Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2020

Course : MULTIVARIATE CALCULUS **Semester : III**
Program : B.Sc. (Hons.) Mathematics **Time : 03 Hour**
Course Code : MATH 2029 **Max. Marks: 100**

SECTION A

Attempt all questions. Each question carries 5 marks. This section contains multiple choice questions. For multiple choice question, only one option is correct.

S.No.		CO
Q1	If $u = e^{xyz}$ then value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$ is (A) $e^{xyz}(1 + 3xyz + x^2y^2z^2)$ (B) $e^{xyz}(1 + 3xyz + 2x^2y^2z^2)$ (C) $e^{xyz}(1 + 3xyz + 3x^2y^2z^2)$ (D) $e^{xyz}(1 + 3xyz + 4x^2y^2z^2)$	CO1
Q2	The directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ is (A) $-\frac{10}{3}$ (B) $\frac{10}{3}$ (C) $\frac{13}{3}$ (D) $-\frac{13}{3}$	CO1
Q3	What is the value of following double integral? $$\int_{x=0}^{x=3} \int_{y=0}^{y=\frac{1}{x}} y e^{xy} \, dy dx$$ (A) $-\frac{1}{3}$ (B) $\frac{1}{3}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{6}$	CO2
Q4	What is the area of the region bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$? (A) $\frac{31}{3} - 4\log 2$ (B) $\frac{31}{3} + 4\log 2$ (C) $\frac{28}{3} - 4\log 2$ (D) $\frac{28}{3} + 4\log 2$	CO2
Q5	What is the volume generated by revolving a quadrant of the circle $x^2 + y^2 = a^2$, about its diameter? (A) $\frac{8}{3}\pi a^3$ (B) $\frac{6}{3}\pi a^3$ (C) $\frac{4}{3}\pi a^3$ (D) $\frac{2}{3}\pi a^3$	CO2
Q6	If $\vec{F} = x^2y\hat{\imath} + xz\hat{\jmath} + 2yz\hat{k}$ then $div(curl\, \vec{F})$ is (A) -1 (B) 0 (C) 1 (D) 3	CO3

SECTION B

Attempt all questions. Each question carries 10 marks. Question 11 has internal choice.

Q7	Evaluate $\iint_R [[x + y]] \, dxdy$ over the rectangle $R = \{(x, y): 0 \le x \le 1, 0 \le y \le 2\}$, where $[[x + y]]$ denotes greatest integer less than or equal to $(x + y)$.	CO2

Q8	Determine whether the line integral $$\int (2xyz^2)dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz$$ is independent of the path of integration ? If so, then evaluate it from (1,0,1) to $(0, \frac{\pi}{2}, 1)$.	**CO3**
Q9	Show that the function $$f(x,y) = \begin{cases} (x+y)\sin\left(\frac{1}{x+y}\right), & x+y \neq 0 \\ 0, & x+y = 0 \end{cases}$$ is continuous at (0,0) but its partial derivatives f_x and f_y do not exist at (0,0).	**CO1**
Q10	Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$.	**CO1**
Q11	Calculate the volume of the solid bounded by the planes $x = 0, y = 0, x + y + z = 1$ and $z = 0$. **OR** Evaluate the following by changing into polar coordinates $$\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2+y^2}\,dxdy.$$	**CO2**
SECTION C **Question of this section carries 20 marks and it has internal choice.**		
Q12	Verify Stokes' theorem for $$\vec{F} = (y - z + 2)\hat{\imath} + (yz + 4)\hat{\jmath} - xz\hat{k}$$ over the surface of a cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy −plane open from the bottom. **OR** Verify Gauss divergence theorem for $$\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$$ taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.	**CO3**