UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Examinations (Online Mode), Jan-Feb 2021

Course: Mathematics   **Semester:** I
Program: BCA-Spl. IoT   **Time:** 3 Hrs
Course Code: MATH1037   **Max. Marks:** 100

	SECTION - A	**6 x 5 = 30 Marks**

1. Each Question will carry 5 Marks
2. Instruction: Select the correct option(s)

Q 1	The real roots of the quadratic equation $x^4 - 8x^2 + 15 = 0$ are A. $-\sqrt{3}, +\sqrt{3}, -\sqrt{5}, +\sqrt{5}$ B. $-\sqrt{2}, +\sqrt{2}, -\sqrt{6}, +\sqrt{6}$ C. $+2\sqrt{2}, -2\sqrt{2}, -6, +6$ D. None of these	CO1
Q 2	If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then the value of $A^2 - 5A + 7I$ is A. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 2 & -2 \\ 3 & -4 \end{bmatrix}$ D. None of these	CO2
Q 3	For what value of k, the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & k \end{bmatrix}$ is 2? A. -1 B. 0 C. 1 D. None of these	CO2
Q 4	The value of $lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ is A. 1 B. 2 C. 3 D. None of these	CO3
Q 5	If $y = u^2 + 1$ and $u = x^3 + x + 4$, then the value of $\frac{dy}{dx}$ is A. $6x^5 + 6x^3 + 4x^2 + 2x + 8$ B. $6x^5 + 8x^3 + 24x^2 + 2x + 8$ C. $6x^5 + 8x^3 + 4x^2 + 2x + 8$ D. None of these	CO3
Q 6	A bag contains 8 white and 6 red balls. The probability of drawing two balls of the same color is A. $41/91$ B. $42/91$ C. $43/91$ D. None of these	CO4

1. Each question will carry 10 marks
2. Instruction: Answer on a separate white sheet, upload the solution as image.

Q 7	(a) Solve the equation $x^2 - 9x + 14 = 0$ by completing the square method. (b) A total of Rs. 3,300 is raised by collecting equal amounts from a certain number of people. If there were 22 people more, each person would have to contribute Rs. 200 less to raise the same amount. How many people actually contributed?	CO1
Q 8	If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, verify that $A(adj\ A) = (adj\ A)A = \|A\|I_3$.	CO2
Q 9	Define the Rank of a matrix. Reduce the following matrix into its Echelon form and hence find its rank. $$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$	CO2
Q10	Prove that $\int e^{ax} \cos bx\ dx = \dfrac{e^{ax}}{a^2 + b^2}[a \cos bx + b \sin bx] + C$.	CO3
Q 11	There are three bags: first containing 1 white, 2 red, 3 green balls; second containing 2 white, 3 red, 1 green balls; third containing 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag.	CO4

Section – C   **1 x 20 = 20 Marks**

1. Each Question carries 20 Marks.
2. Instruction: Answer on a separate white sheet, upload the solution as image.

Q 12	**A:** Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9, 7x + 3y - 2z = 8$, and $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions. **B:** The prices, in rupees per unit, of the three commodities X, Y and Z are x, y and z respectively. A purchases 4 units of Z and sells 3 units of X and 5 units of Y. B purchases 3 units of Y and sells 2units of X and 1 unit of Z. C purchases 1 unit of X and sells 4 units of Y and 6 units of Z. In the process A, B and C earn Rs.6000, Rs.5,000 and Rs.13,000 respectively. Using matrices, find the prices of the three commodities (note that selling the units is positive earning and buying the units is negative earning.)	CO2

(OR)

A: Solve the following homogeneous system of equations for its non-trivial solution

$x + 3y + 2z = 0, 2x - y + 3z = 0, 3x - 5y + 4z = 0, x + 17y + 4z = 0.$

B: An amount of Rs. 4,000 is distributed into three investments at the rate of 7%, 8% and 9% per annum respectively. The total annual income is Rs. 317.50 and the annual income from the first investment is Rs. 5 more than the income from the second. Find the amount of each investment.