Name:

Enrolment No:

END SEMESTER EXAMINATIONS, DECEMBER 2017

Course: NUMERICAL METHODS IN ENGINEERING

Course Code: CHPL 7003 / MATH 701
Programme: M.Tech Pipeline Engineering
Semester: I (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:

Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

Section A

(Attempt all questions)

1. The table below gives the results of an observation: θ is the observed temperature in degrees centigrade of a vessel of cooling water; *t* is the time in minutes from the beginning of observation **[4]** **CO1**

t	1	3	5	7	9
θ	85.3	74.5	67.0	60.5	54.3

Find the approximate rate of cooling at *t=3*.

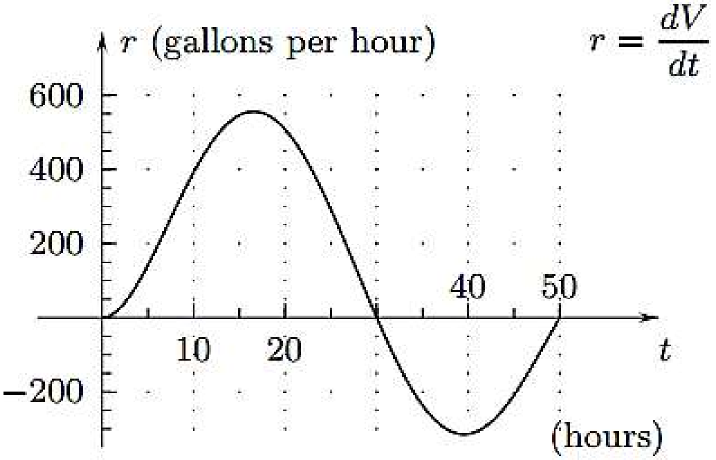
2. Using Newton's method, find a root between *0* and *1* of $x^3 = 6x - 4$ correct to 4 decimals **[4]** **CO2**

3. Solve the following equations by Gauss-elimination method. **[4]** **CO2**
$3x + 4y + 5z = 18, 2x - y + 8z = 13, 5x - 2y + 7z = 20$

4. Using Taylor series method, compute $y(0.2)$ given $\frac{dy}{dx} = 1 - 2xy, y(0) = 0$ by considering the terms up to third derivative. **[4]** **CO3**

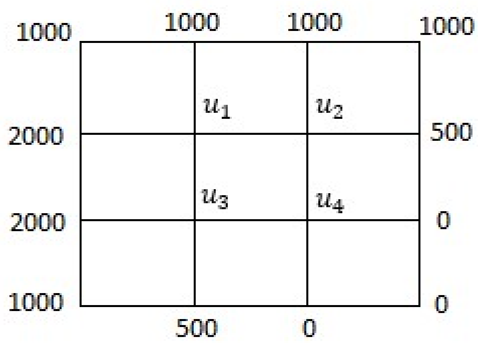
5. Find the regions in which the equation $u_{xx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = \sin(x + y)$ is (i) elliptic (ii) hyperbolic (iii) parabolic. **[4]** **CO4**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	A tank contains 1000 gallons of oil at *t=0* hours. The following figure shows the rate of change of the volume for $0 \leq t \leq 50$. Estimate the total amount of oil in the tank at $t = 50\ hours$.  	**[8]**	**CO1**
7.	Solve the modified radio activity equation $\frac{dN}{dt} = -\alpha N - \gamma$ using Euler's method with step size 0.5 over the interval *t=0 to t=2* for $\alpha = 0.1$ and $\gamma = 10$ where *N(0)=1000.*	**[8]**	**CO3**
8.	Solve the following system of equations using relaxation method. $\begin{bmatrix} 10 & -2 & -2 \\ -1 & 10 & -2 \\ -1 & -1 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$	**[8]**	**CO2**
9.	The following system of equations is designed to determine concentrations in a series of coupled reactors as a function of the amount of mass input to each reactor: $\begin{aligned} -3c_1 + 18c_2 - 6c_3 &= 1200 \\ 15c_1 - 3c_2 - c_3 &= 3800 \\ -4c_1 - c_2 + 12c_3 &= 2350 \end{aligned}$ Obtain the concentration values correct to 2 decimals by using *Gauss-Seidel* iterative technique with initial approximate solution as $\left[c_1{}^{(0)}, c_2{}^{(0)}, c_3{}^{(0)}\right] = [300, 220, 310]$.	**[8]**	**CO2**
10.	Using Crank-Nicholson's scheme, solve $u_{xx} = 16u_t, 0 < x < 1, t > 0$ given $u(x,0) = 0, u(0,t) = 0, u(1,t) = 100t$. Compute u for one time step in t direction taking $h = 0.25$. **OR** Apply Bender-Schmidt recurrence equation to solve $u_{xx} = 32u_t$, taking $h = 0.25$ for $t > 0, 0 < x < 1$ and $u(x,0) = 0, u(0,t) = 0, u(1,t) = t$, up to 5 time steps.	**[8]**	**CO4**

SECTION C
(Q11 is compulsory and Q12A, Q12B have internal choice)

11.A	The equations for the deflection y and rotation z of a simply supported beam with a uniformly distributed load of intensity 2 *kips/ft* and bending moment $M(x) = 10x - x^2$ can be expressed as $$\frac{dy}{dx} = z$$ $$\frac{dz}{dx} = \frac{10x - x^2}{EI}$$ where E is the modulus of elasticity, and I is the moment of inertia of the cross section of the beam. Taking *EI=3600 kips/ft*, $y(0) = 0$, *and* $z(0) = -0.02$, find the deflection at $x = 0.5$ and rotation at $x = 1$ using fourth order Runge-Kutta method with $\Delta x = 0.5$.	**[10]**	**CO3**
11.B	Debye's formula for the heat capacity C_V of copper is given by the formula $C_V = 9 * N * K * g(u)$ where $$g(u) = u^3 \int_{0.1}^{1/u} \frac{x^4 e^x}{(e^x - 1)^2}\, dx$$ The terms in the equation are: N=number of particles in the solid, K=Boltzmann constant= 1.38×10^{-23},$u = T/\emptyset$, T = absolute temperature and $\emptyset$ =Debye temperature=343.5 K. Compute the number of particles if $C_V = 40$ units and $T = 343.5\,K$ by using an appropriate numerical integration technique.	**[10]**	**CO1**
12.A	A steady state heat balance for a rod can be represented as $\frac{d^2T}{dx^2} - 0.15T = 0$. Considering four sub intervals, obtain a solution for a 2 meter rod with $T(0) = 240$ & $T(2) = 150$ by finite difference technique. **OR** Apply Galerkin's method to the boundary value problem $y'' + y + x = 0$, $(0 \leq x \leq 1)$, $y(0) = y(1) = 0$, to find the coefficients of the approximate solution $\bar{y}(x) = c_1 x(1 - x) + c_2 x^2(1 - x)$.	**[10]**	**CO3**

12.B	Given the values of $u(x,y)$ on the boundary of the square in the figure below. Find the initial approximate values of $u(x,y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points by standard/diagonal five point formula and tabulate the values of $u(x,y)$ obtained by perform two iterations of Liebmann's iteration process.  **OR** Solve the partial differential equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y, x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1.	**[10]**	**CO4**

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