Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May-June 2021

Course: Mathematics **Semester: II**
Program: B.Tech Food Technology **Time : 03 hrs.**
Course Code: MATH1038 **Max. Marks: 100**

Instructions:

SECTION A (Type your answers)

S. No.	MCQs or Fill in the blanks (1.5 marks each)	**30 Marks**	**CO**
1	If Lagrange's mean value theorem is applicable on $f(x) = x^2$ in $(1,5)$, then the value of c is (a) 3 (b) 4 (c) 5 (d) None of these	**1.5**	**CO1**
2	If n^{th} term of the series does not tend to zero as $n \to \infty$, then series is (a) Necessarily convergent (b) May or may not be convergent (c) Never convergent (d) None of these	**1.5**	**CO1**
3	The series $\sum \frac{1}{n^{\frac{2}{3}}}$ is (a) Convergent (b) Divergent (c) Oscillatory (d) None of these	**1.5**	**CO3**
4	If $z = f(x + ct) + \emptyset(x - ct)$, then (a) $\frac{\partial^2 z}{dt^2} = c^2 \left(\frac{\partial^2 z}{dx^2}\right)$ (b) $\frac{\partial^2 z}{dt^2} = c \left(\frac{\partial^2 z}{dx^2}\right)$ (c) $\frac{\partial^2 z}{dt^2} = c^4 \left(\frac{\partial^2 z}{dx^2}\right)$ (d) None of these	**1.5**	**CO4**
5	If $w = ln\sqrt{x^2 + y^2}$, the value of $\frac{\partial w}{\partial y}$ is (a) $\frac{x}{x^2+y^2}$ (b) $\frac{y}{x^2+y^2}$ (c) $\frac{x^2}{x^2+y^2}$ (d) None of these	**1.5**	**CO4**
6	The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is (a) 1 (b) 2 (c) 3 (d) None of these	**1.5**	**CO5**
7	The value of $\Gamma(n)\Gamma(1-n)$ is (a) $\frac{\pi}{sinn\pi}$ (b) $\frac{\pi}{cosn\pi}$ (c) $\frac{\pi^2}{sinn\pi}$ (d) all the above	**1.5**	**CO1**
8	If $f(x)$ is odd function, then which of the Euler's coefficients is present in its Fourier series expansion? (a) a_0 (b) a_n (c) b_n (d) all of these	**1.5**	**CO3**
9	For the function $f(x) = x^2$, the value of the Euler's coefficient b_n is (a) zero (b) finite (c) infinite (d) none of these	**1.5**	**CO1**
10	The value of $\beta(\frac{9}{2},\frac{7}{2})$ is (a) $\frac{\pi}{2048}$ (b) $\frac{5\pi}{2048}$ (c) $\frac{7\pi}{2048}$ (d) None of these	**1.5**	**CO1**

11	If the matrix $\begin{bmatrix} x & 2 & x+2 \\ 3 & 5 & 8 \\ x+1 & 7-x & 12 \end{bmatrix}$ is singular, the value of x is ………..	**1.5**	**CO5**
12	For consistent m x n non-homogeneous system of linear equations $AX = B$, if rank of A = number of unknowns, then the system possesses ……….. number of solutions.	**1.5**	**CO5**
13	The system of equations $x + 2y + 3z = 0, 2x + 3y + z = 0, 4x + 5y + 4z = 0$ has ………..number of solutions.	**1.5**	**CO5**
14	If $\bar{A} = 2x^2\boldsymbol{i} - 3yz\boldsymbol{j} + xz^2\boldsymbol{k}$ and $f = 2z - x^3y$, the value of $\bar{A}.\nabla f$ at the point $(1, -1, 1)$ is ………..	**1.5**	**CO4**
15	If $\bar{A} = (bx + 4y^2z)\boldsymbol{i} + (x^3 \sin z - 3y)\boldsymbol{j} - (e^x + 4cosx^2y)\boldsymbol{k}$ is solenoidal, then the value of b is ………	**1.5**	**CO4**
16	The divergence of $(2x^2z\boldsymbol{i} - xy^2z\boldsymbol{j} + 3yz^2\boldsymbol{k})$ at the point $(1,1,1)$ is ……….	**1.5**	**CO4**
17	The maximum value of $f(x, y) = 1 - x^2 - y^2$ is ……….	**1.5**	**CO4**
18	The point where the function is neither minimum nor maximum is called as ……….	**1.5**	**CO4**
19	The value of $lim_{x\to 0} \frac{1-\cos x}{x^2}$ is ……….	**1.5**	**CO1**
20	If $u = x^2 + y^2 + z^2$, where $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$ the total derivative $\frac{du}{dt}$ is ……………	**1.5**	**CO4**
	SECTION B 20 marks 4 questions 5 marks each (scan and upload)		
Q	Short Answer Type Question (5 marks each) Scan and Upload 4 questions 5 marks each	**20 Marks**	**CO**
1	Verify Rolle's theorem on $f(x) = \begin{cases} x^2 + 1, & 0 \le x \le 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$	**5**	**CO2**
2	Define series of positive terms with an example and derive the necessary condition for the convergence of a positive term series.	**5**	**CO1**
3	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.	**5**	**CO4**
4	Prove that $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.	**5**	**CO1**
	SECTION C 30 marks		
Q	**Two case studies 15 marks each subsections (scan and upload)**	**30 Marks**	**CO**
1	Case Study 1: **(Convergence and divergence of infinite series)** (a) Define Geometric series and derive the conditions for its convergence and divergence. [5 marks] (b) Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{2^n+3}{3^n+1}\right)^{\frac{1}{2}}$ [5 marks] (c) Define D'Alembert's ratio test and using this, test the convergence of the series whose n^{th}term is $\frac{(n+3)!}{3!n!3^n}$ [5 marks]	**15**	**CO2**

2	Case Study 2: **(Fourier Series Expansion of functions)** (a). Define Fourier Series of a periodic function $f(x)$ and Dirichlet's conditions for the expansion of $f(x)$ as Fourier series. [4 marks] (b) Derive Euler's formulae. [5 marks] (c) Find the Fourier series of $f(x) = \begin{cases} 0, & when -\pi \leq x \leq 0 \\ x^2, & when\ 0 \leq x \leq \pi \end{cases}$ which is assumed to be periodic with period 2π. [6 marks]	**15**	**CO3**
	SECTION- D 20 marks (scan and upload)		
Q	Long Answer type Questions Scan and Upload (10 marks each)	**20 Marks**	**CO**
1	Solve the system of non-homogeneous equations $x + y - z = 0,\ 2x - y + z = 3$ and $4x + 2y - 2z = 2$.	**10**	**CO5**
2	Diagonalize the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	**10**	**CO5**