Name:

Enrollment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, January 2021

Programme Name: M.Tech Petroleum Engineering **Semester : I**
Course Name : Applied Mathematics in Petroleum Engineering **Time : 03 hrs**
Course Code: MATH 7001 **Max. Marks : 100**

Section A (All questions are compulsory)			
1.	An approximate value of π is given by 3.1428571 and its true value is 3.1415926. The relative error will be approximately. A. 0.402502 B. 0.0402502 C. 0.00402502 D. 0.000402502s	**[5]**	**CO1**
2.	Let $x = s$ be a solution of $x = g(x)$ and suppose that g has a continuous derivative in some interval J containing s. Then a sufficient condition for the fixed point iteration to converge at any point a in J is that, A. g is differentiable at a B. g is continuous at a C. $\lvert g'(x)\rvert < 1$ in J D. $\lvert g'(x)\rvert < a$ in J	**[5]**	**CO3**
3.	Choose the correct approximation to the integral $\int_0^1 x^3 dx$ using Trapezoidal rule considering five intervals. A. 0.26 B. 0.25 C. 0.24 D. 0.30	**[5]**	**CO2**
4.	Using Newton-Raphson method, choose the first iteration for a root of the function $f(x) = 3x - \cos x - 1$. Assume the first guess as $x_0 = 0.6$. A. 0.60 B. 0.6071 C. 0.5921 D. 0.6125	**[5]**	**CO3**
5.	Find a bound on the eigenvalues of the following matrix using Gerschgorin theorem. A. 1 B. 1/2 C. -1/2 D. 5/6	**[5]**	**CO4**

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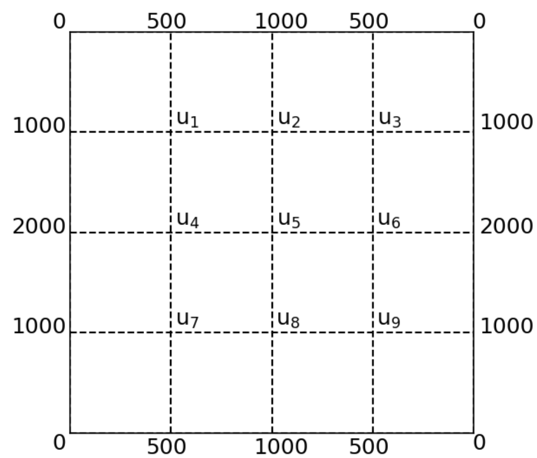
6.	The 5-point approximation with the coefficient scheme or stencil of the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, given by A. $u(x-h,y)+\ u(x,y-h)+\ u(x-h,y)+u(x,y-h)+u(x,y)=0$ B. $u(x-h,y)+\ u(x,y-h)+\ u(x+h,y)+u(x,y+h)+2u(x,y)=0$ C. $u(x+h,y)+\ u(x,y+h)+\ u(x+h,y)+u(x,y+h)+3u(x,y)=0$ D. $u(x+h,y)+\ u(x,y+h)+\ u(x-h,y)+u(x,y-h)+4u(x,y)=0$	**[5]**	**CO5**
	SECTION B **(Q1-Q5 are compulsory and Q5 has internal choices)**		
1.	Use Simpson's $\frac{1}{3}$rd rule to evaluate $\int_0^1 \frac{1}{1+x}\,dx$ by dividing the interval of integration into 8 equal parts. Use the evaluated value to approximate $\log_e 2$.	**[10]**	**CO2**
2.	Compute five iterations of the bisection method to find a root between 2.74 and 2.75 of the function $x\log_{10}x=1.2$.	**[10]**	**CO3**
3.	Compute five iterations for solving the following system of equations using Gauss-Seidel Iteration method with initial choice as $x=0, y=0, z=0$ and $w=0$ $x-0.25y-0.25z=50$ $-0.25x+y-0.25w=50$ $-0.25x+z-0.25w=25$ $-0.25y-0.25z+w=25$	**[10]**	**CO3**
4.	Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ using Liebmann's method for the square mesh of the following figure with boundary values displayed.	**[10]**	**CO5**



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5.	Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, under the condition $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$, using Schmidt method (Take $h = 0.2$, $\alpha = \frac{1}{2}$) **OR** Consider a laterally insulated metal bar of length 1 and such that $c^2 = 1$ in the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. Suppose that the ends of the bar are kept at temperature $u = 0°C$ and the temperature in the bar at some instant call it $t = 0$ is $f(x) = \sin \pi x$. Applying the Crank-Nicolson method with $h = 0.2$ and $r = 1$, find the temperature $u(x,t)$ in the bar for $0 \le t \le 0.2$.	**[10]**	**CO6**
	SECTION C **(Q1 is compulsory and has internal choices)**		
1	Use the fourth-order Runge-Kutta method to solve the following system of ordinary differential equations, assuming $x_0 = 0,\ y_0 = 4,\ z_0 = 6$. Solve the system at $x = 2$ with a step size of 0.5. $\frac{dy}{dx} = -0.5y$ $\frac{dz}{dx} = 4 - 0.3z - 0.1y$ **OR** Use the Euler'ss method to solve the following system of ordinary differential equations, assuming $x_0 = 0,\ y_0 = 4,\ z_0 = 6$. Solve the system at $x = 2$ with a step size of 0.5. $\frac{dy}{dx} = -0.5y$ $\frac{dz}{dx} = 4 - 0.3z - 0.1y$	**[20]**	**CO4**