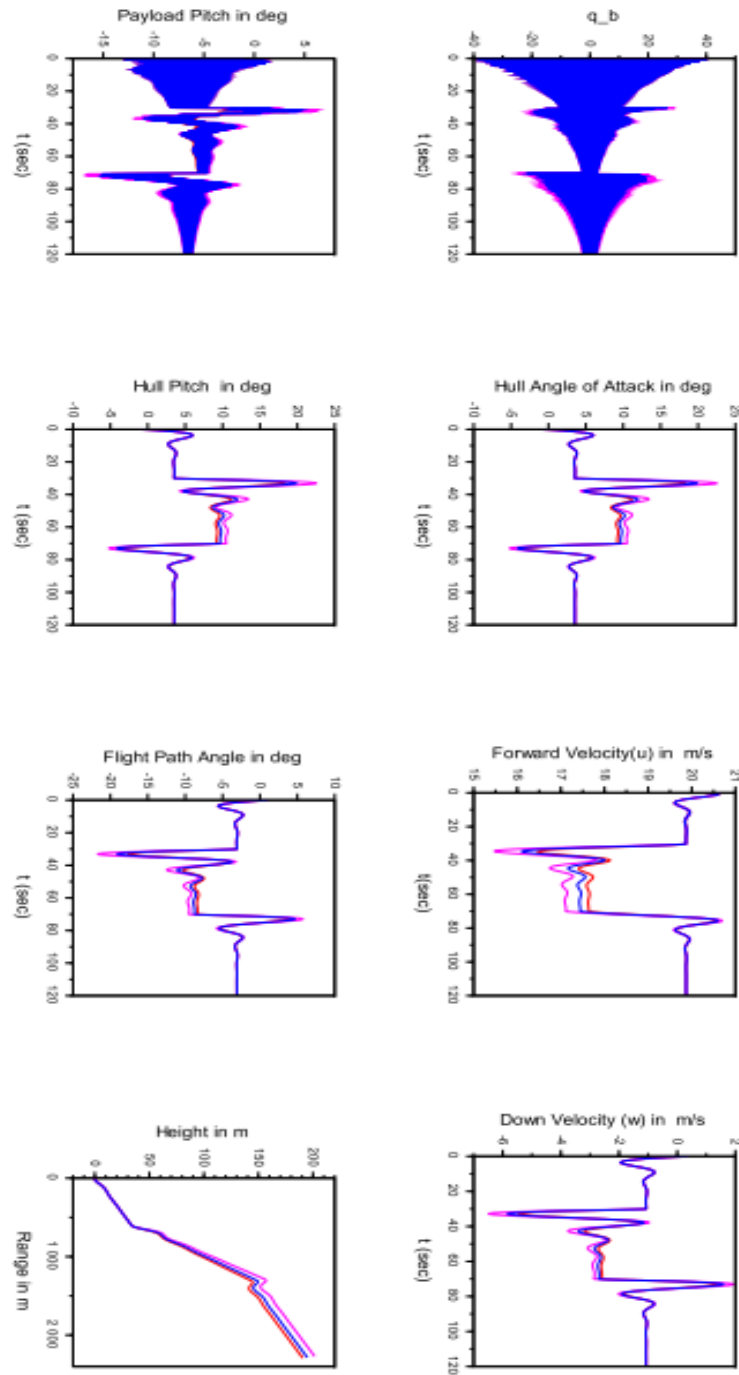


Fig. A4.25 Hybrid Airship response for 20,10,5 deg deflection corresponding to elevator areas of 10,20,30 percent of tail areas

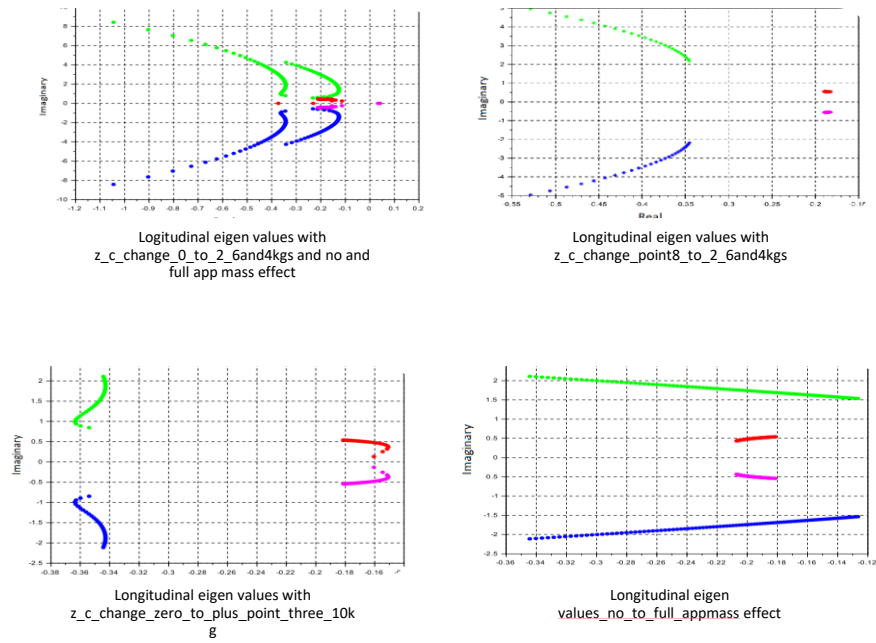


Fig. A4.26-winged hybrid airship Longitudinal Single body eigen values parametric analysis collages of image with varying CG and apparent mass.

APPENDIX A5

Controller Design (Additional work)

A5.1 MPC CONTROLLER DESIGN:

MPC controller has been designed for trajectory tracking for ARX model for lateral dynamics of an aircraft. Models are trained with MPC based on input output information. Similar work is being carried out for Hybrid Airship.

Two inputs and four outputs are used for lateral dynamics of aircraft. Input is aileron and rudder deflection denoted as *ail* and *rdr*. Output is side slip angle, roll rate, yaw rate and roll angle denoted as *beta*, *p*, *r*, and *phi* respectively. For identifying a suitable model for lateral dynamics of aircraft, suitable parameters of F014 flight data for validation purpose and F017 flight data for estimation purpose are loaded to MATLAB Workspace. The Data is imported to System identification Toolbox. *Mydata_e* and *Mydata_v* is dragged into working data and validation data box respectively. And then different options of pre-process operations and estimate operations are used for obtaining a model. Simulation block diagram of model ARX211 with MPC controller is shown in Fig. A5.3. Here the model ARX211 is used to design the MPC controller and is considered as a plant. A LTI model say ARX211 is selected and the properties list displays the number of inputs and outputs, their names and signal types etc. By clicking the import button, this loads ARX211 plant into the design tool as shown in Fig. A5.1. After selecting constraints button, a dialog box is appeared as shown in Fig. A5.2.

