Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2022

Course: Algebra **Semester: I**
Program: B.Sc. (Hons.) Mathematics **Time: 03 hrs.**
Course Code: MATH 1040 **Max. Marks: 100**

Instructions:

1. Section A has 5 questions. All questions are compulsory.
2. Section B has 4 questions. All questions are compulsory. Question 9 has internal choice to attempt any one.
3. Section C has 2 questions. All questions are compulsory. Question 11 has internal choice to attempt any one.

SECTION A

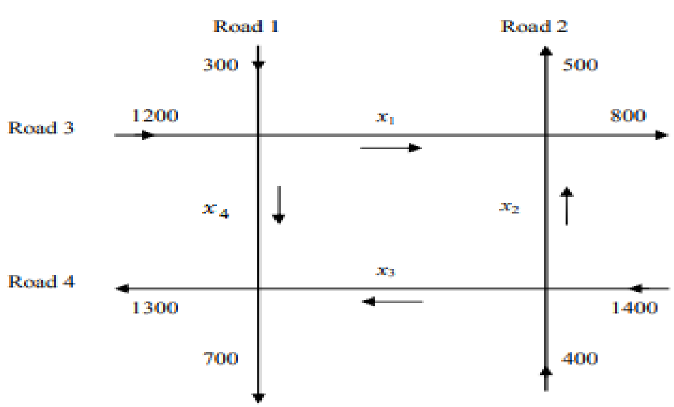
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find all the 5^{th} roots of unity.	**4**	**CO1**
Q 2	Prove that for $a, b, m, c \in I$ if $a \equiv b (mod\ m)$ then $ac \equiv bc (mod\ m)$.	**4**	**CO2**
Q 3	Consider the chemical equation given below. $$\mathrm{CaCl_2} \; + \; \mathrm{AgNO_3} \; = \; \mathrm{Ca(NO_3)_2} \; + \; \mathrm{AgCl}$$ Solve the system of linear equations and obtain the balanced chemical equation.	**4**	**CO3**
Q 4	Find $\det(A)$ given that A has $p(\lambda)$ as its characteristics polynomial, where $p(\lambda) = \lambda^2 - 6\lambda + 1$.	**4**	**CO3**
Q 5	Check if the given set of polynomials form a basis for $\boldsymbol{P}_2$(set of all polynomials of degree less than or equal to 2). $$\{1 - 3x + 2x^2, \; 1 + x + 4x^2, \; 1 - 7x\}.$$	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

Q 6	Solve the equation $z^2 + (i - 2)z + 3 - i = 0$ in complex variable z.	**10**	**CO1**
Q 7	Let $f: Z \to Z$ be a function defined as $f(x) = x + 5$. Determine whether the function is invertible or not. If it is invertible, find its inverse.	**10**	**CO2**

Q 8	In a given electrical network, the equations for the currents i_1, i_2 and i_3 are $3i_1 + i_2 + i_3 = 8$; $2i_1 - 3i_2 - 2i_3 = 5$; $7i_1 + 2i_2 - 5i_3 = 0$. Use Gauss Elimination method to find the currents.	**10**	**CO3**
Q 9	Let $T: P_3 \to P_2$ be a mapping defined by $$T(a_0 + a_1x + a_2x^2 + a_3x^3) = 5a_0 + a_3x^2.$$ Is T is a linear transformation? If yes, find the bases for the range and kernel of T. **OR** Show that the set of all continuous functions **F**[a, b] = $\{f(x), x \in [a,b]: \int_a^b f(x)dx = 0\}$ is a subspace of **C**[a, b]. Find an example of a subset of **C**[a, b] which is not a subspace. Give justification for that.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	In downtown Denver, the traffic flows of a set of four intersecting roads during the rush hours on a typical weekend are shown in the schematic below. The arrow shows the direction and the number besides that is the average number of vehicles leaving and entering per hour. Express the given problem into a system of linear equations and solve the system. 	**20**	**CO3**

Q 11	**a)** Let M_{nn} be the set of all $n\times n$ matrices. Check whether the set of all $n\times n$ upper triangular matrices and set of all $n\times n$ lower triangular matrices are subspaces of M_{nn} or not, with respect to usual matrix addition and scalar matrix multiplication. **b)** Let $V=\mathbb{R}^{\boldsymbol{n}}$ be a vector space, and $W=\{(a_1,a_2,\dots,a_n): a_2={a_1}^2\}$. Check whether W is a subspace of V or not, with respect to usual addition and scalar multiplication **OR** Find the bases for the null space and row space of $P=\begin{bmatrix}1 & -1 & 3\\ 5 & -4 & -4\\ 7 & -6 & 2\end{bmatrix}$	**20**	**CO4**