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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, June 2021

Programme: B.Tech | **Semester – II**
Course Name: Mathematics-II | **Max. Marks: 100**
Course Code: MATH 1027 | **Duration : 3 Hrs**
No. of page/s: 3

Section A

(Attempt all questions)

		MARKS	
1.	If $u(x,y) = 4xy - 3x + 2$ is harmonic then corresponding analytic function $f(z) = u(x,y) + iv(x,y)$ in terms of complex variable z is given by A. $-2iz^2 - 3z + 2 + ic$ B. $-2x^2 + 2y^2 - 3y + c$ C. $-2z^2 + 3iz - 3z + ic$ D. None of these	[5]	CO2
2.	The particular integral of the differential equation $\frac{d^2x}{dt^2} - 4x = \cos^2 t$ is given by A. $-\frac{1}{8} - \frac{1}{16}\cos 2x$ B. $\frac{1}{25}cos^2 t$ C. $-\frac{1}{8} + \frac{1}{16}\cos 2x$ D. $-\frac{1}{8} - \frac{1}{16}\cos 2t$	[5]	CO1
3.	The radius of convergence of the power series $\sum_{n=0}^{\infty}\left(\frac{n\sqrt{2}+i}{1+2in}\right) z^n$ is A. 1 B. $\frac{1}{2}$ C. 0 D. None of these	[5]	CO3

4.	Find the type of singularity of function $e^{-\frac{1}{z^2}}$ A. Isolated Singularity B. Removable Singularity C. No singularity D. Essential Singularity	**[5]**	**CO3**
5.	The residue of $f(z) = \frac{z^3}{z^2-1}$ at $z = \infty$ is given by A. 1 B. -1 C. 0 D. None of these	**[5]**	**CO3**
6.	The solution of the partial differential equation $\left(\frac{y^2 z}{x}\right) p + xzq = y^2$ is given by A. $\phi(x + y, x - z) = 0$ B. $\phi(x^2 + y^2, x - z^2) = 0$ C. $\phi(x^3 - y^3, x^2 - z^2) = 0$ D. None of these	**[5]**	**CO4**
SECTION B **(All questions are compulsory)**			
7.	Evaluate by using Cauchy integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)} dz,$ where c is the circle $\lvert z \rvert = \frac{3}{2}$	**[10]**	**CO1**
8.	Solve the following differential equation: $$(1 - t^2)\frac{d^2 z}{dt^2} + t\frac{dz}{dt} - z = t(1 - t^2)^{3/2}$$	**[10]**	**CO2**
9.	Obtain the Taylor or Laurent series which represents the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $1 < \lvert z \rvert < 2$ and $\lvert z \rvert > 2$.	**[10]**	**CO3**

10.	Form a partial differential equation by eliminating the arbitrary function from the equation $\phi(x^2+y^2+z^2, z^2-2xy)=0$.	**[10]**	**CO4**
11.	Apply the method of calculus of residues to prove that $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \frac{\pi}{6}$ **OR** Apply the method of calculus of residues to evaluate the integral $$\int_{-\infty}^{\infty} \frac{\log(1+x^2)}{(1+x^2)} dx$$	**[10]**	**CO3**
	SECTION C **(Q12A, Q12B are compulsory. Both have internal choice)**		
12.A	A string is stretched and fastened to two points $(0,0)$ and $(l,0)$ and released at rest from the initial deflection given by $$f(x) = \begin{cases} \frac{2k}{l}x & when \quad 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l-x) & when \quad \frac{l}{2} < x < l \end{cases}$$ Find the deflection of the string at any time t. **OR** Find the complete solution of the following partial differential equation $$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos 2y\,(\sin x + \cos x)$$	**[10]**	**CO4**
12.B	Solve the differential equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for the condition of heat along a rod without radiation subject to the following conditions: (i) u is finite when $t \to \infty$ (ii) $u=0$ when $x=l$ for all values of t (iii) $\frac{\partial u}{\partial x}=0$ when $x=0$ for all values of t (iv) $u=u_0$ when $t=0$ for $0<x<l$ **OR** Solve the partial differential equation: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} = \frac{4x}{y^2} - \frac{y}{x^2}$.	**[10]**	**CO4**

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