Name: **Enrolment No:**	

UPES

End Semester Examination, December 2023

Course: Engineering Mathematics I **Semester: I**

Program: B. Tech. [ASE+APE(UP)+ADE+Chemical+E&CE+Civil+ Mechatronics+ Mechanical +Electronics & Communication] **Time : 03 hrs.**

Course Code: MATH 1050 **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Find the rank of matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{bmatrix}$	**4**	**CO1**
Q 2	Evaluate $\int_0^\infty x^{\frac{1}{4}} e^{-\sqrt{x}} dx$.	**4**	**CO2**
Q 3	If $u = x^2 + y^2 + z^2$, prove that $xu_x + yu_y + zu_z = 2u$.	**4**	**CO2**
Q 4	Find $curl\left(curl\vec{V}\right)$ where $\vec{V} = 2xz^2\hat{\imath} - yz\hat{\jmath} + 3xz^3\hat{k}$ at $(1, 1, 1)$.	**4**	**CO3**
Q 5	Evaluate $\int_C \vec{F}.\overrightarrow{dr}$, where $\vec{F} = x^2\hat{\imath} + xy\hat{\jmath}$ and C is the boundary of the square in the plane $z = 0$ and bounded by $x = 0, y = 0, x = 1$ and $y = a$.	**4**	**CO3**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find the modal matrix P such that $P^{-1}AP$ is a diagonal matrix.	**10**	**CO1**
Q 7	Evaluate $\iint_R (x+y)dydx$, where R is the region bounded by the lines $x = 0, x = 2, y = x$ & $y = x + 2$.	**10**	**CO2**
Q 8	If the vector $\vec{F} = (ax^2y + yz)\hat{\imath} + (xy^2 - xz^2)\hat{\jmath} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal, find the value of a. Also find the curl of this solenoidal vector.	**10**	**CO3**

Q 9	Find the Fourier series representing $f(x) = x, \;\; 0 < x < 2\pi$. **OR** Using Maclaurin's series, expand $log(1+x)$. Hence, deduce that $log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{3} + \cdots$	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10A	If $u = x + y + z, v = \; x^2 + y^2 + z^2, w = yz + zx + xy$, prove that $grad\ u, grad\ v$ and $grad\ w$ are coplanar vectors. **OR** Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = \; x^2 + y^2 - 3$ at the point $(2, -1, 2)$.	**10**	**CO3**
Q 10B	If a force $\vec{F} = 2x^2y\hat{\imath} + 3xy\hat{\jmath}$ displace a particle in the xy plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$, find the work done. **OR** Apply the Green's theorem to evaluate $\oint_C (2x^2 - y^2)\, dx + (x^2 + y^2)dy$, where C is the boundary of the region enclosed by $x -$axis and the upper half of the circle $x^2 + y^2 = a^2$	**10**	**CO3**
Q 11	Find the Fourier series for $f(x)$, if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.	**20**	**CO4**