Name:

Enrolment No:

UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2020

Course: Mathematics II **Semester: II**

Course Code: MATH1005 **Time: 03 hrs.**

Programme: B.Tech. (All SoCS Branches) **Max. Marks: 100**

Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 20 multiple choice questions and 5 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 10:00 AM to 1:00 PM on 6th July 2020. The due time for PART A is 1:00 PM on 6th July 2020. After the due time, the PART A will not be available.

S. No.		Marks	CO
Q1 (i)	Change the independent variable x *to* z by the relation $z = f(x)$ in the differential equation, $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ to get a new differential equation $\frac{d^2y}{dz^2} + P_1\frac{dy}{dx} + Q_1y = R_1$ where P_1, Q_1 *and* R_1 are: A. $P_1 = \frac{\left(P\frac{d^2z}{dx^2}+\frac{dz}{dx}\right)}{\frac{dz}{dx}}, Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}\ R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$ B. $P_1 = \frac{\left(\frac{d^2z}{dx^2}+P\frac{dz}{dx}\right)}{\left(\frac{dz}{dx}\right)^2}, Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}\ R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$ C. $P_1 = \frac{\left(P\frac{d^2z}{dx^2}+\frac{dz}{dx}\right)}{\frac{dz}{dx}}, Q_1 = \frac{Q}{\frac{dz}{dx}}\ R_1 = \frac{R}{\frac{dz}{dx}}$ D. $P_1 = \frac{\left(\frac{d^2z}{dx^2}-P\frac{dz}{dx}\right)}{\left(\frac{dz}{dx}\right)^2}, Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}\ R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$	2	CO1
Q1 (ii)	The linear differential equation $\frac{1}{2}\left(\frac{1}{x} - y\right)dx - \frac{1}{2}\left(\frac{1}{y} + x\right)dy = 0$ is Exact differential equation if A. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = -\frac{1}{2}$ B. $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} = -\frac{1}{2}$ C. $M + N = 0$ D. $x\frac{\partial M}{\partial x} = y\frac{\partial N}{\partial y} = -\frac{1}{2}$	2	CO1

Q1 (iii)	The complete solution of $(D^2+1)^2(D-1)y=0$ is A. $y = c_1 cosx + c_2 sinx + c_3 e^x$ B. $y = (c_1 + c_2 x)e^x + (c_3 + c_4 x)e^{-x} + c_5 cosx$ C. $y = (c_1 + c_2 x)cosx + (c_3 + c_4 x)sinx + c_5 e^x$ D. $None\ of\ these$	**2**	**CO1**
Q1 (iv)	In kurtosis, frequency curve that has flatten top than normal curve of bell shaped distribution is classified as A. leptokurtic B. platykurtic C. mega curve D. mesokurtic	**2**	**CO2**
Q1 (v)	The second moment about mean represents A. Mean B. Variance C. Skewness D. Expected Value	**2**	**CO2**
Q1 (vi)	Match the correct sequence of the following a. Newton-Raphson 1. Integration b. Runge-Kutta 2. Root finding c. Gauss-Seidel 3. Ordinary Differential Equations d. Simpson's Rule 4. Solution of system of Linear Equations A. a2-b3-c4-d1 B. a3-b2-c1-d4 C. a2-b1-c3-d4 D. a3-b4-c1-d2	**2**	**CO3**

Q1 (vii)	If $f(x) = x^2 - 166 = 0$, then the iterative formula for Newton Raphson method is A. $x_{n+1} = 0.25\left[x_n + \frac{166}{x_n}\right]$ B. $x_{n+1} = 0.5\left[x_n + \frac{166}{x_n}\right]$ C. $x_{n+1} = 0.5\left[x_n - \frac{166}{x_n}\right]$ D. $x_{n+1} = 0.25\left[x_n - \frac{166}{x_n}\right]$	**2**	**CO3**
Q1 (viii)	The value of $\Delta(x + cosx)$, taking h =1 is A. $1 + 2sin\left(\frac{x+1}{2}\right).sin\left(\frac{1}{2}\right)$ B. $1 - 2sin\left(\frac{2x+1}{2}\right).sin\left(\frac{1}{2}\right)$ C. $1 - 2sin\left(\frac{x-1}{2}\right).sin\left(\frac{1}{2}\right)$ D. $1 + 2sin\left(\frac{x-1}{2}\right).sin\left(\frac{1}{2}\right)$	**2**	**CO3**
Q1 (ix)	To evaluate the integral $\int_a^b f(x)\,dx$ by using Simpson's $\frac{1}{3}rd$ as well as Simpson's $\frac{3}{8}th$ rule, the number of sub intervals must be A. multiple of 6 B. multiple of 3 C. multiple of 2 D. none of these	**2**	**CO3**
Q1 (x)	A river is 80 m wide. The depth y of the river at a distance x from one bank is given by the following table: x: 0, 10, 20, 30, 40, 50, 60, 70, 80 y: 0, 4, 7, 9, 12, 15, 14, 8, 3 The approximate area of cross-section of the river using Simpson's $\frac{1}{3}$ rd rule is A. 710 B. 720 C. 700 D. 701	**2**	**CO3**

