Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2022

Course: PDE and System of ODE **Semester: IV**

Program: B.Sc. (Hons.) Mathematics **Time : 03 hrs.**

Course Code: MATH 2030 **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Check the linearity of the PDE $u_x + 5u = x^2 y$ by using linear transformation	4	CO1
Q 2	For what regions in xy −plane the second order PDE $xu_{xx} + 2xyu_{xy} + yu_{yy} + xu_x + yu_y = 0$ is hyperbolic?	4	CO2
Q 3	Find the characteristic curves for the PDE $x^2 u_{xx} - y^2 u_{yy} - y^2 \cos x + x^2 = 0; x > 0$.	4	CO2
Q 4	Find the solution for the infinite string problem $u_{tt} = 9u_{xx}, t > 0, -\infty < x < \infty$ satisfying the conditions $u(x,0) = x^2, u_t(x,0) = 0$.	4	CO3
Q 5	Show that $X(t) = \begin{bmatrix} e^{3t} \\ -e^{3t} \end{bmatrix}$ is a solution of homogeneous system $\frac{dX(t)}{dt} = \begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix} X(t)$.	4	CO4

SECTION B

(4Qx10M= 40 Marks)

Q 1	Find the solution of the IVP $(x-y)u_x + (y-x-u)u_y = u$ with $u(x,0) = 1$.	10	CO1
Q2	Prove that the solution of the Cauchy's Problem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$ on $D = \{(x,y,u): x^2 + y^2 \neq 0, z > 0\}$ subject to the Cauchy data $x^2 + y^2 = 1, u = 1$ is $u = \sqrt{x^2 + y^2}$.	10	CO1
Q3	Find the solution of $u_{xx} = h^2 u_t$, with $u(0,t) = u(1,t) = 0$ and $u(x,0) = sin\frac{\pi x}{l}$	10	CO3
Q4	Find the solution $X(t) = [x \;\; y]^T$ of the non-homogeneous system: $$\frac{dX}{dt} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} e^t \\ 0 \end{bmatrix}$$ ***OR***	10	CO4

	Consider the system of ODE $\frac{dX}{dt} = AX$, where $X(t) = [x \;\; y]^T$ and A is a square matrix of order 3 with an eigen value of algebraic multiplicity 3. Find all linearly independent solutions and hence write the complete solution of given system of ODE.		
SECTION-C **(2Qx20M=40 Marks)**			
Q 1	Transform the second order initial value problem: $y'' + y = 0$ with $y(0) = 1, y'(0) = 0$ into a system of first order initial value problem, and use the Runge-Kutta method with $h = 0.1$ to find the approximate value of $y(0.2)$.	20	CO4
Q2	Solve Laplace equation $u_{xx} + u_{yy} = 0$ with $u(0,y) = u(1,y) = 0, for\ 0 \le y \le 4, u(x,0) = 0, u(x,4) = x\ cos\frac{\pi x}{2}$ for $0 \le x \le 1$ **OR** Consider the wave equation with a forcing term as follows: $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + e^x$ for $0 < x < 1, t > 0$ $y(0,t) = y(1,t) = 0$ for $t \ge 0$ $y(0,t) = \frac{\partial y}{\partial t}(x,0) = 0$ for $0 \le x \le 1$ Using suitable transformation reduce it into homogeneous wave equation and hence find the solution.	20	CO3