Name:

Enrolment No:

UPES
End Semester Examination, May 2025

Course: Advanced Engineering Mathematics II **Semester: II**
Program: B. Tech. SoCS **Time: 03 hrs.**
Course Code: MATH1065 **Max. Marks: 100**

Instructions: Attempt all questions from Section A, Section B and Section C. There are internal choices in Questions 9 and 10. Use of a scientific calculator is permitted.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$ where Δ and ∇ are forward difference and backward difference operators respectively.	**4**	**CO1**
Q 2	Show that if $f(z)$ is analytic and $Re[f(z)]$ is constant then $f(z)$ is a constant.	**4**	**CO2**
Q 3	Discuss the nature of the singularity of the function $$f(z) = \frac{\sin 4z - 4z}{z^2},$$ at the point $z = 0$.	**4**	**CO2**
Q 4	Find the Laplace transform of the function $f(t) = t^3\delta(t-4)$.	**4**	**CO4**
Q 5	Classify the partial differential equation: $x^2\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u.$	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	A tank is discharging water through an orifice at a of depth x meter below the surface of the water whose area is A meter^2. The following are the values of x for the corresponding values of A:	**10**	**CO1**

A	1.257	1.39	1.52	1.65	1.809	1.962	2.123	2.295	2.462	2.650	2.827
x	1.50	1.65	1.80	1.95	2.10	2.25	2.40	2.55	2.70	2.85	3.00

Using the formula

$$(0.018)T = \int_{1.5}^{3.0} \frac{A}{\sqrt{x}}\,dx\ ,$$

calculate T, the time (in seconds) for the level of the water to drop from 3.0 meter to 1.5 meter above the orifice.

Q 7	Find the value of the integral $\oint_C |z|\bar{z}\,dz$, where C consists of the line segment $-1 \leq x \leq 1$ and the upper half of the circle $|z| = 1$, positively oriented.	**10**	**CO2**
Q 8	Show that $x = 0$ is an ordinary point and $x = 1$ is a regular singular point of the following differential equation: $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$	**10**	**CO3**
Q 9	Express the following function in terms of unit step function and find its Laplace transform: $f(t) = \begin{cases} 0, & 0 < t < 1 \\ t - 1, & 1 < t < 2 \\ 1, & 2 < t \end{cases}$ **OR** Find the function $F(t)$ whose inverse Laplace transform $f(s)$ is given by $\frac{s+4}{s(s-1)(s^2+4)}$.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Apply the method of separation of variables to obtain the solution of the equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ for case of wave motion only. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right).$ **OR** Obtain the solution of the equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ for case of wave motion only using the method of separation of variables. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0\sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x,t)$.	**20**	**CO5**
Q 11	(i) Apply Laplace transform to solve the differential equation: $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 64\sin 2t,\ y(0) = 0,\ \left(\frac{dy}{dt}\right)_{t=0} = 1.$ (ii) Find the Fourier series to represent, $f(x) = x^2$ for $-\pi \leq x \leq \pi$.	**10+10**	**CO4**