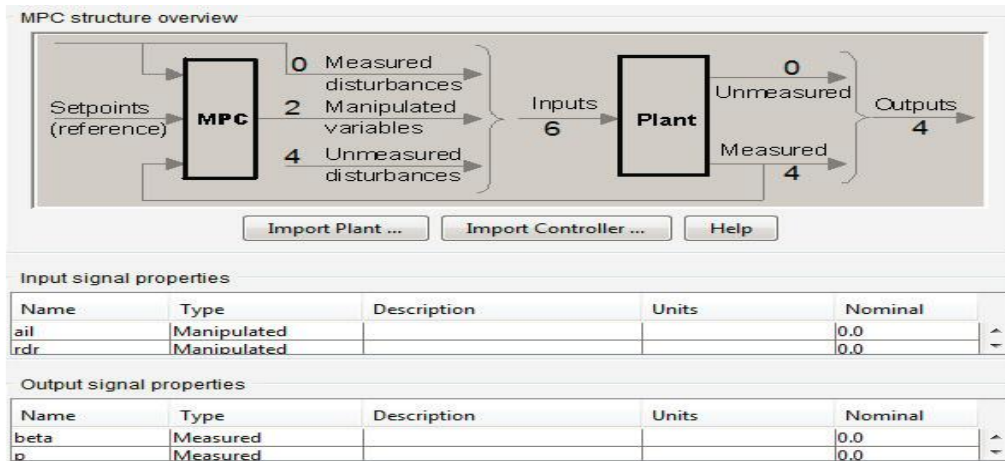


Fig. A5.1 MPC Design Tool

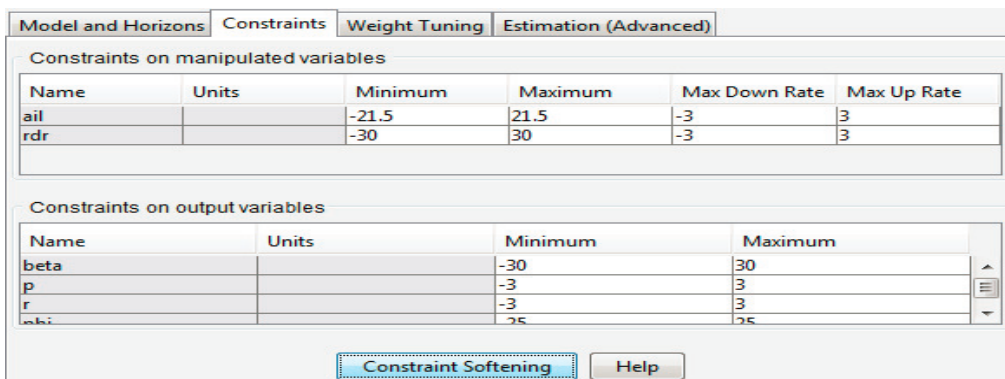


Fig. A5.2 MPC Constraints dialog box

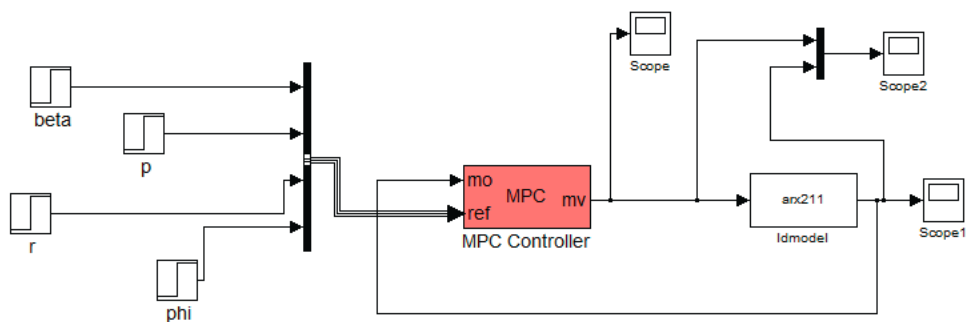


Fig. A5.3 Simulation Block diagram of model with MPC controller

The similar work is being carried out for hybrid airship for trajectory tracking by

using simulation data. Once the flight data for hybrid airship is available will give more accurate result.

A5.1.1 SIMULATION RESULT MPC CONTROLLER FOR ARX MODEL OF AIRCRAFT FOR LATERAL DYNAMICS:

