Name: **Enrolment No:**	

UPES

End Semester Examination, Dec 2025

Course: Advanced Engineering Mathematics- I **Semester : I**

Program: B.Tech.(SoAE) **Time : 03 hrs.**

Course Code: MATH1059 **Max. Marks: 100**

Instructions:

- Attempt all questions from Section A (each carrying 4 marks).
- Attempt all questions from Section B (each carrying 10 marks) and Q7 has an internal choice.
- Attempt all questions from Section C (each carrying 20 marks) and Q11 have internal choice.
- Calculator is allowed

SECTION A

(5Q x 4M = 20Marks)

S. No.		**Marks**	**CO**
Q 1	If $x^2 = au + bv;\ y^2 = au - bv$, then prove that $$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$$ where $\left(\frac{\partial u}{\partial x}\right)_y$ denotes the partial differentiation of u w. r. t. x, when y is constant.	**4**	**CO1**
Q 2	If $\alpha(x), \beta(x), \gamma(x)$ are differentiable in (a, b), prove that there exists at least one point c in (a, b), such that $$\begin{vmatrix} \alpha(a) & \beta(a) & \gamma(a) \\ \alpha(b) & \beta(b) & \gamma(b) \\ \alpha'(\mathrm{c}) & \beta'(\mathrm{c}) & \gamma'(\mathrm{c}) \end{vmatrix} = 0$$	**4**	**CO1**
Q 3	Find the complementary function of the ordinary differential equation $$\frac{d^2y}{dx^2} - 4\,\frac{dy}{dx} + 4y = 0.$$	**4**	**CO4**
Q 4	Let the rate of increase of a country's population be given by the differential equation $\frac{dy}{dt} = ky$, where k is specified as the proportional constant and $y(t)$	**4**	**CO4**

	represents the population at time t. Determine when the population of the country will triple, given that it has already doubled in 50 years.		
Q 5	Evaluate the integral $\int_0^{\pi/4} cos^3 2\theta \, sin^4 2\theta \, d\theta$, using the relation between Beta & Gamma functions.	**4**	**CO2**
	SECTION B **(4Q x 10M = 40 Marks)**		
Q 6	Find the Taylor series expansion of the function $\tan\left(x+\frac{\pi}{4}\right)$ as far as 4th term of the expansion. Hence evaluate $\tan(46.5^o)$.	**10**	**CO1**
Q 7	If the motion of a fluid is given by $\vec{V} = (y \sin z - \sin x)\,\hat{\imath} + (x \sin z + 2yz)\,\hat{\jmath} + (xy \cos z + y^2)\,\hat{k},$ then show that the motion is irrotational and hence find its velocity potential. **OR** Find the work done in moving a particle in a force field $\vec{F} = (2x - y + z)\hat{\imath} + (x + y - z^3)\hat{\jmath} + (3x - 2y + 4z)\hat{k}$ once around the curve in $xy -$ plane $\frac{x^2}{25} + \frac{y^2}{16} = 1$.	**10**	**CO3**
Q 8	Evaluate the integral by changing the order of integration $\int_0^6 \int_{x/3}^2 x\sqrt{(1+y^3)}\, dy\, dx$	**10**	**CO2**
Q 9	Find all the critical points for the competitive interaction as described by the Lotka-Volterra competition model $x' = 0.05x(70 - x - 2y)$ $y' = 0.01y(80 - 2x - y).$	**10**	**CO5**
	SECTION-C **(2Q x 20M = 40 Marks)**		
Q 10	Verify the Green's theorem for the integral given by $\oint_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the area enclosed by the $x -$ axis and the upper half of circle $x^2 + y^2 = a^2$.	**20**	**CO3**

Q 11	Test whether the following differential equations are exact or not. If not, then using suitable integrating factor solve it. (i) $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0.$ (ii) $x\frac{dy}{dx} + \frac{y^2}{x} = y$ **OR** Find the general solution of the following ordinary differential equations, (i) $(D^2 + 1)y = \text{cosec}x.$ (ii) $(D^2 + 6D + 9)y = 5^x - \sin x \cos x.$ Where $D \equiv \frac{d}{dx}$.	**10 + 10**	**CO4**