Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Course: Finite Volume Methods for Conservation Laws **Semester: I**

Program: M. Tech CFD **Time 03 hrs.**

Course Code: ASEG 7021 **Max. Marks: 100**

No. of pages: 04

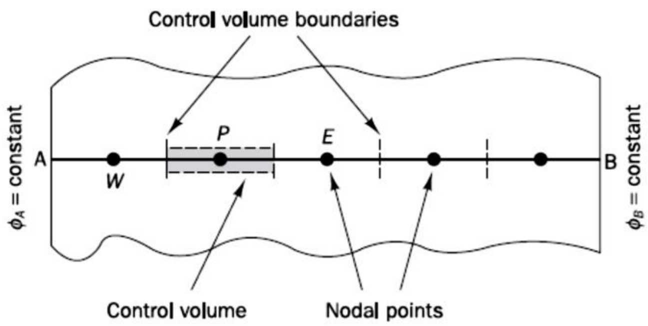
Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory

SECTION A (20 marks)

S. No.		Marks	CO
Q 1	Classify the steady two-dimensional velocity potential equation: $$\left(1-M^2\right)\phi_{xx}+\phi_{yy}=0$$ where *M* is mach number. Explain the physical meaning of various classifications based on *M*.	**[04]**	**CO2**
Q 2	Explain the algorithm of the Jacobi Iteration method applied to a parabolic partial differential equation.	**[04]**	**CO1**
Q 3	What is the stability requirement of an explicit equation produced from the model equation $\frac{\partial u}{\partial t} = \propto \left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x^2}\right]$.	**[04]**	**CO2**
Q 4	Explain the explicit MacCormack time marching algorithm for the solution of transient Euler equations in 2-dimensions.	**[04]**	**CO1**
Q 5	What do you mean by initial and boundary conditions? Define various types of boundary conditions which are usually encountered in CFD problems.	**[04]**	**CO1**

SECTION B (40 marks)

Q 6	In compressible viscous flows the energy equation is completely decoupled from the continuity and momentum equations, i.e. the solution of energy equation is not required for obtaining pressure and velocity fields. Prove it.	**[10]**	**CO2**

Q 7	Consider the steady state diffusion of a property ϕ in a one-dimensional domain defined in figure. The process is governed by $$\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right)+S=0$$ where Γ is the diffusion coefficient and *S* is the source term. Boundary values of ϕ at points A and B are prescribed.  Explain the several steps involved in discretizing the geometry and the equation to obtain the appropriate solutions of the governing differential equation.	**[10]**	**CO3**
Q 8	Consider the model equation $$\frac{\partial u}{\partial t}=-a\frac{\partial u}{\partial x}$$ Following the Lax-Wendroff method, find the value of the dependent variable at the next time step.	**[10]**	**CO4**
Q 9	Explain with proper example how a pentadiagonal coefficient matrix can be reduced to two sets of tridiagonal coefficient matrix to be solved in sequence. **OR** Explain in details the philosophy of the SIMPLE method. What is the need for a staggered grid?	**[10]**	**CO3**

SECTION-C (40 marks)

Q 10

Use each of the following methods to solve the Burgers equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x}$$

a) Lax-Wendroff
b) MacCormack

The initial condition is specified as:

$u(x,0) = 5.0 \qquad 0.0 \leq x \leq 20.0$

$u(x,0) = 0.0 \qquad 20.0 < x \leq 40.0$

Find the solution at intervals of 0.4 sec up to t = 2.4 sec. Choose appropriate step size.

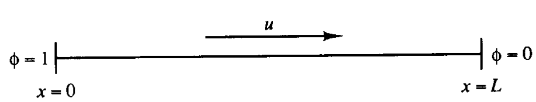
OR

A property $\emptyset$ is transported by means of convection and diffusion through the one-dimensional domain sketched in figure below. The governing equation below;

$$\frac{d}{dx}(\rho u \emptyset) = \frac{d}{dx}\left(\tau \frac{d\emptyset}{dx}\right)$$

boundary conditions are $\emptyset_0 = 1$ at x = 0 and $\emptyset_L = 0$ at x = L . Using five equally spaced cells (for first two cases) and the central differencing scheme for convection and diffusion calculate the distribution of $\emptyset$ as a function of x for cases:

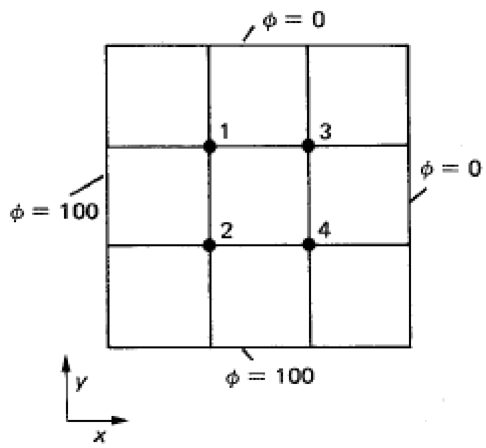
(i) Case 1: u = 0.1 m/s, using 5 cells
(ii) Case 2: u = 2.5 m/s, using 5 cells
(iii) Case 3: u = 2.5 using 10 cells



and compare the results with the analytical solution given below. The following data apply: length L = 1.0 m, p = 1.0 kg/m3, Γ = 0.1 kg/m/s.

$$\frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\rho u x/\Gamma) - 1}{\exp(\rho u L/\Gamma) - 1}$$

[20] **CO5**

Q 11	In a steady two-dimensional situation, the variable ϕ is governed by $$div\,(\rho \boldsymbol{u}\phi) = div(\Gamma grad\phi) + a - b\phi$$ where $\rho = 1$, $\Gamma = 1$, $a = 10$, and $b = 2$. The flow field is such that u = 1 and v = 4 everywhere. For the uniform grid shown in the figure $\Delta x = \Delta y = 1$. The values of ϕ are given for the boundaries. Adopting the control volume design calculate the values of $\phi_1, \phi_2, \phi_3,$ and ϕ_4 by the use of: (a) The central-difference scheme (b) The upwind scheme (c) The hybrid scheme (d) The power-law scheme  	**[20]**	**CO5**