Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2022

Course: Linear Algebra **Semester: II**
Program: B.Sc. (Hons.) Mathematics & Int. B.Sc. - M.Sc. Mathematics **Time : 03 hrs.**
Course Code: MATH 1047 **Max. Marks: 100**

Instructions:

1. Section A has 5 questions. All questions are compulsory.
2. Section B has 4 questions. All questions are compulsory. Question 9 has internal choice to attempt any one.
3. Section C has 2 questions. All questions are compulsory. Question 11 has internal choice to attempt either (a), (b) or (c), (d).

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Show the vectors of the set $\{p_0(x), p_1(x), \dots, p_n(x)\}$ are linearly independent in $P_n(F)$, where $p_k(x) = x^k + x^{k+1} + \cdots + x^n$ for $k = 0,1, \dots, n$.	**4**	**CO1**
Q 2	Is the set of linearly independent vectors $\{(1,0,0,0), (0,2,3,0), (0,0,1,4)\}$ form the basis of vector space R^3? If not then give your justification.	**4**	**CO1**
Q 3	In R^3, let $\alpha = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $\beta = \{(1,1,1), (1,1,0), (1,0,0)\}$ be the ordered bases of R^3. Calculate change of coordinate matrix.	**4**	**CO2**
Q 4	Let A be the matrix of linear operator T with respect to the basis β. Find the minimal polynomial of T if A is given as below $$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}.$$	**4**	**CO3**
Q 5	Show that the operator $T \in L(C^2)$ defined by $T(w,z) = (z,0)$ is not diagonalizable. (Here, C represents set of complex numbers).	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	For the data $\{(1,2), (2,3), (3,5), (4,7)\}$, use least square approximation to find the best fit linear function.	**10**	**CO4**
Q 7	Show that the set $W = \{(a,b,c): a,b,c \in R \text{ and } a + b + 2c = 0\}$ is a subspace of vector space R^3.	**10**	**CO1**

Q 8	Let $T: P_2(R) \to P_3(R)$ be the linear transformation defined by $$T\big(f(x)\big) = 2f'(x) + \int_0^x 3f(t)dt.$$ Show that T is one-to-one but not onto.	**10**	**CO2**
Q 9	Let $T: R^2 \to R^3$ be the linear transformation defined by $$T(x, y) = (x + 3y, 0, 2x - 4y).$$ Find the matrix representation of T with respect to standard ordered bases for R^2 and R^3. OR Find the null(T)and range(T) of the linear transformation $T: R^3 \to R^2$ defined by $T(x, y, z) = (x - y, 2z)$.	**10**	**CO2**
SECTION-C (2Qx20M=40 Marks)			
Q 10	(a) The linear operator $T: R^2 \to R^2$ defined by $T(x, y) = (3x + 2y, x)$. Let β be the standard ordered basis for R^2. Find the matrix of dual map T' with respect to the dual basis β'. (b) Let T be the linear operator on R^2 defined by $T(a, b) = (a + 2b, -2a + b)$ and β be the standard ordered basis for R^2. Verify Cayley-Hamilton theorem for operator T.	**20**	**CO3**
Q 11	(a) Give the definition of self-adjoin operator on an inner product space. (b) Let $V = P(R)$ with the inner product $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(t)g(t)dt$, and consider the subspace $P_2(R)$ with the standard ordered basis β. Apply Gram-Schmidt process to replace β by an orthogonal basis $\{v_1, v_2, v_3\}$ for $P_2(R)$, and then use this orthogonal basis to obtain an orthonormal basis for $P_2(R)$. **OR** (c) Give the definition of normal operator on an inner product space. (d) Calculate the minimal solution of the following system of equations $$x + 2y - z = 1$$ $$2x + 3y + z = 2$$ $$4x + 7y - z = 4$$	**20**	**CO4**