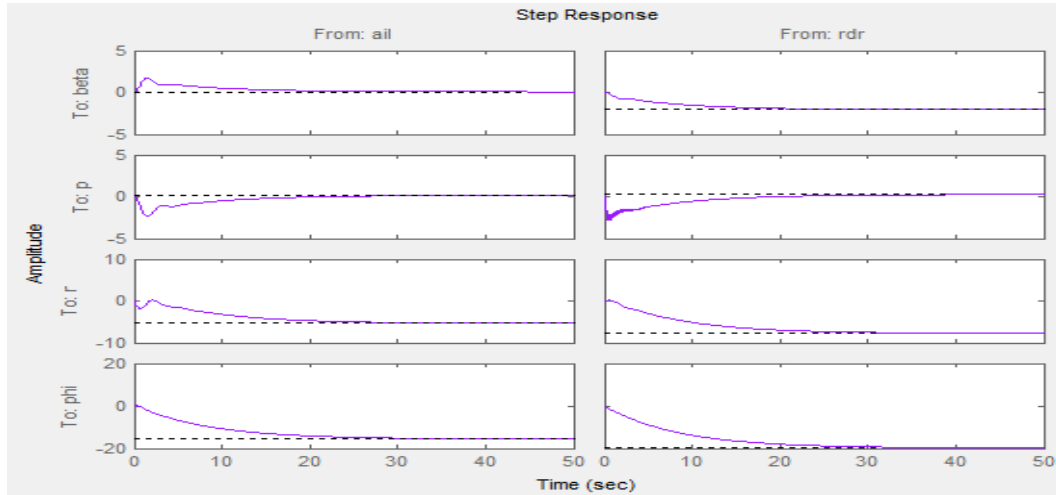


Fig. A5.4 Step response of ARX model of lateral dynamics of aircraft F014

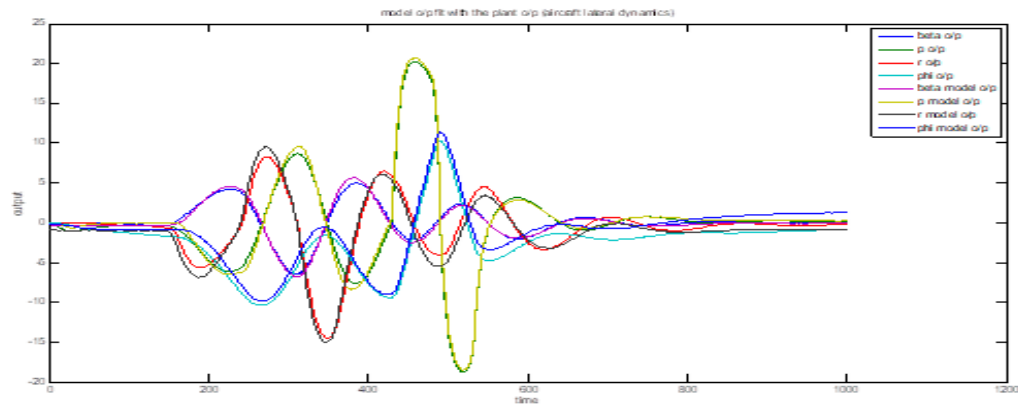


Fig. A5.5 validation result of model ARX vs flight data for different inputs and outputs in Simulink.

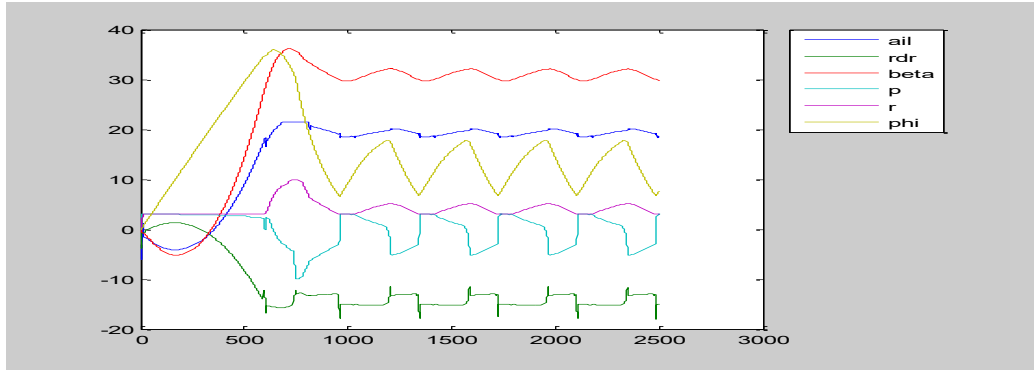


Fig. A5.6 Output of ARX with MPC in Simulink

A5.1.2 SIMULATION RESULT OF ARX MODEL OF 4DOF HYBRID AIRSHIP WITH SUSPENDED PAYLOAD FOR LONGITUDINAL DYNAMICS:

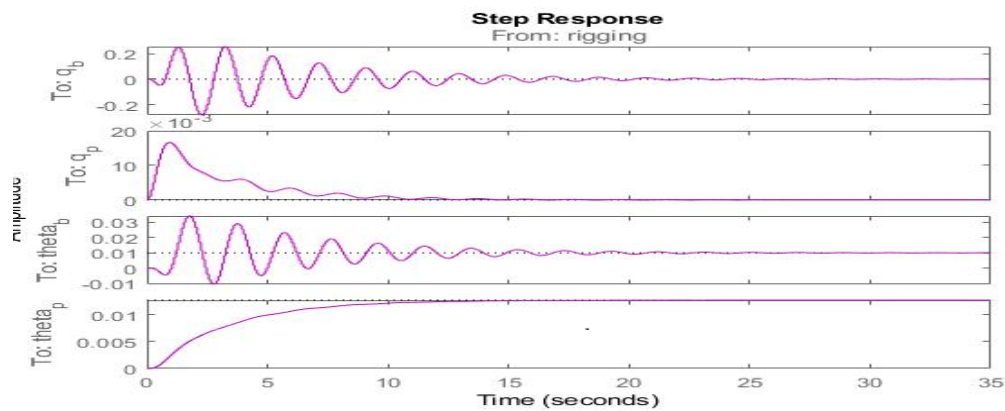


Fig A5.7 Step response of ARX model of longitudinal dynamics of 4 DOF hybrid airship with suspended payload.

At the starting of the research work, understanding the concept of aircraft dynamics was required. Hybrid Airship modelling is primarily based on the dynamic equation of Aircraft. Here a model predictive controller is designed for lateral dynamics of an aircraft model and is published as additional publication in ICONAT, IEEE Xplore. This gives a good insight into deriving a state space longitudinal dynamic model of hybrid airship from 6DOF dynamic equation of Aircraft. The same work has been extended to hybrid airship for designing a MPC controller and published in MAC 2024 , IEEE Xplore.

A5.2 LQG (LINEAR QUADRATIC GAUSSIAN) CONTROLLER DESIGN:

G11 is the system G defined as:

A =					B =				
	u	w	q	theta		elevator			
u	-0.3339	0.3763	0.0633	-4.87	u	-0.00032			
w	-1.254	-0.5901	34.2	0	w	0.0056			
q	-0.1094	0.0809	-0.0286	1.198	q	-0.0022			
theta	0	0	1	0	theta	0			
C =					D =				
	u	w	q	theta		elevator			
u	1	0	0	0	u	0			

Designing of LQG Controller for Hybrid Airship (G11 transfer function):

