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UNIVERSITY OF PETROLEUM & ENERGY STUDIES

End Semester Examination, December 2017

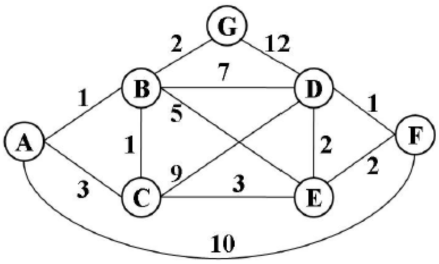
Program: B.Tech. (CSE) **Semester – III**
Subject (Course): Discrete Mathematical Structures **Max. Marks : 100**
Course Code : MATH 231 **Duration : 3 Hrs**
No. of page/s: 02

Section A (5 X 4 = 20)

1. Using principle of mathematical induction, prove that term $\mathbf{7^{2n} + 2^{3n-3}.3^{n-1}}$ is divisible by 25 for all positive integers.
2. (a) Define ***Konigsberg Bridge problem***. The *Euler Construction Company* has been contacted to construct an extra bridge in Konigsberg so that an Eulerian path through the town exists. Can this be done, and, if so, where should the bridge be built?
   (b) Prove that the number of pendant vertices in an **n**-vertices binary tree are ***(n+1)/2.***
3. Find the standard matrix for the linear operator that first rotates a vector counterclockwise about the z-axis through an angle 30 degree, and then reflects the resulting vector about the yz -plane, and then projects that vector orthogonally onto the xy-plane.
4. Find the equation of the plane spanned by the vectors **u** = (-1, 1, 1) and **v** = (3, 4, 4). Show that a bipartite graph with ***n*** vertices cannot have more than $\mathbf{n^2/4}$ edges.

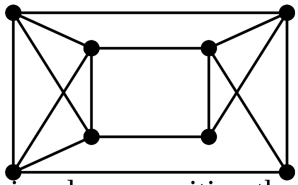
Section B (5 X 8 = 40)

5. Solve the recurrence relation $\mathbf{a_r - 7a_{r-1} + 12\, a_{r-2} = r.\, 2^r}$
6. Using Dijkastra's algorithm, find out the shortest cost path between node **A** to node **F** in the given graph. All intermediate steps must be clear.

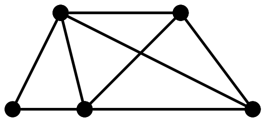


7. Consider the vectors **u** = (1, 2,-1) and **v** = (6, 4, 2) in $\mathbf{R}^3$. Show that **w** = (9, 2, 7) is a linear combination of **u** and **v** and that **w'**= (4,-1, 8) is not a linear combination of **u** and **v**.

8. Define Incidence Matrix and Adjacency matrix representations of a graph. Write these two matrix representation for the following graph after suitably labelling its edges and vertices.



9. Write the chromatic polynomial of the following graph using decomposition theorem.



Section C (2 X 20 = 40)

11. State and prove Euler's formula **(f = e-n+2)** for planar graphs, having **n** vertices, **e** edges and **f** regions.

12. (a) Each questions carries <u>*one*</u> mark.

(i) What is the number of edges in a complete graph of ***n*** vertices, K_n?

(ii) Write down the number of edges in any spanning tree of a wheel graph of **n** vertices W_n.

(iii) Write down the number of edges in a cycle graph of ***n*** vertices C_n.

(iv) Write down the maximum number of edges in a complete bipartite graph of ***n*** vertices.

(v) What will be the maximum height of an ***n*** vertices binary tree?

(vi) Write down the rank and nullity of a complete graph of ***n*** vertices, K_n

(vii) How many pendant vertices will be there in an ***n*** vertices binary tree?

(viii) What will be the vertex connectivity of a wheel graph of ***n*** vertices W_n.

(ix) A complete graph of ***7*** vertices K_7 is an Euler graph also.

(x) Name the **Kuratowski's** two most fundamental non-planar graphs.

(b) Let ***f:*** **R→R** be a function defined as ***f(x)*** **= 7x - 3** and ***g:*** **R→R** be another function defined as ***g(x)*** **= 3x**. Find $(g \circ f)^{-1}$ and $f^{-1} \circ g^{-1}$ and thus verify $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$