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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2017

Program: M. Tech. CFD	**Semester – I**
Subject (Course): Finite Elements & Boundary Elements Analysis	**Max. Marks : 100**
Course Code: ASEG7006	**Duration : 3 Hrs**
No. of page/s: 02	

Note: Make use of sketch/plots to elaborate your answer. All sections are compulsory

Section-A [5 X 4 = 20 Marks]

1) State the finite element equation for a two dimensional triangular element placed in the Cartesian coordinate with origin on one side of the element.

2) State the advantages of Gaussian elimination technique.

3) Explain the importance of shape function. Give its properties.

4) List any two sources of errors in finite element method.

5) Derive the stiffness matrix for a 1-D two node linear element.

Section-B [4 X 10 = 40 Marks]

6) Explain the various shapes of finite elements that can be utilized with classification for one, two and three dimensional elements. Sketch clearly giving details of the corner and side nodes.

7) Describe the polynomial expansions in finite elements for the following structure:
   a) Pascal Triangle
   b) Pascal tetrahedron
   c) Two-dimensional hypercube
   d) Three-dimensional hypercube

8) Derive the finite element formulation for a one-dimensional problem, using Galerkin method. Demonstrate this process directly from the global from the global form using the Boolean algebra, with the local properties then arising indirectly as a consequence

9) Explain the usage of the Lagrange and Hermite interpolation function to desire the continuity of the derivative of a variable at common nodes and to avoid the inversion of the coefficient matrix for higher order approximations.

Section-C [2 X 20 = 40 Marks]

10) Solve the differential equation $\emptyset'' + \emptyset' - 2\emptyset = 0$ with boundary conditions $\emptyset(x = 0) = 0$ and $\emptyset(x = 1) = 1$ by the below given methods:

a) Point Collation Method

b) Galerkin's Method

The general solution is $\emptyset = \propto_1 x + \propto_2 x^2 + \propto_3 x^3$. Choose $x = 0.25$ and $x = 0.5$ as collocation points. Compare the solution in terms of percentage error.

11) Derive the Euler-Lagrange equation for a functional given by,

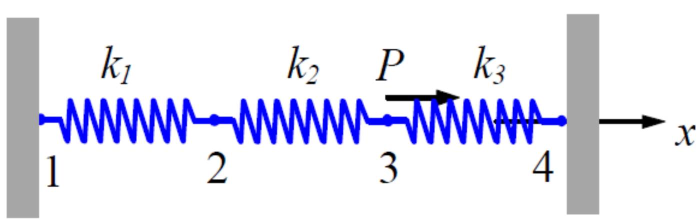
$$I(u) = \int_a^b F\left(u, \frac{du}{dx}, x\right) dx$$

Thus, obtain the corresponding Euler-Lagrange for the functional given below,

$$I = \frac{1}{2}\int_0^L \left[\propto \left(\frac{dy}{dx}\right)^2 - \beta y^2 + ryx^2\right] dx - y(L)$$

OR

For the spring system shown below,



k_1 = 200 N / mm, k_2 = 100 N / mm, k_3 = 200 N / mm

P = 10 N (applied at point 3).

The fixed boundary leads to the displacement $U_1 = U_4 = 0$

Find: (a) Global stiffness matrix

(b) Displacements of nodes 2 and 3

(c) Reaction forces at nodes 1 and 4

(d) Force in the spring 2