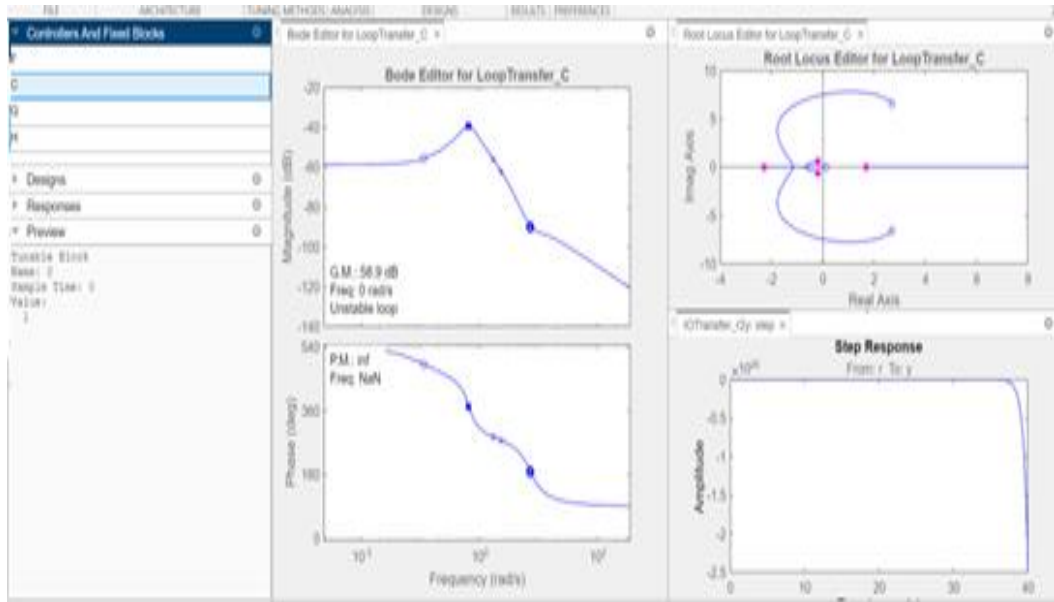


Fig. A5.8 without controller response for G11 transfer function of hybrid airship.

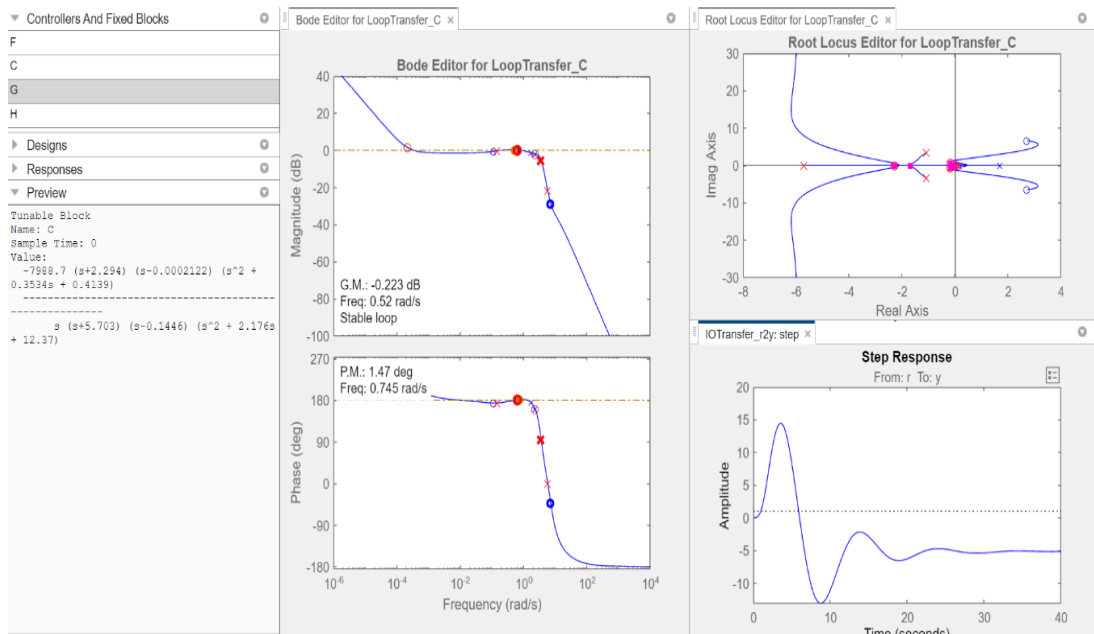


Fig. A5.9 With LQG controller response for G11 transfer function of hybrid airship.

G12 is the system G defined as:

A = u w q theta u w q theta -0.3339 0.3763 0.0633 -4.87 -1.254 -0.5901 34.2 0 -0.1094 0.0809 -0.0286 1.198 0 0 1 0	B = elevator u w q theta -0.00032 0.0056 -0.0022 0
C = u w q theta u 0 1 0 0	D = elevator w 0

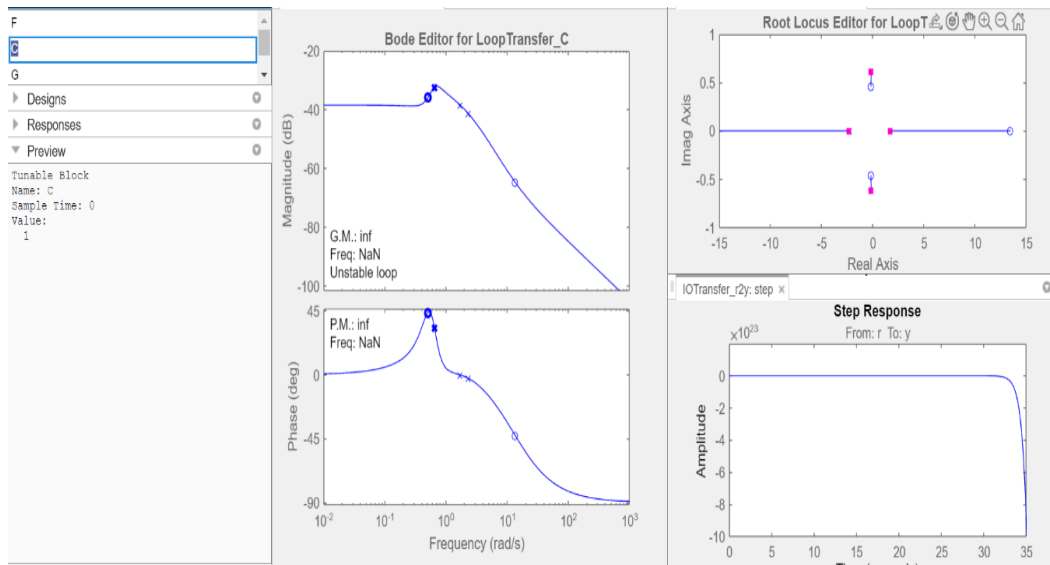


Fig. A5.10 without controller response for G12 transfer function of hybrid airship.

