UPES SAP ID No.: _________________

UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

Examination, July 2020

Programme: B.Tech Mechanical Engg / ADE | **Semester : VI**
Course Name: Applied Numerical Techniques | **Max. Marks : 100**
Course Code: MATH3001 | **Attempt Duration : 3 Hrs.**
No. of page/s:

Note:

1. Read the instruction carefully before attempting.
2. This question paper has two section, Section A and Section B.
3. There are total of six questions in this question paper. **One** in **Section A** and **five** in **Section B**
4. **Section A** consist of multiple choice based questions and has the total weightage of 60%.
5. **Section B** consist of long answer based questions and has the total weightage of 40%.
6. The maximum time allocated to **Section A** is 90 minutes.
7. **Section B** to be submitted within 24 hrs from the scheduled time i.e. if the examination starts at 10:00 AM, the long answers must be submitted by 09:59:59 AM next day. Similarly, if the examination starts at 2:00 PM it must be submitted by 01:59:59 PM next day. (*Exceptional provision due extraordinary circumstance due to COVID-19 and due to internet connectivity issues in the far-flung areas*).
8. No submission of **Section B** shall be entertained after 24 Hrs.
9. **Section B** should be attempted after **Section A**
10. **The section B** should be attempted in blank white sheets (hand written) with all the details like programme, semester, course name, course code, name of the student, Sapid at the top (as in the format) and signature at the bottom (right hand side bottom corner)
11. Both section A & B should have questions from entire syllabus.
12. The COs mapping, internal choices within a section is same as earlier

Section – A (Attempt all the questions)

1. Answer all the questions

<table>
<tr><td>(a)</td><td>Which of the following methods are used for interpolation of unequally spaced data? (Select all that apply)

(a) Newton Gregory forward interpolation formula
(b) Lagrange interpolation formula
(c) Newton divided difference interpolation formula
(d) Newton Gregory backward interpolation formula</td><td>2M
CO1</td></tr>
<tr><td>(b)</td><td>The value of $\nabla^3 y_3$ from the following data is

<table><tr><td>x</td><td>2.6</td><td>2.65</td><td>2.7</td><td>2.75</td></tr><tr><td>y</td><td>32.5980</td><td>34.4750</td><td>36.4470</td><td>38.5210</td></tr></table>
(a) 0.085
(b) 0.092
(c) 0.007
(d) None of these</td><td>2M
CO1</td></tr>
<tr><td>(c)</td><td>Consider the following table

<table><tr><td>x</td><td>x_0</td><td>x_1</td><td>x_2</td><td>x_3</td></tr><tr><td>y</td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td></tr></table>
While finding $\frac{dy}{dx}$ at the tabulated point x_0 using the first derivative formula derived from Newton-Gregory forward interpolation, the coefficient of $\Delta^3 y_0$ is

(a) $\frac{1}{2}$
(b) $-\frac{1}{3h}$
(c) $\frac{1}{3h}$
(d) None of these</td><td>2M

CO1</td></tr>
</table>

(d)	While evaluating $\int_1^3 \frac{1}{1+x^2}\, dx$ with step size $h = 0.2$ which of the following methods cannot be applied? (a) Trapezoidal rule (b) Simpson's $\frac{1}{3}$ rule (c) Simpson's $\frac{3}{8}$ rule (d) None of these	**2M** CO2
(e)	While solving a transcendental equation $f(x) = 0$, if $f(a).f(b) < 0$ in the interval $[a\ b]$, then the equation has (a) exactly one root in $[a\ b]$ (b) at least one root in $[a\ b]$ (c) no root in $[a\ b]$ (d) None of these	**2M** CO3
(f)	The fixed point iteration method defined as $x_{n+1} = \emptyset(x_n)$ converges in the interval $I = [a\ b]$ if (a) $\lvert\emptyset'(x)\rvert = 0$ in I (b) $\lvert\emptyset'(x)\rvert < 1$ in I (c) $\lvert\emptyset'(x)\rvert > 1$ in I (d) None of these	**2M** CO3
(g)	Which of the following is a diagonally dominant system? (a) $27x + 6y - z = 85; 6x + 15y + 2z = 72; x + y + 54z = 110$ (b) $3x - 4y - z = 40; x - 2y + 12z = -86; x - 6y + 2z = -32$ (c) $4x = 2y - z - 1; x + z = -4; 3x - 5y + z = 3$ (d) None of these	**2M** **CO4**
(h)	While decomposing the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ as a product LU where $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$ and $U = \begin{bmatrix} p & q & r \\ 0 & s & t \\ 0 & 0 & u \end{bmatrix}$, the L matrix is obtained as (a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix}$	**2M** **CO4**

