Name: **Enrolment No:**	

UPES

End Semester Examination, May 2025

Course: **Partial Differential Equations** **Semester : IV**

Program: **B. Sc. (Hons.) Geology/Physics by Research** **Time : 03 hrs.**

Course Code: MATH2054 **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 6 and 11 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Calculate the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the function $f(x,y)$ where the function $f(x,y) = \left(\log_e x\right) e^{x \cos y}$.	**4**	**CO1**
Q 2	Under what condition(s) of α and β, the function $u(x,y) = e^{\alpha x + \beta y}$ satisfies the equation $$u_{xx} - 7u_{xy} + 12u_{yy} = 0.$$	**4**	**CO1**
Q 3	Form a partial differential equation by eliminating the arbitrary constants a and b from the following relation $$z(x,y) = axe^y + \frac{1}{2}a^2e^{2y} + b.$$	**4**	**CO2**
Q 4	Find the general solution of Lagrange's equation $$x^2p + y^2q = z^2, \text{(where } p \equiv \frac{\partial z}{\partial x} \text{ and } q \equiv \frac{\partial z}{\partial y}\text{)}.$$	**4**	**CO2**
Q 5	Solve the linear partial differential equation $$(D - D')(D + D' + 1)z = 0,$$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Examine the functional dependence of the functions $$u(x,y) = y\sqrt{1-x^2} + x\sqrt{1-y^2}\,,$$ $$v(x,y) = \sqrt{(1-x^2)(1-y^2)} - xy.$$ If functionally dependent, find the relation between them.	**10**	**CO1**

	OR A torpedo has the shape of a cylinder with conical ends. For a given surface area, show that the dimensions which give maximum volume are $l = h = \frac{2}{\sqrt{5}} r$, where l is the length of the cylinder, r its radius, and h the altitude of the cone.		
Q 7	Apply Charpit's method to find the complete solution of the non-linear partial differential equation $yzp^2 - q = 0$ (where $p \equiv \frac{\partial z}{\partial x}$ and $q \equiv \frac{\partial z}{\partial y}$).	**10**	**CO2**
Q 8	Determine the region(s) in the xy-plane where the following equation: $(1 + x)u_{xx} + 2xyu_{xy} - y^2 u_{yy} = 0,$ is elliptic, hyperbolic or parabolic.	**10**	**CO3**
Q 9	Solve the partial differential equation: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$ with the conditions $u(x, 0) = 3 \sin n\pi x; \quad u(0, t) = 0 = u(l, t), 0 < x < l.$	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**(i)** Find the general solution of the partial differential equation $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y} + \sin(x - 2y)$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. **(ii)** Reduce the linear partial differential equation $3\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0.$ to canonical form.	**20**	**CO3**
Q 11	A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = A \sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x, t) = A \sin(\pi x/l) \cos(\pi ct/l).$ **OR** A tightly stretched string with fixed endpoints $x = 0$ and $x = \pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity given as $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.03 \sin x - 0.04 \sin 3x,$ then find the displacement $y(x, t)$ at any point of the string at any time t.	**20**	**CO4**