Roll No: ---------------------------

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Program: BA(H) Specialization in Energy Economics — **Semester – II**
Subject (Course): Mathematical Methods II — **Max. Marks: 100**
Course Code : DSQT1006 — **Duration: 3 Hrs**
No. of page/s: 3

Instructions:
Answer all the questions from **Section A** (each carrying 2 marks), **Four** questions from **Section B** (each carrying 5 marks), **Three** Questions from **Section C** (each carrying 10 marks) and **Two** Questions from **Section D** (each carrying 15 marks)**.**

Section A (Total: 20 Marks)

Specify the order and degree of the following differential equations (Q1 and Q2).

Q1. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 4x$

Q2. $\left(\frac{\mathrm{d}^3\mathrm{y}}{\mathrm{dx}^3}\right)^2 + \left(\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}\right)^3 = 2$

Specify the order of the following difference equation.

Q3. $Q_s = \alpha + \beta P_{t-1}$

Integrate of the following functions (Q4 and Q5).

Q4. $f(x) = 4/\sqrt[3]{x}$

Q5. $f(x) = x^2 + 2x$

Evaluate the following definite integrals (Q6 and Q7).

Q6. $\int_1^6 3x^3 dx$

Q7. $\int_0^4 2e^{2x} dx$

Q8. Determine the rank ρ of the following matrix.

$$A=\begin{bmatrix} -3 & 6 & 2 \\ 1 & 5 & 4 \\ 4 & -8 & 2 \end{bmatrix}$$

Q9. Find the partial derivatives (w.r.t. *x* and *y*) of the following function.

$$z = 3x^2 - xy + 2y^2 - 4x - 7y + 12$$

Q10. Find the general solution of the following differential equation.

$$\frac{dy}{dx} = 90(1 - 0.3y)$$

SECTION B (Total: 20 Marks) Answer Any Four Questions

Q1. (a) Find the determinant of the following matrix.

$$A=\begin{bmatrix} 8 & 3 & 2 \\ 6 & 4 & 7 \\ 5 & 1 & 3 \end{bmatrix}$$

(b) Assume that the rate of net investment is $I = 60t^{3/5}$, and capital stock at $t = 0$ is 75.

Find the capital function *K*.

Q2. (a) Assume that the marginal propensity to consume (MPC) is given as

$\frac{dC}{dY} = 0.6 + 0.1/\sqrt[3]{Y}$, and $C = 45$ when $Y = 0$. Find the consumption function.

(b) Marginal cost is given as $MC = 16e^{0.4Q}$, and fixed cost is 100.

Find total cost (TC) function.

Q3. Find the general solution for the differential equation $y''(t) = 4$.

Q4. Find the demand function $Q = f(P)$ if elasticity of demand, $e = -c$, a constant.

Q5. Solve the following difference equation and comment on the nature of the time path.

$$5y_t + 2y_{t-1} - 140 = 0,\ y_0 = 30$$

SECTION C (30 Marks) Answer Any Three Questions

Q1. Assume that the supply function is $P_s = (Q + 1)^2$ and the demand function is

$P_d = 113 - Q^2$ under perfect competition. Find consumers' surplus (CS) and producers' surplus.

Q2. Let the consumption function be $C_t = 90 + 0.8Y_{t-1}$, $I_t = 50$ and $Y_0 = 1200$.

In equilibrium, $Y_t = C_t + I_t$

(a) Find the time path of national income Y_t
(b) Comment on the stability of the time path

Q3. (a) The rate at which the population (P) of a country is growing is given by the equation

$\frac{dP}{dt} = 0.02(400 - P)$, given that $P = 100$ at $t = 0$ (t is time in years).

(a) Solve the differential equation to obtain an expression for P in terms of t.
(b) Calculate the time taken for the population to reach 1000.

Q4. Let the function be $z(x, y) = 3x^3 - 5y^2 - 225x + 70y + 23$

Find the critical points and determine if at these points the function is at a relative maximum, relative minimum, or saddle point.

SECTION D (30 Marks) (2*15)

Q1. Using Cramer's rule solve for the unknowns in the system of linear equations given below.

$$2x_1 + 4x_2 - 3x_3 = 12$$
$$3x_1 - 5x_2 + 2x_3 = 13$$
$$-x_1 + 3x_2 + 2x_3 = 17$$

Q2. Maximize the utility function $U = x^{0.25}y^{0.4}$ subject to the budget constraint

$2x + 8y = 104.$