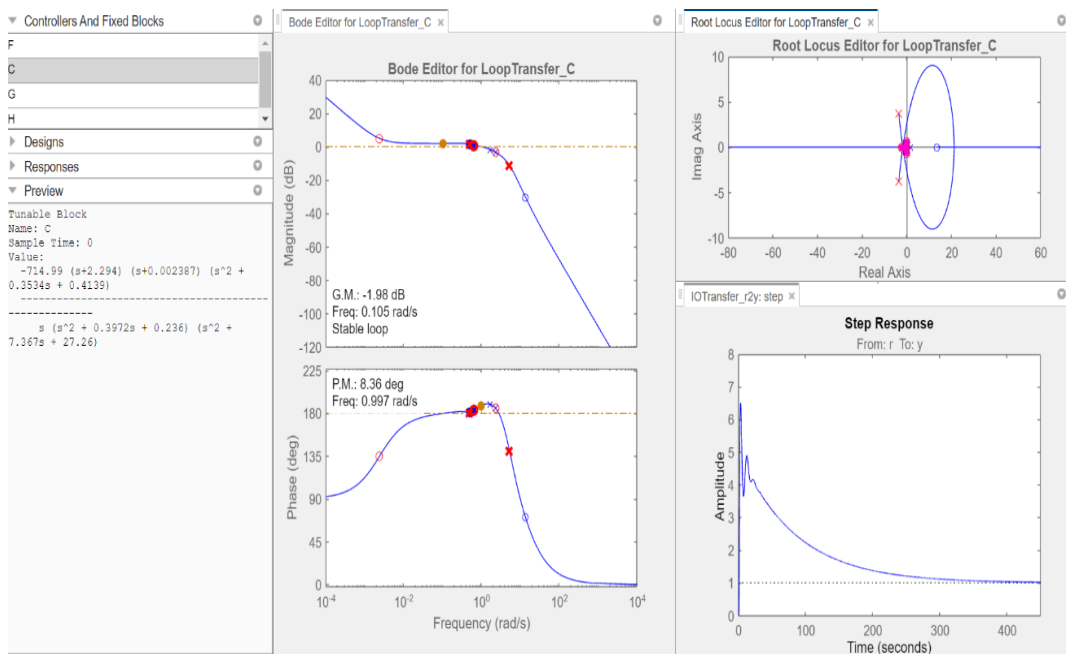


Fig. A5.11 With LQG controller response for G12 transfer function of hybrid airship.

G13 is the system G defined as:

A =					B =				
	u	w	q			elevator			
theta					u	-0.00032			
u	-0.3339	0.3763	0.0633	-4.87	w	0.0056			
w	-1.254	-0.5901	34.2	0	q	-0.0022			
q	-0.1094	0.0809	-0.0286	1.198	theta	0			
theta	0	0	1	0					

C =					D =				
	u	w	q	theta		elevator			
u	0	0	1	0	q	0			

Because of unstable modes LQG can not be formed for this sub system G13. PID controller has been designed here for G13. PID controller design was not possible to for G11, G12 and G14.

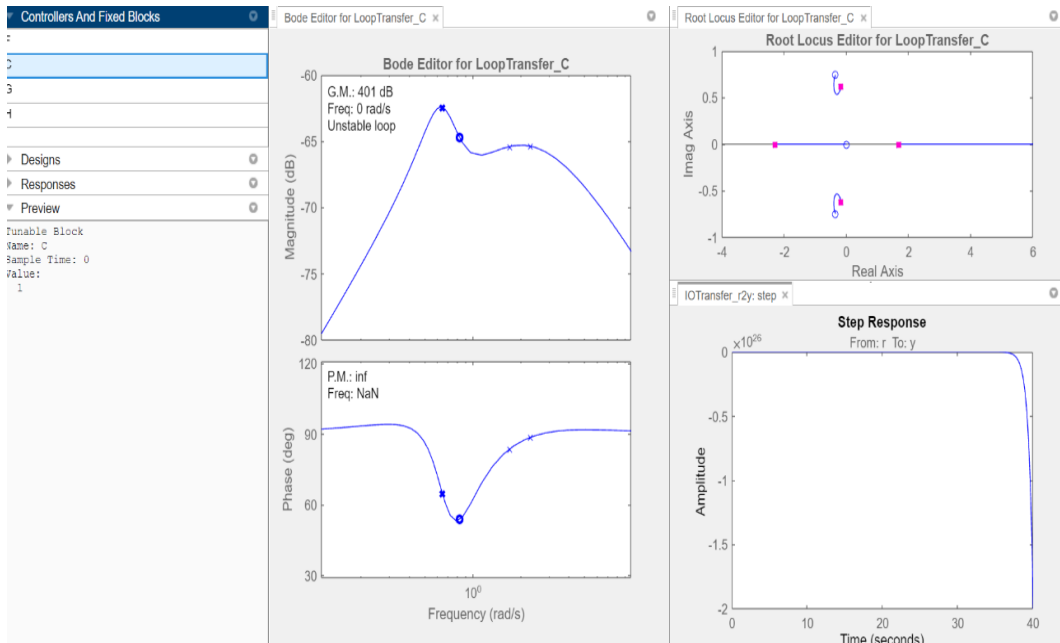


Fig. A5.12 without controller response for G13 transfer function of hybrid airship.

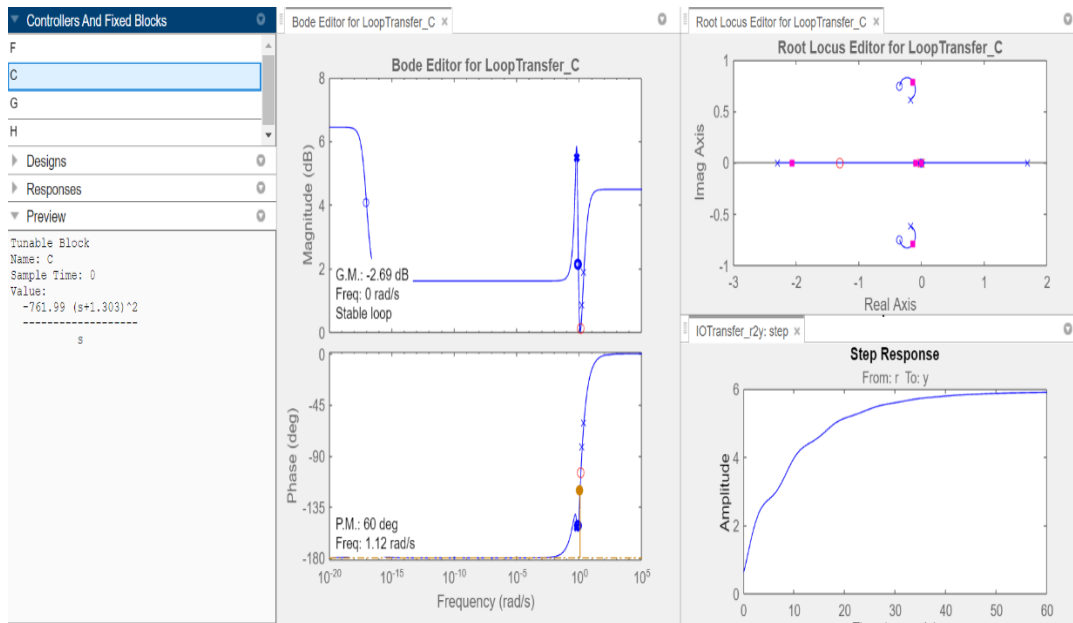


Fig. A5.13 With PID controller response for G13 transfer function of hybrid airship.