	(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -3/2 & 7 & 1 \end{bmatrix}$ (c) $L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 3/2 & 7 & 1 \end{bmatrix}$ (d) None of these	
(i)	Given $3\frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5$. Using a step size $h = 0.3$, the value of $y(0.9)$ using Euler's method is most nearly (a) -35.318 (b) -36.458 (c) -600.213 (d) None of these	**2M** CO5
(j)	What are the methods to solve ODE? **(Select all that apply)** (a) Finite difference method (b) Taylor series method (c) Runge-Kutta method (d) Euler method	**2M** CO5
(k)	The Picard's solution in two approximations of the equation $\frac{dy}{dx} = x - y,\ y(0) = 1$ is (a) $1 - x + x^2 - \frac{x^3}{6}$ (b) $1 + x - x^2 + x^3$ (c) $1 + x - x^2 + \frac{x^3}{6}$ (d) None of these	**2M** CO5
(l)	Given y_0, y_1, y_2 and y_3. The Milne's corrector formula to find y_4 for $\frac{dy}{dx} = f(x, y)$ is (a) $y_4 = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4)$ (b) $y_4 = y_0 + \frac{h}{3}(f_2 + 4f_3 + f_4)$ (c) $y_4 = y_2 + \frac{h}{3}(f_2 + 2f_3 + f_4)$ (d) None of these	**2M** CO5

(m)	The finite difference scheme for the equation $2y'' + y = 5$ is (a) $y_{i+1} + 2y_i + y_{i-1} + h^2 y_i = 5h^2$ (b) $y_{i+1} - 2y_i + y_{i-1} + h^2 y_i = 5h^2$ (c) $y_{i+1} - 2y_i + y_{i-1} - h^2 y_i = 5h^2$ (d) None of these	**2M** CO6
(n)	The equation $x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} = x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y}$ is classified as (a) Parabolic (b) Elliptic (c) Hyperbolic (d) None of these	**2M** CO6
(o)	To solve $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ by Bendre-Schmidt recurrence relation with $h = 1$, the value of k is (a) $1/2$ (b) $1/8$ (c) 4 (d) None of these	**2M** CO6
(p)	The following boundary value problem is solved using finite difference method by taking number of subintervals $n = 3$ $x \frac{d^2 y}{dx^2} + y = 0; \quad y(1) = 1, \ y(2) = 2$ Then, the values of $y\left(\frac{4}{3}\right)$ and $y\left(\frac{5}{3}\right)$ are (a) $y\left(\frac{4}{3}\right) = \frac{408}{487}$, $y\left(\frac{5}{3}\right) = \frac{570}{487}$ (b) $y\left(\frac{4}{3}\right) = \frac{508}{487}$, $y\left(\frac{5}{3}\right) = \frac{670}{487}$ (c) $y\left(\frac{4}{3}\right) = \frac{608}{487}$, $y\left(\frac{5}{3}\right) = \frac{770}{487}$ (d) $y\left(\frac{4}{3}\right) = \frac{708}{487}$, $y\left(\frac{5}{3}\right) = \frac{870}{487}$	**3M** CO6
(q)	Given the differential equation $2\frac{dy}{dx} = (1 + x^2)y^2$ and $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21.$ The approximate value of $y(0.4)$ using Milne's predictor corrector method is (a) 1.27	**3M** CO5

(b) 1.37
(c) 1.41
(d) None of these

(r) From the following table, the approximate value of $y(10)$ is — **3M** CO1

x	5	6	9	11
y	12	13	14	16

(a) 8.23
(b) 14.66
(c) 20
(d) None of these

(s) Consider the following table — **3M** CO2

x	1	2	3	4	5	6
$y(x)$	198669	295520	389418	479425	564642	644217

The value of $y''(1)$ is approximately equal to
(a) -710
(b) -3098
(c) -1986
(d) None of these

(t) While solving the system $\begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ -18 \\ 25 \end{bmatrix}$ using Gauss-Jacobi iteration with initial values $\{x^{(0)}, y^{(0)}, z^{(0)}\} = \{0,0,0\}$, the solutions obtained in the second iteration $\{x^{(2)}, y^{(2)}, z^{(2)}\}$ are approximately equal to — **3M** CO4

