Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES
End Semester Examination, December 2023

Course: Mathematical Physics (Generic) — **Semester: I**
Program: BSc (Mathematics, Chemistry, Geology) — **Time: 03 hrs.**
Course Code: PHYS1031 — **Max. Marks: 100**

Instructions: All questions are compulsory

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Show that the differential equation, $(2xy^3 + xy)dx + \left(3x^2y^2 + \frac{x^2}{2}\right)dy = 0$ is exact	**4**	**CO1**
Q 2	Evaluate, $\int_0^{\infty} x^3 e^{-x^2} dx$ using Gamma function	**4**	**CO1**
Q 3	Verify that the Hermite polynomial of degree 4 has the form $\mathrm{H}_4(x) = 16x^4 - 48x^2 + 12$ [where nth degree Hermite polynomial, $\mathrm{H_n}(x) = (-1)^{\mathrm{n}}\, \mathrm{e}^{\mathrm{x}^2} \frac{\mathrm{d^n}\left(\mathrm{e}^{-\mathrm{x}^2}\right)}{\mathrm{dx^n}}$]	**4**	**CO2**
Q 4	Prove that for Legendre polynomial of degree l, $P_l(-x) = (-1)^l\, P_l(x)$	**4**	**CO2**
Q5	Evaluate the complex integral, $\oint \frac{z^2+1}{(z+1)+(z+2)} dz$ where, $\lvert z \rvert = \frac{3}{2}$	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q6	What is De Moivre's theorem? Prove that $(\cos\theta + i\sin\theta)^{\mathrm{n}} = \cos \mathrm{n}\theta + i\sin \mathrm{n}\theta$	**10**	**CO1**

Q7	Fourier function is defined as, $f(x) = x^2$, $0 < x < 2\pi$ Evaluate, a_0 and a_n OR Fourier function is defined as, $f(x) = \pi$, $0 < x < 2\pi$ Evaluate, a_0 and a_n	**10**	**CO2**
Q8	Using beta function, Show that $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$	**10**	**CO3**
Q9	Hermite polynomial differential equation for 1D harmonic oscillator is given by $H_n''(\xi) - 2xH_n'(\xi) + (\lambda - 1)H_n(\xi) = 0$ Applying the concept of Hermite polynomial as a solution, deduce the recurrence relation. [Consider, $\lambda = \frac{2E}{\hbar\omega}$ and $\lambda - 1 = 2n$, $\xi = \alpha x$, $\alpha = \sqrt{\frac{m\omega}{\hbar}}$. Symbols have their usual meaning]	**10**	**CO4**
SECTION C **(2Qx20M=40 Marks)**			
Q10	(a) Find the roots of the complex number, $x^4 + i = 0$	**10**	**CO2**
	(b) Use Cauchy's integral formula to show $\oint \frac{e^{zt}}{z^2+1} dz = 2\pi i \sin t$ where, $t > 0$ *and* $\lvert z \rvert = 3$	**10**	**CO2**
	OR		
	(a) Show that $\frac{(\cos\theta + i\sin\theta)^8}{(\sin\theta + i\cos\theta)^4} = \cos 12\theta + i\sin 12\theta$	**10**	**CO2**
	(b) Use Cauchy's integral formula to show $\oint \frac{dz}{z^2+1} = 0$ where, $\lvert z \rvert = 2$	**10**	**CO2**

Q11	(a) A voice signal curve is best fitted with ordinary polynomial $8x^3 - 4x^2 + 2x + 2$. Convert it into Hermite polynomial (b) Verify the Legendre polynomial recurrence relation, $(2l+1)xP_l(x) - lP_{l-1}(x) = (l+1)P_{l+1}(x)$ where $g(x,t) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum_{l=0}^{\infty} P_l\,(x)\,t^l$	**10** **10**	**CO3** **CO3**