Name: **Enrolment No:**	UPES

Course: CSAI 7002-System Modelling and Identification
Programme: M.Tech (A&RE)
Semester: I (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Perform three iterations of Picard's method to find an approximate solution of the initial value problem $\frac{dy}{dx} = -xy, \quad y(0) = 1$.	**[4]**	**CO1**
2.	Construct a Liapunov function for the system $\frac{dx}{dt} = -x + y^2, \frac{dy}{dt} = -y + x^2$.	**[4]**	**CO2**
3.	Find the orthogonal trajectories of the family of curves $y^2 = 4ax$.	**[4]**	**CO3**
4.	A vector $\boldsymbol{p}$ is 5 units long and is in the direction of unit vector $\boldsymbol{q}$ described below. Express the vector in matrix form; $$\boldsymbol{q}_{\text{unit}} = \begin{bmatrix} 0.371 \\ 0.557 \\ q_z \\ 0 \end{bmatrix}$$	**[4]**	**CO4**
5.	Calculate the inverse of the given transformation matrix: $\begin{bmatrix} 0.5 & 0 & 0.866 & 3 \\ 0.866 & 0 & -0.5 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	**[4]**	**CO4**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	A point $p(2,3,4)^T$ is attached to a rotating frame. The frame rotates 90^0 about the x- axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation.	**[8]**	**CO4**
7.	Solve the equation $y'' - 4y' + 4y = e^{3x}$ with the boundary conditions $y(0) = 0$, $y(1) = -2$ taking $n = 4$.	**[8]**	**CO1**
8.	Solve $x'' + 8x' + 36x = 24\cos(6t)$ and discuss the behavior of the solution as t	**[8]**	**CO3**

	approaches infinity.		
9.	Determine the nature of the critical point (0,0) of the system $\frac{dx}{dt}=x-y, \frac{dy}{dt}=x+5y$ and determine whether or not the point is stable.	**[8]**	**CO2**
10.	Find the solution of $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to $u(x,\ 0)=\sin \pi x,\ 0 \le x \le 1,\ t>0$ $u(0,\ t)=u(1,\ t)=0$ using Bender-Schmidt method. **OR** Using Crank-Nicolson's method, solve $\frac{\partial u}{\partial t}=\frac{1}{16}\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1,\ t>0$ given that $u(x,\ 0)=0,\ u(0,\ t)=0,\ u(1,\ t)=50t$. Compute u for two steps in t direction taking $h=1/4$.	**[8]**	**CO1**
	SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**		
11.A	A point $p(7,3,1)^T$ is attached to a frame F_{noa} and is subjected to the following transformations. Find the coordinates of the point relative to the reference frame at the conclusion of the transformations. 1. Rotation of 90^0 about $z-$ axis, 2. Followed by a rotation of 90^0 about the $y-$ axis, 3. Followed by a translation of $[4,-3,7]$.	**[10]**	**CO4**
11.B	Discuss the nature and stability of the critical point of the non-linear autonomous system $\frac{dx}{dt}=x+4y-x^2, \frac{dy}{dt}=6x-y+2xy$.	**[10]**	**CO2**
12.A	Solve the equation $\nabla^2 u=0$ over the square with the boundary conditions $u(0,y)=0$, $u(x,0)=0$, $u(3,y)=100$, $u(x,3)=100$ on the boundary and mesh length =1. **OR** Find two parameter solution of the following differential equation $\frac{d^2u}{dx^2}+1+x^2=0$, $u(0)=u(1)=0$ by Galerkin method.	**[10]**	**CO1**
12.B	A spring with a mass of 2 kg has natural length 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.7 m. If the spring is stretched to a length of 0.7 m and then released with initial velocity 0, find the position of the mass at any time *t*.	**[10]**	**CO3**

	OR A series circuit consists of a resistor with $R = 20\ \Omega$, an inductor with $L = 1\ H$, a capacitor with $C = 0.002\ F$, and a 12-V battery. If the initial charge and current are both 0, find the charge and current at time *t*.		