Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December- 2019

Course: Theory of Real Functions — **Semester: III**

Program: B.Sc. (Hons.) Mathematics — **Time 03 hrs.**

Course Code: MATH- 2010 — **Max. Marks: 100**

SECTION A

(Attempt all questions)

S. No.		Marks	CO
Q 1	Examine the existence of $\lim_{x\to 1}(x-\lfloor x\rfloor)$, where $\lfloor x\rfloor$ is greatest integer function.	**4**	**CO1**
Q 2	In each of the following give an example of functions f, g and a cluster point x_0 of $D(f)\cap D(g)$ satisfying the following properties: (a) $\lim_{x\to x_0}[f(x)+g(x)]$ exists, but $\lim_{x\to x_0} f(x)$ and $\lim_{x\to x_0} g(x)$ do not. (b) $\lim_{x\to x_0}[f(x)g(x)]$ exists, but $\lim_{x\to x_0} f(x)$ and $\lim_{x\to x_0} g(x)$ do not.	**4**	**CO1**
Q 3	Investigate the Lipschitz continuity for the following functions on $[0,\infty)$ 1. A polynomial of degree at least 2 2. e^x 3. $xsinx$ 4. $f(x)=\begin{cases} x^2, & if\ 0\le x\le 1\\ x^{1/2}, & if\ 1\le x<\infty\end{cases}$	**4**	**CO2**
Q 4	Discuss the uniform continuity of $\sin x^2$ over $\mathbb{R}$.	**4**	**CO2**
Q 5	Find the values of a_0,a_1,a_2,a_3 for which $a_0+a_1\lvert x\rvert+a_2\lvert x\rvert^2+a_3\lvert x\rvert^3$is differentiable at $x=0$	**4**	**CO3**
Q 6	If $(x)=x^3-3x+\lambda$, λ is real constant and $x\in(0,1)$ then how many values of λ are there for which $f(x)$ has distinct roots?	**10**	**CO3**
Q 7	If $f(x)=\begin{cases} x^2, & x\in\mathbb{Q}\\ 0, & otherwise\end{cases}$, then prove or disprove the following statement " $\lim_{x\to 0} f(x)$ does not exist but $f(x)$ is continuous and differentiable at 0"	**10**	**CO2**
Q 8	Suppose $f:[0,1]\to\mathbb{R}$ is a function satisfying $\lvert f(x)-f(y)\rvert\le(x-y)^2\ \forall x,y\in[0,1]$, then prove or disprove the following statement "f is necessarily continuous but need not be differentiable"	**10**	**CO2**

Q 9	Let $f:[0,1]\to\mathbb{R}$ defined by **(Thomae's Function)** $f(x) = \begin{cases} \frac{1}{q}, x \in \mathbb{Q}\ and\ x = \frac{p}{q} \text{ in the reducible form } (i.e.\ p \in \mathbb{Z}\ and\ q \in \mathbb{N}\ are\ coprime) \\ 0, \quad x \in \mathbb{Q}^C \end{cases}$ Show that $\lim_{x\to a} f(x) = 0$, for any $a \in (0,1)$.	**10**	**CO1**
SECTION-C **(Q10 is compulsory and Q11 has internal choice)**			
Q 10	a. Consider the function $\lvert cosx\rvert + \lvert\sin(2-x)\rvert$. At which of the points f is not differentiable? b. Test for the continuity of $f(x) = \lim_{n\to\infty}\lim_{m\to\infty} cos((m!)\pi x)^{2n}$	**10 +10**	**CO3** **CO1**
Q 11	a. Show that $f(x) = x^2$ is not uniformly continuous on $\mathbb{R}$. **b.** Consider the limit statement: $\lim_{x\to 2}(5x-4) = 6$. Find a value of $\delta > 0$ that will guarantee that whenever x is within distance δ from 2 (but not equal to 2) $5x-4$ will approximate the limit accurately to 3 decimal places. **OR**	**10+10**	**CO2** **CO1**
Q 11	a. Let $I = \{1\} \cup \{2\} \subset \mathbb{R}$. For $x \in \mathbb{R}$, let $\Phi(x) = dist\ (x, I) = \inf\{\lvert x-y\rvert : y \in I\}$. Then find the points where $\Phi(x)$ is continuous but not differentiable. b. Using sequential criterion for limits of functions, prove the following limit statements $\lim_{x\to -4}(2x+13) = 5$.	**10+10**	**CO2** **CO1**