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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
DEHRADUN
End Semester Examination - Dec 2021

Program/Course : M. Tech Chemical (Spl. in Process Design)
Subject: Advanced Transport Phenomena
Code: CHPD 7018
No. of pages: 03

Semester: II
Max. Marks: 100
Duration: 3 hrs

NOTE:

(A) **OPEN BOOK cum OPEN NOTES EXAMINATION**

(B) Assume all missing data. State your assumptions clearly. Draw sketches wherever necessary.

SECTION A: ANSWER ALL QUESTIONS

2 x 30 = 60 marks

1. A relationship often used to model fluid flow in porous media is *Darcy's law*,

$$\underline{v} = -\frac{k}{\mu}\left(\nabla p - \rho g\right) = -\frac{k}{\mu}\nabla P \qquad (1)$$

where k is the Darcy permeability, $\underline{g}$ is the gravitational acceleration vector, and P is the dynamic pressure. The Darcy permeability has units of m^2 and is a constant for a given material. The velocity and pressure in Darcy's law are averaged over a length scale large compared to the size of the individual pores, but compared to the dimensions of the object. That is, micro structural details are not considered. Pore structure enters only through its effect on the value of k, which is usually determined by experiment.

   (a) For a material of average porosity ε (volume fraction of pores), use Darcy's law to obtain expression of conservation of mass in the fluid, first in integral form and then in differential form. [10 marks]

   (b) Show that for an incompressible fluid, the differential equation from part (a) reduces to

$$\nabla^2 P = 0 \qquad (2)$$

   [10 marks]

   (c) Consider an idealised porous material consisting of straight, cylindrical pores of diameter d. Assume that all pores are parallel to the x, y or z axes and they intersect at points described by a simple cubic lattice of dimension $l >> d$. The overall dimensions of a sample of this material are of order of magnitude

L, where $L >> l$. The volume flow rate Q in a single pore segment is described by *Poiseuille's law*,

$$Q = \frac{\pi|\Delta P_l|d^4}{128\mu l} \tag{3}$$

where $|\Delta P_l|$ is the pressure drop for a segment of length l. Evaluate k for this material. [10 marks]

2. Two solid plates of differing thermal properties are in contact, as shown in Fig. 1. The bottom surface of plate 1 is at constant temperature T_0, and at the top surface of plate 2 there is convective heat transfer to the ambient air (temperature T_∞ heat transfer coefficient h). The plate dimensions in the x and z directions are sufficiently large that T= T(y) only. The rate of frictional heating at the contact surface between two solids may be expressed as

$$H_s = c\gamma U \tag{4}$$

where U is the relative velocity of the solids, γ is the force per unit area holding the solids in contact (i.e., normal to the interface), and c is the coefficient of dry friction for the materials involved.

   (a) Determine the steady-state temperature profile in the plates when both are at rest. [15 marks]

   (b) Assume now that plate 2 moves horizontally at speed U, while plate 1 remains stationary. The plates are held in contact only by gravity. Determine the rate of interfacial energy generation and its effect on the steady-state temperature profile in the plates. [15 marks]

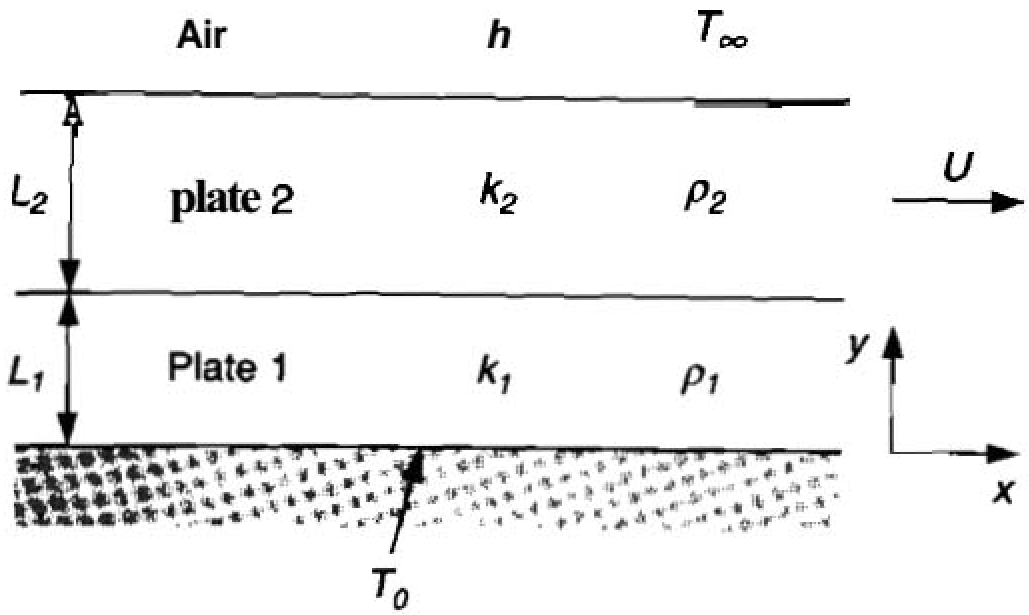

Figure 1: Two solid plates of differing thermal properties

SECTION B: ANSWER ALL QUESTIONS

1 x 40 = 40 marks

1. After a candle is lit the wax next to the flame soon reaches its melting temperature (T_m), and the candle begins to melt. As shown in Fig.2, assume that after a certain time 't' a candle of radius R has a length L(t). There is a net heat flux q_0 to the top surface of the candle, which represents radiant energy transfer from the flame minus convective losses. Assume that q_0 is constant. There is also convective heat transfer from the sides of the candle to the surrounding air (ambient temperature T_∞, heat transfer coefficient h). The base of the candle is at the ambient temperature. For simplicity, assume that the melting occurs slowly enough that the time derivative in the energy conservation equation is negligible; this is a pseudo-steady approximation.

   (a) Find the pseudosteady temperature profile T(z, t) in the candle, assuming that the temperature is approximately independent of radial position.

   [15 marks]

   (b) Evaluate dL/dt at a given instant. Assume that the layer of melted wax on top of the candle is thin enough that the heat flux toward the candle on the liquid side of the melt-solid interface is approximately equal to q_0.

   [15 marks]

   (c) Assuming that the initial length is L(0)=L_0, what is the time (t_m) required for the candle to melt completely?

   [10 marks]

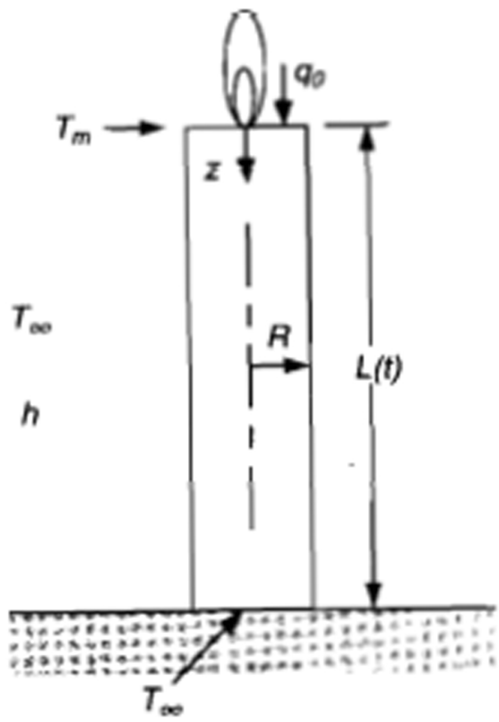

Figure 2: Melting of a candle