Q1 (xi)

Consider the following table.

x	5	7	11	13	17
y	15	41	141	241	541

The entries in the divided difference table corresponding to the first divided difference are (respectively from top to bottom):.

A. 12, 24, 25, 30

B. 13, 25, 50, 75

C. 14, 26, 40, 80

D. none of these

2 **CO3**

Q1 (xii)

Consider the following table.

x	5	7	11	13	17
y	15	41	141	241	541

The entries in the divided difference table corresponding to the second divided difference are (respectively from top to bottom):.

A. 2, 4, 6

B. 1, 2.5, 5.1

C. 2, 4.16..., 4.16...

D. none of these

2 **CO3**

Q1 (xiii)

A relation is said to be partial order relation if it is

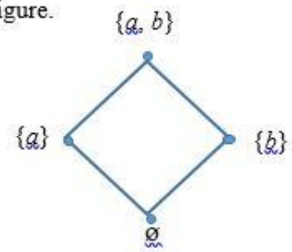
A. symmetric, reflexive and transitive

B. anti-symmetric, reflexive and transitive

C. anti- symmetric, reflexive but not transitive

D. None of these

2 **CO4**

Q1 (xiv)	The Hasse diagram associated with the partial order on the power set of the two element set, $\{a, b\}$ is shown in the figure.  Which one is correct A. The minimal element is ø and maximal element is $\{a, b\}$. B. The maximal element is ø and minimal element is $\{a, b\}$. C. The minimal element is $\{a\}$ and maximal element is $\{b\}$. D. The minimal element is $\{b\}$ and maximal element is $\{a\}$.	**2**	**CO4**
Q1 (xv)	A lattice $(S, \wedge, \vee)$ which is bounded and every element in the lattice $(S, \wedge, \vee)$ has a complement, then the lattice $(S, \wedge, \vee)$ is known as a A. Bounded lattice B. Modular lattice C. Distributive lattice D. Complemented lattice	**2**	**CO4**
Q1 (xvi)	The value of $\left(\frac{1}{D+1} - \frac{1}{D+2}\right) e^{e^x}$ is A. $e^{-2x}e^{e^x}$ B. $e^{2x}e^{e^x}$ C. $e^{x}e^{e^x}$ D. $e^{-x}e^{e^x}$	**3**	**CO1**
Q1 (xvii)	The complete solution (C.F & P.I) of the differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x$ is given by (choose all options that apply) A. $C.F. = c_1 + (c_2 + c_3x)e^{-x}$ B. $P.I. = \frac{e^{2x}}{18} + \frac{x^3}{3} + \frac{3x^2}{2} + 4x$ C. $C.F. = c_1 + (c_2 + c_3x)e^{x}$ D. $P.I. = \frac{e^{2x}}{18} + \frac{x^3}{3} - \frac{3x^2}{2} + 4x$	**3**	**CO1**

Q1 (xviii)	In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for more than 2150 hours: (Given: Area against $z = 1.83$ is equal to 0.4664) A. 97 B. 67 C. 17 D. 7	**3**	**CO2**
Q1 (xix)	It is given that $\frac{dy}{dx} = \sqrt{x+y}$ and $y(0.4) = 0.41$ then the approximate value of $y(0.6)$ using Runge Kutta fourth order method with the step length $h = 0.2$ is A. 0.6103476 B. 0.6203476 C. 0.6003476 D. 0.5923476	**3**	**CO3**
Q1 (xx)	The positive root of the equation $3x - cosx - 1 = 0$, using Regula-Falsi method is A. 0.6701 B. 0.5071 C. 0.6071 D. 0.5701	**3**	**CO3**
Q1 (xxi)	The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table: t: 0, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 v: 0, 3.60, 10.08, 18.90, 21.60, 18.54, 10.26, 5.40, 4.50, 5.40, 9.00 Using Simpson's rule, the distance travelled by the car in 2 minutes is A. 1236.96 B. 1296.96 C. 1296.36 D. 1336.96	**3**	**CO3**
Q1 (xxii)	In which of the following methods, we approximate the curve of solution by the tangent in each interval (Select all the correct answers) A. Picard's method B. Euler's method C. Newton's method D. Modified Euler's Method	**3**	**CO3**

