| Name:<br><br>Enrolment No: | UPES |
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**End Semester Examination, Dec 2017**
**Course: MATH 306-Applied Numerical Methods**
**Programme: B. Tech. (Civil)**
**Semester: III (ODD-2017-18)**
**Time: 03 hrs.** **Max. Marks:100**

**Instructions:**
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

**Section A**
**( Attempt all questions)**

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| 1. | Write general form of a second order linear Partial Differential Equation. Classify it in Parabolic, Elliptic and Hyperbolic equations. | **[4]** | **CO6** |
| 2. | Mention two non - iterative methods to solve an Ordinary Differential Equation. | **[4]** | **CO6** |
| 3. | Write down one - dimensional Heat Equation and its finite difference scheme. | **[4]** | **CO6** |
| 4. | Can we apply Simpson one third and Simpson three eight rule for Numerical Integration for any number of sub intervals? If not then what is to be taken into consideration. | **[4]** | **CO3** |
| 5. | Solve $\frac{dy}{dx} = x^2 - y;\ y(0) = 1$ by Taylor Series method for y (0.1) in single step. | **[4]** | **CO6** |

**SECTION B**
**(Q6-Q9 are compulsory and Q10 has internal choice)**

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| 6. | Solve by Picard's method: $\frac{dy}{dx} = x,\ \frac{dz}{dx} = x^3(y+z)$, where $y = 1$ & $z = \frac{1}{2}$ at $x = 0$. Obtain the values of $y$ and $z$ when $x = 0.2$ correct up to two places of decimal. | **[8]** | **CO6** |
| 7. | Solve the equation $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 1$ by Runge-Kutta method of 4$^{th}$ order from $x = 0$ to $x = 0.2$ with $h = 0.1$. | **[8]** | **CO6** |
| 8. | Define $D, E, \Delta$ & $\nabla$ operators of the finite difference and deduce the following.<br>$(a)\ \Delta\, log f(x) = log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$ $(b)\ e^x = \left(\frac{\Delta^2}{E}\right) e^x \frac{Ee^x}{\Delta^2 e^x}$ where the interval of differencing being unity. | **[8]** | **CO1** |
| 9. | Use the method of fixed point iteration to find a positive root of the equation $x e^x = 1$ between 0 and 1. | **[8]** | **CO4** |

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| 10. | Show that the third divided difference with arguments $x_0, x_1, x_2$ and $x_3$ of the function $\frac{1}{x}$ is $(-1)^3 \frac{1}{x_0 x_1 x_2 x_3}$.<br><br>**OR**<br><br>Using Newton's divided difference formula for Interpolation find the value of $y$ for $x = 9.5$ for a function $y = f(x)$ which has following set of values .<br>$x =$ 7 8 9 10<br>$y =$ 3 1 1 9 | **[8]** | **CO2** |
| **SECTION C**<br>**(Q11 is compulsory and Q12A, Q12B have internal choice)** | | | |
| 11.A | Use LU decomposition method of Crout to solve the system of equations<br>$2x + y + 4z = 12;\ 8x - 3y + 2z = 20;\ 4x + 11y - z = 33$ | **[10]** | **CO5** |
| 11.B | Solve by Horner's method to find the root correct to two places of decimal<br>$x^3 + 9x^2 - 18 = 0.$ | **[10]** | **CO4** |
| 12.A | Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x}$ satisfying the conditions $u(0,t) = 0, t \geq 0$;<br>$u(5,t) = 0,\ t \geq 0$ and $u(x,0) = 10x(5-x), 0 \leq x \leq 5$. Compute $u$ for five time-step by Bender Smith scheme taking $h = 1$ and $k = 1$.<br><br>**OR**<br>Solve the above problem by Crank Nicholson Method for two time steps. | **[10]** | **CO6** |
| 12.B | Solve steady state 2-D heat flow problem $u_{xx} + u_{yy} = 0$ with following conditions using Liebmann's iteration process: $0 \leq x \leq 4, 0 \leq y \leq 4, u(0,y) = 0,\ u(4,y) = 8 + 2y$,<br>$u(x,0) = \frac{x^2}{2}, u(x,4) = x^2$ where $u(x,y)$ is temperature at the point $(x,y)$.<br><br>**OR**<br><br>Solve the equation $u_{xx} + u_{yy} = -10\,(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, x = 3, y = 0$ and $y = 3$ with $u = 0$ on the boundary and mesh length equal to 1. | **[10]** | **CO6** |