G14 is the system G defined as:

A =<table><tr><td></td><td>u</td><td>w</td><td>q</td><td></td></tr><tr><td>theta</td><td></td><td></td><td></td><td></td></tr><tr><td>u</td><td>-0.3339</td><td>0.3763</td><td>0.0633</td><td>-4.87</td></tr><tr><td>w</td><td>-1.254</td><td>-0.5901</td><td>34.2</td><td>0</td></tr><tr><td>q</td><td>-0.1094</td><td>0.0809</td><td>-0.0286</td><td>1.198</td></tr><tr><td>theta</td><td>0</td><td>0</td><td>1</td><td>0</td></tr></table>		u	w	q		theta					u	-0.3339	0.3763	0.0633	-4.87	w	-1.254	-0.5901	34.2	0	q	-0.1094	0.0809	-0.0286	1.198	theta	0	0	1	0	B =<table><tr><td></td><td>elevator</td></tr><tr><td>u</td><td>-0.00032</td></tr><tr><td>w</td><td>0.0056</td></tr><tr><td>q</td><td>-0.0022</td></tr><tr><td>theta</td><td>0</td></tr></table>		elevator	u	-0.00032	w	0.0056	q	-0.0022	theta	0
	u	w	q																																						
theta																																									
u	-0.3339	0.3763	0.0633	-4.87																																					
w	-1.254	-0.5901	34.2	0																																					
q	-0.1094	0.0809	-0.0286	1.198																																					
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q	-0.0022																																								
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C =<table><tr><td></td><td>u</td><td>w</td><td>q</td><td>theta</td></tr><tr><td>u</td><td>0</td><td>0</td><td>0</td><td>1</td></tr></table>		u	w	q	theta	u	0	0	0	1	D =<table><tr><td></td><td>elevator</td></tr><tr><td>theta</td><td>0</td></tr></table>		elevator	theta	0																										
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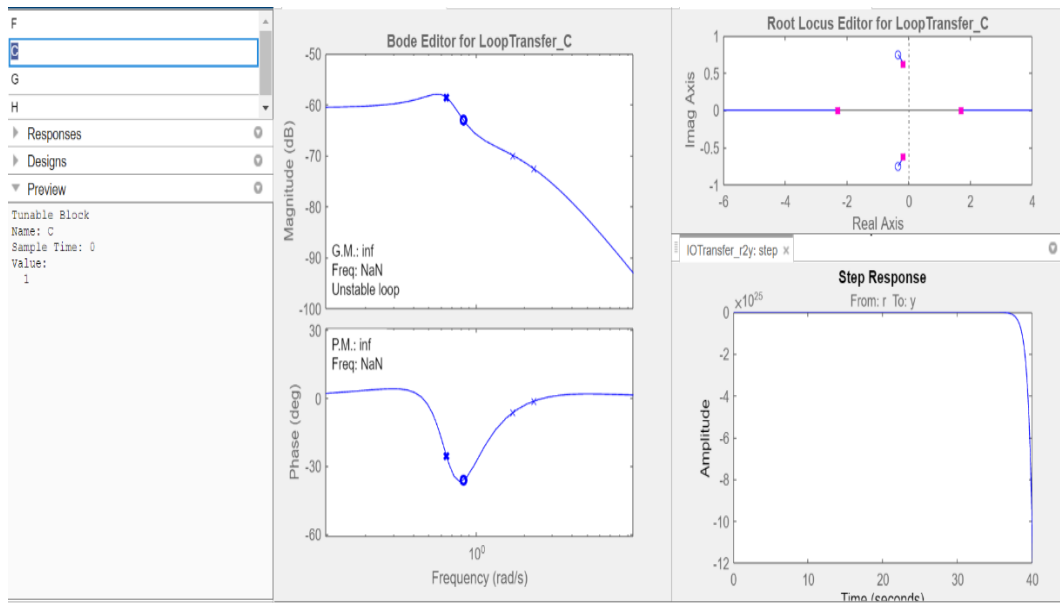


Fig. A5.14 without controller response for G14 transfer function of hybrid airship.

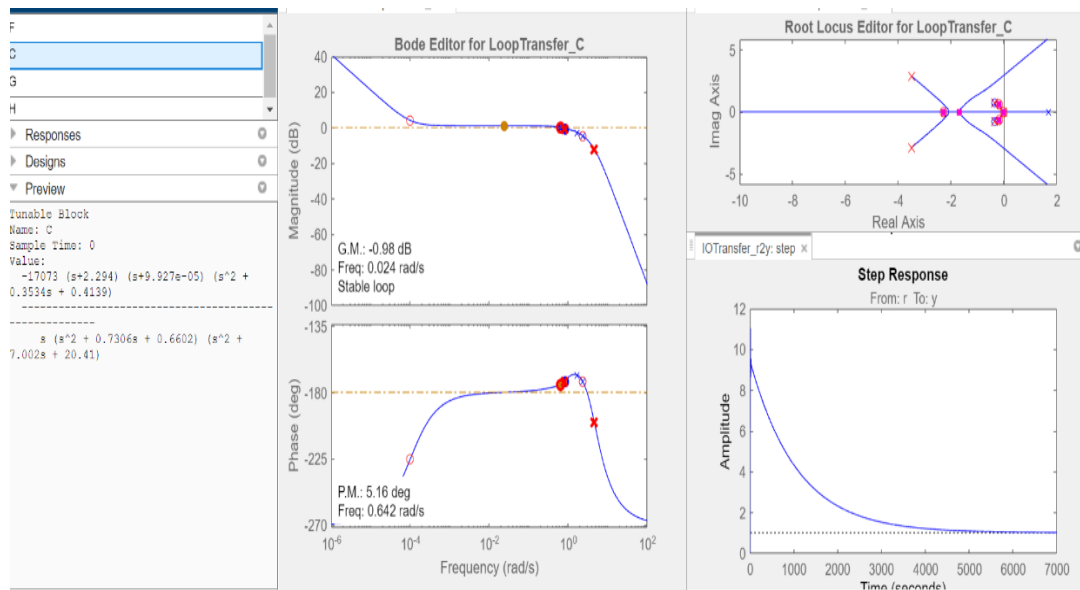


Fig. A5.15 with LQG controller response for G14 transfer function of hybrid airship.

