Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Course: Mathematical Modeling and Simulation **Semester: VIII**

Program: B.Tech ASE+AVE **Time 03 hrs.**

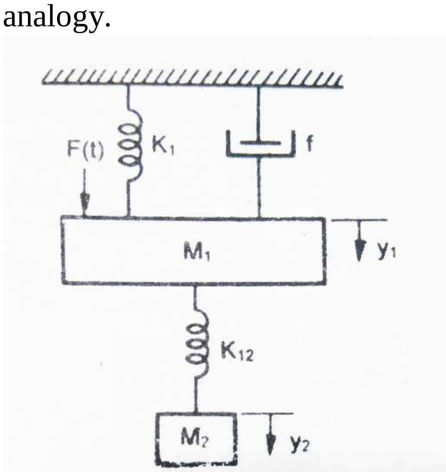
Course Code: AVEG 4003 **Max. Marks: 100**

Instructions: Choice between Q8 and Q9 as well as between Q10 and Q11

SECTION A (4x5=20 Marks)

S. No.		Marks	CO
Q 1	Explain *Through* and *Across* variables of Mechanical and Electrical system.	**5**	**CO1**
Q 2	What are state variables of a system? Compare state variables of *phugoid* and *short - period* motion of aircraft.	**5**	**CO2**
Q 3	Given the characteristics equation $$s^3 + 3s^2 + 3s + 1 + k = 0$$ Find the range of values of k for which the system is stable	**5**	**CO2**
Q 4	The longitudinal motion of an airplane is approximated by the differential equations $\dot{w} = -2\,w + 179\,\dot{\theta} - 27\,\delta$ $\ddot{\theta} = -0.25\,w - 15\,\dot{\theta} - 45\,\delta$ Re-write the equation in state space form	**5**	**CO4**

SECTION B (4x10=40 Marks)

Q 5	Write differential equations governing the behavior of the mechanical systems shown below. Also, obtain the analogous electrical circuit base on force current analogy. 	**10**	**CO1**
Q 6	Write mathematical expression for pure roll and pure yaw motion of aircraft. Compare dynamic characteristics (damping ratio and natural frequency) of an aircraft in pure roll and pure yaw using data given in Table 1.	**10**	**CO2**

Q7	Determine the response of an aircraft to a 5 deg step change in aileron deflection. Plot the roll rate versus time. Assume sea level standard condition and the airplane is flying at M=0.4. What is the steady-state roll rate and time constant for this motion? **Clp=-0.26, $C_{l_{\delta a}} = 0.04$; $S = 20\ m^2$, $b = 7\ m^2$ and I_x =4500 kg.m^2**	**10**	**CO4**
Q 8	The lag compensator also can be constructed from a simple electrical circuit as shown below. Show that the transfer function for this circuit can be written as: $$G(s) = \frac{e_0}{e_1} = \frac{T_2 + 1}{(T_2/b)s + 1}$$ Where $b = R_2/(R_1 + R_2)$ and $T_2 = R_2C$	**10**	**CO3**
	or		
Q 9	Examine the resulting root locus response of system transfer function $$G(s)H(s) = \frac{k}{s(s+2)(s+5)}$$ a) with addition of a simple pole (s+3) b) with addition of simple zero (s+3)	**10**	**CO3**
SECTION-C [2x20=40 Marks]			
Q 10	A) Derive governing equation and transfer function for electric motor servo and aircraft roll dynamic [10 Marks] B) Using above transfer functions design control for a roll attitude control system to maintain a wings level altitude for an aircraft having the following characteristics: $L_{\delta_a} = 3.0/s^2$, $L_p = -1.5/s$.The system performance is to have a damping ratio ζ=0.707, and an undamped natural frequency, $\omega_n = 10\ rad/s$. [10 Marks]	**20**	**CO2** **CO3**
	OR		
Q 11	A) Derive governing equation and transfer function for a wind-tunnel aircraft model constrained to motion in only the vertical direction; that is, pure plunging motion. Also, assume that the model is equipped with direct lift flaps.[10 Marks] B) Design a control system for the wind tunnel model so that the vertical velocity is held near 0. The aerodynamic characteristics are $Z_w = 2.0$, and $Z_{\delta f} = -40$. Assume the direct lift actuator can be represented by the transfer function $\frac{\delta_f}{e} = \frac{k}{s+10}$. [10Marks]	**20**	**CO2** **CO3**
Q12	Given the linear-time invariant dynamical system that is governed by the equation $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}[\eta]$$ where $\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix}$ Determine the state transition matrix and the response of the system if the input signal is a unit step function.	**20**	**CO4**

Table 1: Lateral derivatives for the general aviation airplane

$Y_v = -0.254\ (\mathrm{s}^{-1})$	$L_v = -0.091\ (\mathrm{ft{\cdot}s})^{-1}$
$Y_\beta = -45.72\ (\mathrm{ft/s^2})$	$L_\beta = -16.02\ (\mathrm{s}^{-2})$
$Y_p = 0$	$L_p = -8.4\ (\mathrm{s}^{-1})$
$Y_r = 0$	$L_r = 2.19\ (\mathrm{s}^{-1})$
$N_v = 0.025\ (\mathrm{ft{\cdot}s})^{-1}$	
$N_\beta = 4.49\ (\mathrm{s}^{-2})$	
$N_p = -0.35\ (\mathrm{s}^{-1})$	
$N_r = -0.76\ (\mathrm{s}^{-1})$	

Source: R.C. Nelson, “Flight Stability and Automatic Control “

Table 2: Laplace transform relations

$f(t)$	$F(s)$	$f(t)$	$F(s)$
$u(t)$	$1/s$	$\sin \omega t$	$\omega/(s^2+\omega^2)$
t	$1/s^2$	$\cos \omega t$	$s/(s^2+\omega^2)$
t^n	$n!/s^{n+1}$	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\delta(t)$ Unit impulse	1	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$\int_{-\varepsilon}^{+\varepsilon} \delta(t)\,dt = 1$		$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
e^{-at}	$1/(s+a)$	$t\cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$t\sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$t^n e^{-at}$	$n!/(s+a)^{n+1}$		