Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Program: B. Sc. (Hons.) Mathematics **Semester: IV**
Course: Riemann Integration and Series of Functions **Time : 03 hrs.**
Course Code: MATH 2014 **Max. Marks: 100**

Instructions: All questions are compulsory

SECTION A
(All questions are compulsory)

S. No.		**Marks**	**CO**
Q 1	Prove that the function defined as $f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is not Riemann integrable on [0, 1].	**4**	**CO1**
Q 2	Examine the convergence of $\int_0^1 \frac{1}{x^2} dx$.	**4**	**CO2**
Q 3	Show that the series $x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \frac{x^4}{(1+x^4)^3} + \cdots$ is not uniformly convergent on [0, 1].	**4**	**CO3**
Q 4	Suppose that f is a function defined on $[a, b]$ and that f is Riemann integrable on each closed subinterval of (a, b). Give an example to show that f may not be Riemann integrable on $[a, b]$.	**4**	**CO1**
Q 5	Prove that $\Gamma(m+1) = m\Gamma(m)$.	**4**	**CO2**

SECTION B
(All questions are compulsory Q9 has internal choices)

S. No.		**Marks**	**CO**
Q 6	Prove that if f is integrable on $[a, b]$ then f^2 is also integrable on $[a, b]$.	**10**	**CO4**
Q 7	The function f is defined on $[0, \infty)$ by $f(x) = (-1)^{n-1}$, $n-1 \leq x \leq n$, $n \in N$ then show that the integral $\int_0^\infty f(x)dx$ does not converge.	**10**	**CO2**
Q 8	Show that the function $f(x) = x^2$ is Riemann integrable on any interval $[0, k]$.	**10**	**CO1**
Q 9	Show that the series $\sum_{n=1}^{\infty} \frac{x}{(nx+1)\{(n-1)x+1\}}$, is uniformly convergent on any interval $[a, b]$, $0 < a < b$, but only pointwise on $[0, b]$. **OR** Show that the serie$\sum_1^\infty \frac{(-1)^{n-1}}{n+x^2}$s is uniformly convergent but not absolutely for all real values of x.	**10**	**CO3**

SECTION-C (All questions are compulsory, Q 10 has two parts each of 10 marks, Q11 has internal choices)			
Q 10	**A.** Let $< f_n >$ be a sequence of functions such that $\lim_{n\to\infty} f_n(x) = f(x)$, $x\epsilon[a,b]$ and let $M_n = \sup_{x\epsilon[a,b]} \lvert f_n(x) - f(x)\rvert$ then $f_n \to f$ uniformly on $[a, b]$ if and only if $M_n \to 0\ as\ n \to \infty$. **B.** If a power series $\sum a_n x^n$ converges for $x = x_0$ then it is absolutely convergent for every $x = x_1$, when $\lvert x_1\rvert < \lvert x_0\rvert$.	**20**	**CO4**
Q 11	**A.** Show that $\sum \frac{1}{n^2} = \frac{\pi^2}{6}, n \epsilon N$. **B.** Show that $\sum \frac{\log n}{n^x}$ converges uniformly for all real $x \geq 1 + \alpha > 1$. **OR** **A.** Prove that the power series $1 + \frac{a.b}{1..c}x + \frac{a(a+1)b(b+1)}{1.2.c.(c+1)}x^2 + \cdots$, has unit radius of convergence. **B.** Discuss the uniform convergence with respect to x of the series, $\sum_{n=1}^{\infty} \frac{x^n}{n^\alpha}$.	**20**	**CO3**