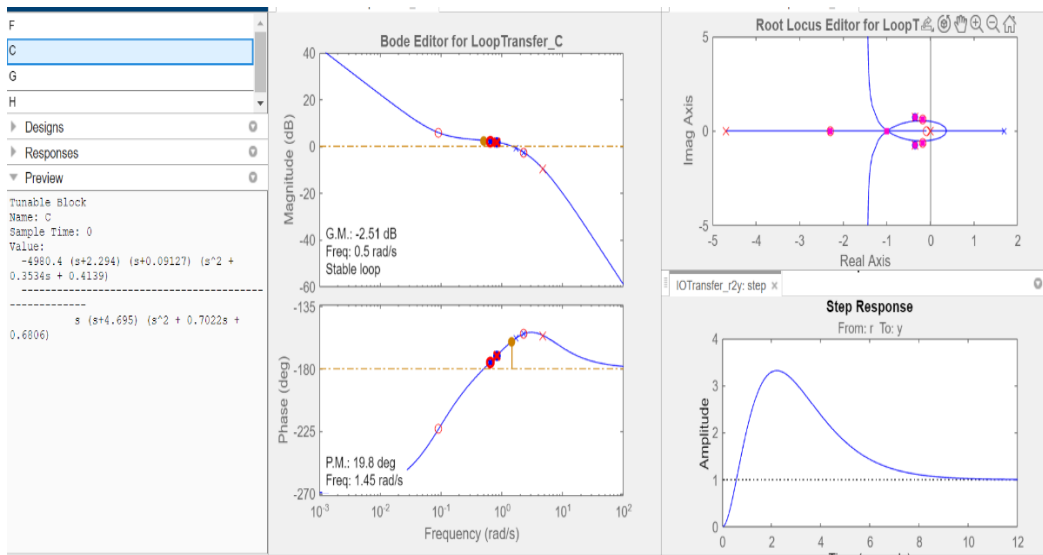


Fig. A5.16 with IMC response for G14 transfer function of hybrid airship.

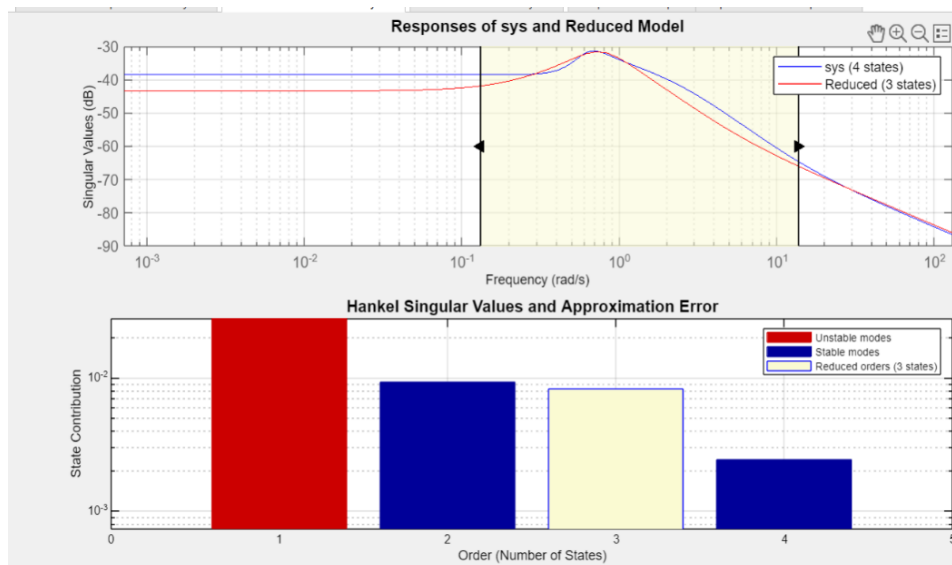


Fig. A5.17 Model Reducer

A5.3 POLE PLACEMENT TECHNIQUES (LQR – Linear quadratic regulator):

LQR controllers possess inherent robustness with guaranteed gain and phase margin, and they also are part of the solution to the LQG (linear-quadratic-Gaussian) problem.

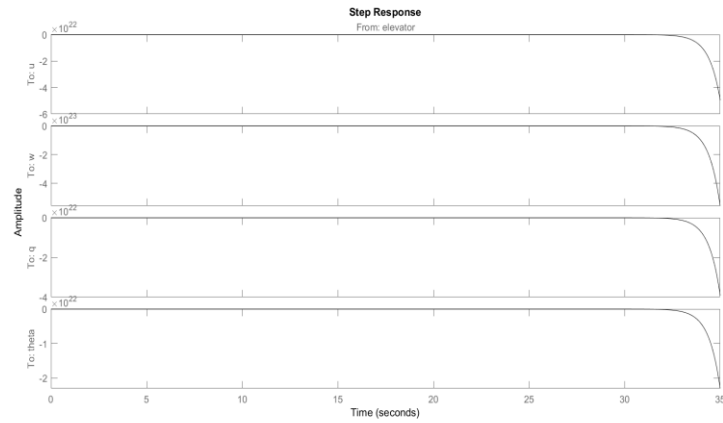


Fig. A5.18 Step response of HA

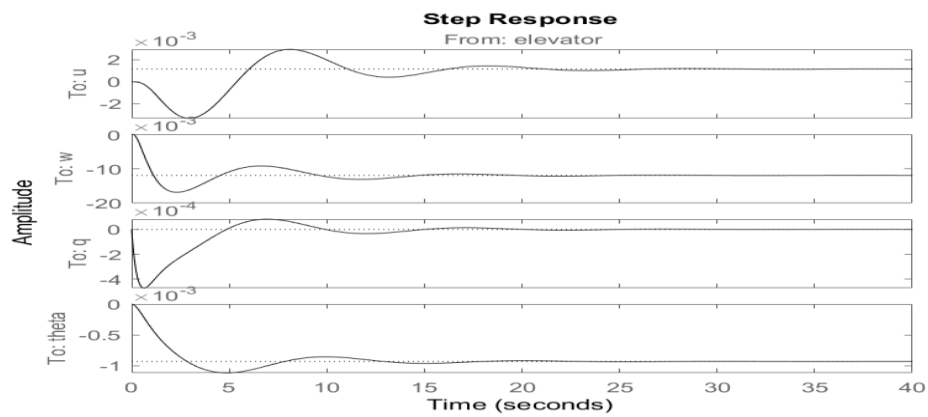


Fig. A5.19 step response of HA with pole placement (LQR)