(a) $x^{(2)} = 1.02\,, y^{(2)} = -0.965, z^{(2)} = 1.1515$
(b) $x^{(2)} = -1.02\,, y^{(2)} = 0.965, z^{(2)} = 1.1515$
(c) $x^{(2)} = 1.02\,, y^{(2)} = 0.965, z^{(2)} = -1.1515$
(d) None of these

(u) The value of the integral $\int_0^2 e^{x^2}\,dx$ estimated by using Trapezoidal rule with 10 intervals is — **3M** CO2

	(a) 17.0621 (b) 17.9983 (c) 18.3464 (d) None of these	
(v)	Consider $y' = e^x + y$ with $y = 0$ at $x = 0$. Then the value of $y(0.2)$ using Modified Euler's method correct to 3 decimals is (Hint: Use step size $h = 0.2$) (a) 0.2 (b) 0.2421 (c) 0.2468 (d) None of these	**3M** CO5
(w)	Using Taylor's series expansion (considering the terms up to 4 derivatives), the value of $y(0.1)$ from $y' = -1 + x^2 y, y(0) = 1$ is approximately equal to (a) 0.7865 (b) 0.8665 (c) 0.9003 (d) None of these	**3M** CO5
(x)	The value of $y(0.2)$ obtained by solving $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}, y(0) = 1$ using Runge-Kutta method of fourth order is (a) 0.096 (b) 1.196 (c) 2.164 (d) None of these	**3M** CO5
(y)	While solving the parabolic equation $u_{xx} = 2u_t$, $u(0,t) = u(4,t) = 0$ and $u(x,0) = x(4-x)$ with step size $h = 1$, The values of $u(2,3)$ and $u(3,4)$ obtained by Bendre-Schmidt recurrence relation are given by (a) $u(2,3) = 1.5$ and $u(3,4) = 0.75$ (b) $u(2,3) = 0.75$ and $u(3,4) = 1.5$ (c) $u(2,3) = 0.25$ and $u(3,4) = 0.95$	**3M** CO6

	(d) None of these	

Section – B (Attempt all the questions)
(5 × 8 marks)

2. The following table shows how the relative density ρ of air varies with altitude h. Using the *Gauss forward* central difference interpolation technique, estimate the relative density of air at 5 km.

h, (km)	0	1.525	3.050	4.575	6.10	7.625	9.150
ρ	1	0.8617	0.7385	0.6292	0.5328	0.4481	0.3741

[CO1,8 Marks]

3. A gas turbine is used to produce power as a gas flows through it. If the process is isothermal, the equation for work is given by

$$\dot{W} = -\dot{n}RT\int_{inlet}^{outlet}\frac{dP}{P}$$

where

$\dot{n}$=molar flow rate, in kmol/s
R= universal gas constant, 8.314 kJ/kmol K
T= temperature, in K
P= pressure, in kPa
$\dot{W}$=power, in kW.

Using *Simpson's 1/3rd rule* on 10 subintervals, estimate the power produced in an isothermal gas turbine if $\dot{n} = 0.1, T = 400, P_{inlet} = 500$ and $P_{outlet} = 100$.

[CO2, 8 Marks]

4. The following system of equations is designed to determine concentrations in a series of coupled reactors as a function of the amount of mass input to each reactor:

$$\begin{aligned} -3c_1 + 18c_2 - 6c_3 &= 1200 \\ 15c_1 - 3c_2 - c_3 &= 3800 \\ -4c_1 - c_2 + 12c_3 &= 2350 \end{aligned}$$

Obtain the concentration values correct to 3 decimals by using *Gauss-Seidel* iterative technique with initial approximate solution as $\left[c_1^{(0)}\ c_2^{(0)}\ c_3^{(0)}\right] = [300\ 220\ 310]$.

[CO4, 8 Marks]

5. Determine the value of $y(0.4)$ using Milne's predictor-corrector method given $y' = xy + y^2, y(0) = 1$; Find the initial values by Taylor series method.
[CO5, 8 Marks]

6. Compute u for one time step by Crank Nicholson's method if $16u_t = u_{xx}; 0 < x < 1$, $t>0$;$u(x,0) = u(0,t) = 0$ & $u(1,t) = 50t$ by taking $h=0.25$
[CO6, 8 Marks]

***