Name: **Enrolment No:**	UPES

END SEMESTER EXAMINATION, DECEMBER 2017

Course: MATH 1002-Mathematics-I

Programme: B. Tech. (All SCE Branches)
Semester: I (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

Section A
(Attempt all questions)

1.	Show that the system of equations $x+y+z=-3, 3x+y-2z=-2, 2x+4y+7z=7$ is not consistent.	**[4]**	**CO 3**
2.	Show that the set of vectors [1, 1, 0], [1, 0, 1], [0, 1, 1] are linearly independent.	**[4]**	**CO 3**
3.	Construct a truth table for the proposition $\sim(p \vee q) \vee (\sim p \wedge \sim q)$.	**[4]**	**CO 2**
4.	Find n[th] derivative of $sin^2 x\ cos^3 x$.	**[4]**	**CO 1**
5.	Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy\, dx\, dz$.	**[4]**	**CO 1**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	Using Cayley Hamilton theorem find the inverse of the matrix $$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$	**[8]**	**CO 3**
7.	Show that t is a valid conclusion from the premises $p \to q\ ,\ q \to r,\ r \to s\ ,\ \sim s$ and $p \vee t$.	**[8]**	**CO 2**
8.	Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum.	**[8]**	**CO 1**
9.	Show that the set $G=\{1, -1, i, -i\}$, where i is a fourth root of unity is a group with respect to multiplication.	**[8]**	**CO 4**

10.	If x is an element of a cyclic group of order 15 and two of x^3, x^5 and x^9 are equal, determine o(x^{13}) where o denotes the order. **OR** Let $U(n)$ be a group defined as $U(n) = \{m \in N: 1 \leq m \leq n$ and $gcd\ (m, n) = 1\}$. Is $U(8)$ isomorphic to $U(12)$? Justify your answer.	**[8]**	**CO 4**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11.A	Evaluate $\iint \frac{x^2y^2}{x^2+y^2} dxdy$ by changing it to polar co-ordinates over the annular region between circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$; $a > b > 0$.	**[10]**	**CO 1**
11.B	Let G be the group of integers under addition and let N be the set of all integral multiples of 3. Prove that N is a subgroup of G and determine all the cosets of N in G.	**[10]**	**CO 4**
12.A	Is the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ diagonalizable? Justify your answer. **OR** Given that $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are roots of $x^3 + x^2 + k = 0$ (k is a constant). Prove that A is orthogonal.	**[10]**	**CO 3**
12.B	Find the order of each element in the cyclic group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ where e being the identity element. **OR** Show that the set R of real numbers is a commutative ring with unity with respect to addition and multiplication of real numbers.	**[10]**	**CO 4**

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Section A
(Attempt all questions)

1.	Find the rank of the matrix $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ by reducing it to Echelon form.	**[4]**	**CO 3**
2.	Using Cayley Hamilton Theorem find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	**[4]**	**CO 3**
3.	Verify that the proposition $p \wedge (q \wedge \sim p)$ is a contradiction.	**[4]**	**CO 2**
4.	Find n^{th} derivative of $\frac{ax+b}{cx+d}$ with respect to x.	**[4]**	**CO 1**
5.	Evaluate $\int_0^1 \int_0^{x^2} \int_0^{x+y} (x-2y+z)dz\,dy\,dx$.	**[4]**	**CO 1**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	Determine the values of λ and μ such that the system $2x-5y+2z=8,\ 2x+4y+6z=5,\quad x+2y+\lambda z=\mu$ has (i) no solution (ii) a unique solution (iii) infinite number of solutions.	**[8]**	**CO 3**
7.	Show that s is a valid conclusion from the premises $p \to q\ ,\ p \to r, \sim(q \wedge r)$ and $s \vee p$.	**[8]**	**CO 2**
8.	Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}\ and\ \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$	**[8]**	**CO 1**

9.	Show that the set $G = \{1,\ \omega,\ \omega^2\}$ where ω is an imaginary cube root of unity is a group with respect to multiplication.	**[8]**	**CO 4**
10.	If x is an element of a cyclic group of order 21 and two of x^3, x^5 and x^9 are equal, determine $\mathrm{o}(x^{19})$ where o denote the order. **OR** Consider the elements $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ from a group G with respect to matrix multiplication. Find $\mathrm{o}(A)$, $\mathrm{o}(B)$, $\mathrm{o}(AB)$, where o denote the order.	**[8]**	**CO 4**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11.A	Evaluate $\iint \frac{1}{\sqrt{xy}}\,dx\,dy$ by changing it to polar coordinates, over the region of integration bounded by $x^2 + y^2 - x = 0$ and $y \geq 0$.	**[10]**	**CO 1**
11.B	Prove that the set $G = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7.	**[10]**	**CO 4**
12.A	Is the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ diagonalizable? Justify your answer? **OR** The Eigen vectors of a 3×3 matrix A corresponding to eigen values $1, 1, 3$ are $[1, 0, -1]^T$; $[0, 1, -1]^T$ and $[1, 1, 0]^T$ respectively, find the matrix A.	**[10]**	**CO 3**
12.B	Find the inverse of the following permutations:: (i) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ Also determine which of the following are even permutations: a. $g = (1\ \ 2\ \ 3\ \ 4\ \ 5)(1\ \ 2\ \ 3)(4\ \ 5)$ b. $h = (1\ \ 2)(1\ \ 3)(5\ \ 6\ \ 7)$ **OR** Show that the set Q of rational numbers is a commutative ring with unity with respect to addition and multiplication of real numbers.	**[10]**	**CO 4**