Table A5.1 Response analysis of HA with LQR (pole placement):

pole	Damping	Frequency(rad/Sec)	Time constant (Sec)
$-1.77e-01 + 6.19e-01i$	$2.75e-01$	$6.43e-01$	$5.66e+00$
$-1.77e-01 - 6.19e-01i$	$2.75e-01$	$6.43e-01$	$5.66e+00$
$-1.69e+00$	$1.00e+00$	$1.69e+00$	$5.90e-01$
$-2.29e+00$	$1.00e+00$	$2.29e+00$	$4.36e-01$

Table A5.2 Response analysis of HA:

pole	Damping	Frequency(rad/Sec)	Time constant (Sec)
$-1.77\text{e-}01 + 6.19\text{e-}01\text{i}$	$2.75\text{e-}01$	$6.43\text{e-}01$	$5.66\text{e+}00$
$-1.77\text{e-}01 - 6.19\text{e-}01\text{i}$	$2.75\text{e-}01$	$6.43\text{e-}01$	$5.66\text{e+}00$
$1.69\text{e+}00$	$-1.00\text{e+}00$	$1.69\text{e+}00$	$-5.90\text{e-}01$
$-2.29\text{e+}00$	$1.00\text{e+}00$	$2.29\text{e+}00$	$4.36\text{e-}01$

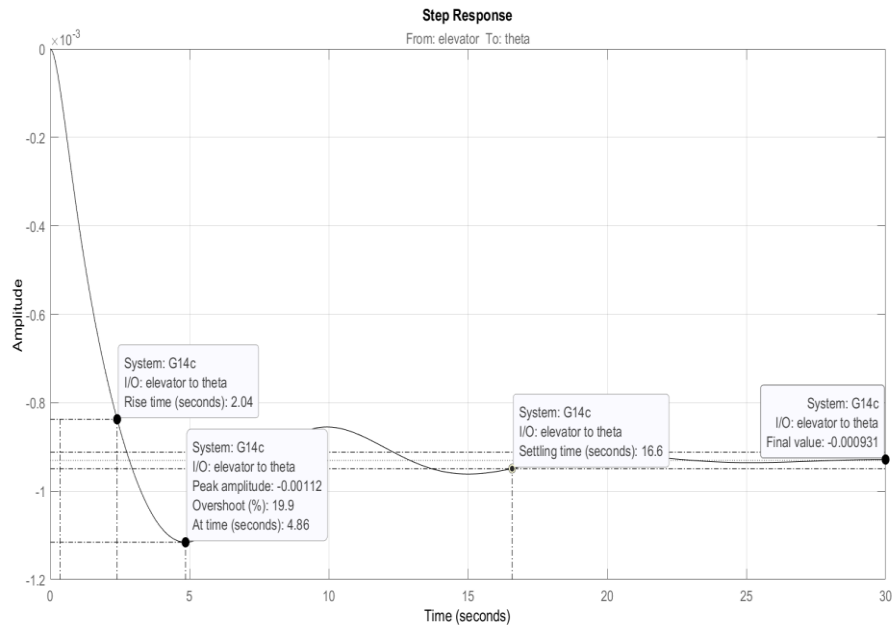


Fig. A5.20 Step response of G14 with LQR

TABLE: A5.3 Response Analysis of HA with LQR

Transfer Function	Rise Time	TransientTime	SettlingTime	SettlingMin	SettlingMax	Overshoot	Undershoot	Peak	Peak Time
G11c	0.4985	24.5864	30.3332	4.0319e-04	0.0029	157.7625	293.6032	0.0033	2.9711
G12c	0.7281	18.5621	18.5785	-0.0168	-0.0091	41.4372	1.3949	0.0168	2.3287
G13c	0	18.4698	NaN	-4.7159e-04	8.2525e-05	Inf	Inf	4.7159e-04	0.6250
G14c	2.0434	16.5871	16.5871	-0.0011	-8.3939e-04	19.8960	0	0.0011	4.8581

$$K = 1.0e+03 * [0.1141 \quad -0.0399 \quad -1.6590 \quad -1.5004]$$

The updated system with LQR is:

A =

	u	w	q	theta
u	-0.2974	0.3635	-0.4676	-5.35
w	-1.893	-0.3664	43.49	8.402
q	0.1417	-0.006977	-3.678	-2.103
theta	0	0	1	0

B =

elevator
u -0.00032
w 0.0056
q -0.0022
theta 0

For pitch control different controllers are designed and comparison are given below in table A5.4:

TABLE A5.4 Response Analysis of controllers for pitch control:

System	Controllers	GM (dB)	PM (dB)	Rise time	Settling Time	Overshoot	Peak	Stability
$G_{14} = \frac{\theta(s)}{\delta_e}$	PID	-20.6 dB @ 2.02 rad/s	69 deg @ 20.1 rad/s	0.064 s	1 s	15.7 %	1.16	YES
	IMC	-2.51 dB	19.8 deg	0.377 s	10.1 s	232	3.32	YES
	LQG	-0.98 dB	5.16 deg	0.346s	6.49 e+03 s	1.01e+03	11.1	YES
	LQR	1.0747e+03	inf	2.0434s	16.5871 s	19.8960	0.0011	YES

TABLE A5.5. Qualitative Comparisons among different controllers:

Criteria	PID	NDI	SMC	MPC	LQR	LQG	Pole Placement
Nonlinearity Handling	Low	High	High	High	Low	Low	Low
Model Dependency	Low	High	Medium	High	High	High	High
Robustness	Low-Medium	Low-Medium	High	Medium-High	Medium	Medium	Low-Medium
Computational Load	Very Low	Medium	Medium	High	Low	Medium	Low
Constraint Handling	Poor	Poor	Fair	Excellent	Poor	Poor	Poor
Ease of Implementation	Easy	Medium	Medium	Difficult	Medium	Medium	Medium
Stochastic Noise Handling	Poor	Poor	Fair	Fair	Poor	Excellent	Poor
Optimality (Cost Function)	No	No	No	Yes	Yes	Yes	No
Output Feedback Support	Yes	Limited	Yes	Yes	No	Yes	Limited

ADDITIONAL PUBLICATION:

[1] A. Kumar and O. Prakash, "Analysis of Lateral Dynamics of ARX Model of an Aircraft with Model Predictive Controller," 2022 International Conference for Advancement in Technology (ICONAT), 2022, pp. 1-6, doi: <https://doi.org/10.1109/ICONAT53423.2022.9726117>.

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