Table for Q1 (xxi):

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Q1 (xxiii)	Consider the following table. | x | 100 | 150 | 200 | 250 | 300 | 350 | 400 | | y | 10.63 | 13.03 | 15.04 | 16.81 | 18.42 | 19.90 | 21.27 | Use forward difference table to choose the correct options. A. $\Delta^2 y$ at $x = 100$ is -0.39 B. $\Delta^2 y$ at $x = 200$ is -0.39 C. The value of y when $x = 218$ is approximately between 15 and 16 D. The value of y when $x = 218$ is approximately between 16 and 17	**3**	**CO3**
Q1 (xxiv)	Consider the $(P(S), \subseteq)$, where $S = \{a, b, c\}$ and the partial order relation $(\subseteq)$ is 'inclusion'. Then (select all the correct options) A. It is not a bounded lattice B. It is a complemented lattice C. Neither it is a bounded nor a complemented lattice D. It is bounded as well as complemented lattice.	**3**	**CO4**
Q1 (xxv)	Consider the set $S = \{2, 4, 5, 8, 10, 15, 20, 30, 40, 60\}$ with the partial order relation | defined as **a** | b i.e. "a divides b". Then choose the correct options (select all) A. The minimal and maximal elements do not exist. B. First and last elements do not exist. C. The minimal elements are 2, 5 and maximal elements are 40, 60. D. The first element is 2 and the last element is 60.	**3**	**CO4**

PART B

The link for PART B will be available from 10:00 AM on 6th July 2020 to 10:00 AM on 7th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.

Q2 (A)	Determine the solution of $\left(1 + e^{\frac{x}{y}}\right) dx + \left(1 - \frac{x}{y}\right) e^{\frac{x}{y}} dy = 0.$	**4**	**CO1**
Q2 (B)	If $y = e^{x^2}$ is a solution of the differential equation $y'' - 4x\,y' + (4\,x^2 - 3)\,y = 0$, then determine a second independent solution.	**4**	**CO1**
Q3 (A)	Out of 320 families with 5 children each, what percentage would be expected to have (*i*) 2 boys and 3 girls, and (***ii***) at least one boy? Assuming equal probability for boys and girls.	**4**	**CO2**

Q3 (B)	Perform two iterations to determine the real root of $\cos x - 3x + 1 = 0$ by Bisection method in the interval [0.60, 0.61].	**4**	**CO3**
Q4 (A)	If δ and μ denote the central and average difference operators respectively, then prove the relation $1 + \delta^2\mu^2 \cong \left(1 + \frac{\delta^2}{2}\right)^2$.	**4**	**CO3**
Q4 (B)	Perform two iteration to solve the system of linear equations $2x + y - z = 4,\ x - y + 2z = -2,\ -x + 2y - z = 2$ by Gauss Seidel's method correct up to three places of decimal with the initial guess $x = 0.75, y = 0.75$ and $z = -0.75$.	**4**	**CO3**
Q5 (A)	The value of the integral $\int_1^9 x^2 dx$ by Trapezoidal rule is $2\left[\frac{1}{2}(1 + 9^2) + \alpha^2 + \beta^2 + 7^2\right]$ for $n = 4$. Determine the value of α and β.	**4**	**CO3**
Q5 (B)	Using Runge-Kutta fourth order method, evaluate $y(0.1)$ of the differential equation $\frac{dy}{dx} = x + y^2$, with $y(0) = 1$, taking $h = 0.1$.	**4**	**CO3**
Q6	Draw the Hasse diagram for the poset $P = (\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, \vert)$, where "a | b " means " a divides b". Answer the following questions: (***i***) Find the maximal elements. (***ii***) Find the minimal elements. (***iii***) Find the greatest lower bound of {2, 9}, if it exists. (***iv***) Find the least upper bound of {2, 9}, if it exists.	**8**	**CO4**