Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2021

Programme Name: B.Tech ASE and B.Tech ASE+AVE **Semester : III**
Course Name : Numerical Methods **Time : 03 hrs**
Course Code : MATH 2044 **Max. Marks : 100**
Nos. of page(s) : 03

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Instructions: Attempt all questions from Section A (Q1-Q5, each carrying 04 marks); Section B (Q6-Q9, each carrying 10 marks); Section C (Q10 & Q11, each carrying 20 marks). Scientific calculators are allowed for the examination.

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q1.	Let $x_0 = 1.5$ be the initial approximation of a root of the equation $$x^2 + \log_e x - 2 = 0.$$ Find an approximate root of the equation using fixed point iteration method (iteration method), correct upto three significant digits.	[4]	CO1
Q2.	Establish the operator relation $E \equiv e^{hD}$, where E and D denote the Shifting and Differential operators respectively. (h is the step-length).	[4]	CO2
Q3.	Show that the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix}$ is not factorable in form of $A = LU$ by Doolittle's method. Find a new matrix B by rearranging the rows of the matrix A so that B is factorable by that method. Give reason for your answer.	[2+2]	CO4
Q4.	Show that the partial differential equation $x^2 u_{xx} - 2xy\, u_{xy} - 3y^2 u_{yy} + u_y = 0$ is hyperbolic type at every points on xy-plane except for the coordinate axes $x = 0$ and $y = 0$. Identify the characteristic of the equation on coordinate axes.	[3+1]	CO6
Q5.	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is $$\frac{dy}{dx} = -ky, \text{ where } k = 0.01.$$ Given that $x_0 = 0$ and $y_0 = 100$. Determine how much substance will remain at the moment $x = 100$, using Modified Euler's method with the step-length $h = 100$.	[4]	CO5

SECTION B

(Q6-Q8 are compulsory and Q9 has internal choice)

Q6. Fit a polynomial of degree three, which takes the following values, by Newton forward interpolation formula, and find $y(3.5)$. **[8+2]** **CO2**

x	3	4	5	6
y	6	24	60	120

Q7. Use fourth order Runge-Kutta method to solve for $y(0.4)$ taking $h = 0.2$, for the following initial value problem. **[10]** **CO5**

$$\frac{dy}{dx} = 1 + y^2, \text{ with the initial condition } y(0) = \lim_{x\to\infty} \frac{x^2}{2^x}.$$

Q8. Let $x_0 = 1.6$ be an initial approximation of the root of the following equation. **[10]** **CO1**

$$10xe^{-x^2} = 1$$

Use Newton-Raphson method to find a positive root of that equation, correct to six decimal places.

Q9. The speeds of an electric train at various times after leaving one station are given in the following table. **[10]** **CO3**

Time t(in hour)	0	$\frac{1}{120}$	$\frac{1}{60}$	$\frac{1}{40}$	$\frac{1}{30}$
Speed v(in mph)	0	13	33	39.5	40

Find the distance (in mile), travelled by the train, and acceleration of the train in 2 minutes.

OR

Evaluate the integration $\int_0^{\frac{\pi}{2}} \sqrt{1 - k\sin^2\phi}\, d\phi, k = 0.162$, by Simpson's $\frac{1}{3}$ rule, dividing the interval $0 \le \phi \le \frac{\pi}{2}$ into six equal subintervals.

SECTION-C

(Q10 is compulsory, and Q11.A and Q11.B have internal choices)

Q10.A Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then apply Gauss-Seidel method to evaluate an approximate solution, taking the initial approximation as $x_1^{(0)} = 1, x_2^{(0)} = 1, x_3^{(0)} = 1$. Perform three iterations. **[3+7]** **CO4**

$$x_1 - x_2 + 5x_3 = 7$$
$$6x_1 - x_2 + x_3 = 20$$
$$x_1 + 4x_2 - x_3 = 6.$$

Q10.B	Show that the matrix $A = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 82 & 39 \\ 6 & 39 & 26 \end{bmatrix}$ is decomposable by Cholesky method. Hence, find the solution of the following system of equations by that method. $$4x_1 + 2x_2 + 6x_3 = 16$$ $$2x_1 + 82x_2 + 39x_3 = 206$$ $$6x_1 + 39x_2 + 26x_3 = 113$$	**[3+7]**	**CO4**
Q11.A	Solve the Laplace equation $u_{xx} + u_{yy} = 0$ with $h = \frac{1}{3}$ over the boundary of a square of unit length with $u(x, y) = 16x^2y^2$ on the boundary by Liebmann's iteration process. Perform three iterations using Gauss Seidel method. **OR** Solve the Poisson's equation $u_{xx} + u_{yy} = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length 1. Perform one iteration by Gauss Seidel method to solve the linear equations in u assuming initial solution as (0,0,0,0).	**[10]**	**CO6**
Q11.B	Solve $\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 - x)$ taking $h = 1$ and employing Bender-Schmidt method. Continue the solution through five time steps. **OR** Using Crank-Nicholson's method, solve $u_{xx} = 16u_t, 0 < x < 1, t > 0$, given that $u(x, 0) = 0, u(0, t) = 0, u(1, t) = 50t$. Compute u for two steps in t direction taking $h = \frac{1}{4}$.	**[10]**	**CO6**