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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2017

Program: M. Tech. CFD | **Semester – I**
Subject (Course): Finite Difference and Finite Volume Methods | **Max. Marks : 100**
Course Code: ASEG7005 | **Duration : 3 Hrs**
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Section-A [5 X 4 = 20 Marks]

1. Find a forward difference approximation of $O(\Delta x)$ for $\frac{\partial^4 f}{\partial x^4}$.
2. What is the stability requirement of an explicit finite difference equation produced from the model equation $\frac{\partial u}{\partial t} = \propto \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2}\right]$.
3. Define the following terms:
   a) Diffusion number
   b) Approximate factorization
4. Explain the algorithm of the Jacobi Iteration method applied to a parabolic partial differential equation.
5. Differentiate between explicit and implicit methods for converting partial differential equations into finite differential equations.

Section-B [4 X 10 = 40 Marks]

6. Determine an approximate backward difference representation for $\frac{\partial^3 f}{\partial x^3}$ which is of order (Δx), given evenly spaced grid points by means of:
   (a) Taylor series expansions.
   (b) A backward difference reccurence formulae
   (c) A third-degree polynomial passing throug four points.

7. Explain with proper example how a pentadiagonal coefficient matrix can be reduced to two sets of tridiagonal coefficient matrix to be solved in sequence.

8. Given the function $f(x) = \frac{1}{4}x^2$, compute the first derivative of f at $x = 2$ using forward and backward differencing of order (Δx). Compare the results with a central differencing of $O(\Delta x)^2$ and the exact analytical value. Repeat the computations for a step size of 0.4.

9. Given the following data, compute $f'(5)$, $f'(7)$ and $f'(9)$. Use finite difference of order (Δx). Compare the results to the values obtained by finite differencing of order $(\Delta x)^2$.

x	5	6	7	8	9
$f(x)$	25	36	49	64	81

Section-C [2 X 20 = 40 Marks]

10. Derive and explain the following methods used for solving the parabolic equation:

$$\frac{\partial u}{\partial t} = \propto \frac{\partial^2 u}{\partial x^2}$$

a) The FTCS method.
b) The Richardson method
c) The DuFort-Frankel method
d) The Laasonen Method
e) The Crank-Nicolson method

State clearly the advantages and the stability criteria for each of the methods.

11. Derive the explicit Mac-Cormack time marching algorithm for the solution of transient Euler equations in 2-Dimensions.

OR

Consider the model equation:

$$a\frac{\partial u}{\partial x} = \vartheta \frac{\partial^2 u}{\partial y^2}$$

(a) Write an explicit formulation using a first-order forward differencing in x and a second-order central differencing in y.

(b) Use von Neumann stability analysis to determine the stability requirement of the scheme.