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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination – December 2017

Program/course: M. Tech Chemical Engineering (PD) **Semester – I**
Subject: Chemical Engineering Computing **Max. Marks : 100**
Code : CHPD7002 **Duration : 3 Hrs**
No. of page/s: 04

*The question paper consists of three sections. Answer the questions section wise in the answer booklet.

Note: 1) Attempt ***all*** the questions.
2) Assume suitable data wherever necessary. The notations used here have the usual meanings.

SECTION – A (Total Marks: 2 x 10 = 20)

Q.1 Obtain the eigenvalues and eigenvectors for the symmetric matrix

$$\begin{bmatrix} 3 & 1 \\ 1 & 3/2 \end{bmatrix}$$

Use $x_2 = 1$ as an additional condition. Check if the eigenvalues are real for this case and if the eigenvectors are orthogonal. **[10]**

Q.2 Consider the following set of coupled ODE-IVPs:

$$\frac{dy_1}{dt} = -0.2\,y_1 + 0.2\,y_2 = f_1(t, y);\; y_1(0) = 0$$

$$\frac{dy_2}{dt} = 10\,y_1 - (60 + 0.125t)\,y_2 + 0.124\,t = f_2(t, y);\; y_2(0) = 0$$

(a) Obtain the appropriate eigenvalues at t = 0, 1, 5 and 10 analytically
(b) State whether the ODEs are stable and suggest the largest value of h for which stable solutions are expected using the 4th order explicit RK technique. **[10]**

SECTION – B (Total Marks: 4 x 15 = 60)

Q.3 Consider a separation system as shown in Fig. 1. By setting up the mass balances, calculate the mass flow rates of each outlet streams (x_1, x_2 and x_3) using Gauss elimination with backward sweep. **[15]**

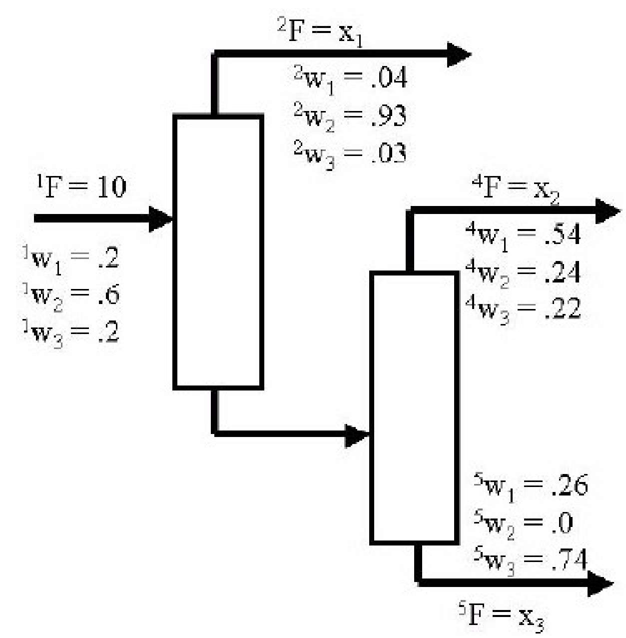

Fig. 1: A Separation system

Q.4 Consider the following non-linear ODE-BVP

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 3y^3 = 0$$

$$x = 0 : \frac{dy}{dx} = 0; \ \ x = 1 : y(x = 1) = 3 \quad ;$$

Using the OC for the non-symmetric case, and with $N + 2 = 4$:

(a) What are the numerical values of x_2 and x_3 **[03]**

(b) Give the simplified OC equation for $x_1 = 0$ (take ALL the constants and the non-linear terms to the right hand side) **[04]**

(c) Give the simplified OC equation for point 2 and 3 (keep B_{ij} and A_{ij} terms separate) **[08]**

Q.5 (a) Solve using the Newton-Raphson technique:

$$y=1+2(1.2-y)^2 \exp\left[10\left(1-\frac{1}{y}\right)\right]$$

Use $y^{(1)} = 1.05$. Obtain $y^{(2)}$ and $y^{(3)}$. **[12]**

(b) Explain in brief about the condition for convergence for successive substitutions. **[03]**

Q.6 The desorption of a gas from a liquid stream in a wetted-wall column can be described by the following PDE:

$$(1-\eta^2)\frac{f}{\square}=\frac{\square^2 f}{\eta^2}$$

$$\frac{f}{\eta}=0 \; at \; \eta=1$$

$$f=o \; at \; \eta=0$$

$$f=1 \; at=0$$

Note that the problem as defined above is not symmetric about η=0. Carry out an orthogonal collocation solution in the η direction, using N = 2 (i.e. two internal OC points) and simplify to obtain a set of two ODE-IVPs. Do not solve these. **[15]**

SECTION – C (Total Marks: 1 x 20 = 20)

Q.7 The concentration of salt x in a homemade soap maker is given as a function of time by

$$\frac{dx}{dt}=37.5-3.5x$$

At the initial time, $t=0$, the salt concentration in the tank is 50 g/L. Using Runge-Kutta 4th order method and a step size of, $h=1.5 \text{ min}$, what is the salt concentration after 3 minutes?

The exact solution of the ordinary differential equation is given by

$$x(t) = 10.714 + 39.286\, e^{-3.5t}$$

Compare the exact solution with the numerical solution and comment on the result. **[20]**

Data:

TABLE 6.1

Matrices for the Orthogonal Collocation Technique [5] (non-symmetric)*

$[0 \le x \le 1, \quad W(x) = 1$ in Eq. 6.19]

N + 2	x	w	A	B
3	$\begin{pmatrix} 0 \\ 0.5 \\ 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{6} \\ \frac{2}{3} \\ \frac{1}{6} \end{pmatrix}$	$\begin{pmatrix} -3 & 4 & -1 \\ -1 & 0 & 1 \\ 1 & -4 & 3 \end{pmatrix}$	$\begin{pmatrix} 4 & -8 & 4 \\ 4 & -8 & 4 \\ 4 & -8 & 4 \end{pmatrix}$
4	$\begin{pmatrix} 0 \\ 0.21132 \\ 0.78868 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -7 & 8.196 & -2.196 & 1 \\ -2.732 & 1.732 & 1.732 & -0.7321 \\ 0.7321 & -1.732 & -1.732 & 2.732 \\ -1 & 2.196 & -8.196 & 7 \end{pmatrix}$	$\begin{pmatrix} 24 & -37.18 & 25.18 & -12 \\ 16.39 & -24 & 12 & -4.392 \\ -4.392 & 12 & -24 & 16.39 \\ -12 & 25.18 & -37.18 & 24 \end{pmatrix}$
5	$\begin{pmatrix} 0 \\ 0.11270 \\ 0.50000 \\ 0.88730 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0.27778 \\ 0.44444 \\ 0.27778 \\ 0 \end{pmatrix}$	use program in Ref. 12 to generate these	

ompiled from Ref. 5)

given by Eq. 6.23

[Ref: Numerical Methods for Engineers, Gupta S. K., 3rd Edition, New Age International]