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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme: B.Tech ECE and Electrical	**Semester – II**
Course Name: Mathematics II	**Max. Marks : 100**
Course Code: MATH 1013	**Duration : 3 Hrs**
No. of page/s:2	

Instructions: Use of non-programmable Scientific Calculator is allowed

Section A

(Attempt all questions)

		MARKS	
1.	Find the Fourier transform of $f(x)=\begin{cases}1, \lvert x\rvert<a \\ 0, \lvert x\rvert>a\end{cases}$	**[4]**	**CO5**
2.	Solve the following IVP using Euler's method with $h=0.1$ for $x\in[11.4]$ given that $y'=x+y+xy$, $y(1)=0$.	**[4]**	**CO3**
3.	Find an approximate value of y when $x=0.1$ with $h=0.1$, if $y'-2y=3e^x$, $y(0)=0$ using Taylor's series second order method.	**[4]**	**CO3**
4.	Solve $u_t=\frac{1}{16}u_{xx}, 0\le x\le 1$,with $u(x,0)=x(1-x)$ and $u(0,t)=0=u(1,t)$ for all $t>0$. Use Bender-Schmidt method with $h=\frac{1}{4}$. Compute for two time steps.	**[4]**	**CO3**
5.	Construct an equivalent form $x=\phi(x)$(where $\phi(x)$ is called an iterative function) for the equation $3x^4+x^3+12x+4=0$ such that $\lvert\phi'(x)\rvert<1$ in $x\in(-1,0)$.	**[4]**	**CO2**

SECTION B

(All questions are compulsory, Q10 has internal choice)

6.	Write the second order equation $\frac{d^2y}{dx^2}+sinx\left(\frac{dy}{dx}\right)-\left(\frac{dy}{dx}\right)^2+xy=e^x$ as an equivalent pair of first order equations and hence solve using Runge-Kutta method of fourth order for $x=0.2$. Initial conditions are $y(0)=1$, $y'(0)=1$. Consider the step length $h=0.2$.	**[08]**	**CO3**
7.	Solve $u_{xx}+u_{yy}=0$ numerically under the boundary conditions $u(x,0)=2x, u(0,y)=-y, u(x,1)=2x-1, u(1,y)=2-y$ with square mesh of width $h=\frac{1}{3}$.	**[08]**	**CO4**
8.	Calculate the area bounded by the curve $y=x^2+4$, and the lines $y=-1, x=1$ and $x=4$ by Trapezoidal rule, taking number of subintervals as 6.	**[08]**	**CO2**

9.	Evaluate L^{-1}¿using convolution theorem	**[08]**	**CO5**
10.	Show that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\,sinx$ **OR** Solve the Lagrange's equation $a(x,y,z)p+b(x,y,z)q=c(x,y,z)$ where $a(x,y,z)\equiv 2x^2+y^2+z^2-2yz-zx-xy;$ $b(x,y,z)\equiv x^2+2y^2+z^2-yz-2zx-xy;$ $c(x,y,z)\equiv x^2+y^2+2z^2-yz-zx-2xy;\ p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.	**[08]**	**CO1**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice**			
11. A	Solve the heat equation $u_t=u_{xx}, 0\le x\le 1$, subject to the initial and boundary conditions $u(x,0)=\sin(\pi x), 0\le x\le 1, u(0,t)=0, u(1,t)=0, t>0$ using Crank-Nicolson method with $h=\frac{1}{3}, k=\frac{1}{36}$. Integrate for one time step.	**[10]**	**CO4**
11.B	Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$.	**[10]**	**CO5**
12. A	Solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-3)y=e^{x^2}$. **OR** Find a complete integral of $pxy+pq+qy=yz$ using Charpit's method where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.	**[10]**	**CO1**
12.B	Solve the differential equation $y''+y=t, y(0)=1, y'(0)=-2$ by Laplace transform method. **OR** Evaluate $\int_0^\infty \frac{e^{-2t}\,sinht\ sint}{t}dt$ using Laplace transforms.	**[10]**	**CO5**

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Section A
(Attempt all questions)

		MARKS	
1.	Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, \lvert x\rvert\le a \\ 0, \lvert x\rvert>a \end{cases}$	**[4]**	**CO5**
2.	Solve $y'=x^2-y^2$, with $x_0=2, y_0=1$ to find an approximate value of $y(2.4)$ using Euler's method with 0.1step length.	**[4]**	**CO3**
3.	Using Taylor's series second order method, find y for $x=3.1$ given that $y'=2xy+3y$, $y(3)=1$, $h=0.1$.	**[4]**	**CO3**
4.	Solve $u_t=u_{xx}, 0\le x\le 1$, with $u(x,0)=\sin(2\pi x)$ and $u(0,t)=0=u(1,t)$ for all $t>0$. Use Bender-Schmidt method with $h=\frac{1}{4}$. Compute for two time steps.	**[4]**	**CO3**
5.	Construct an equivalent form $x=\phi(x)$(where $\phi(x)$ is called an iterative function) for the equation $x^3+x^2-1=0$ such that $\lvert\phi'(x)\rvert<1$ in $x\in(0,1)$.	**[4]**	**CO2**

SECTION B
(All questions are compulsory, Q10 has internal choice)

6.	Find the values of $y(2.2)$ and $y'(2.2)$ using Runge-Kutta method of fourth order by considering the step size $h=0.2$. Given that $x^2y''-e^x(y')^2+y=x^2e^x$ with $y(2)=3, y'(2)=0.8$.	**[08]**	**CO3**
7.	Find the solution of the Laplace's equation $u_{xx}+u_{yy}=0$ in the region R, where R is a square of side 3 units. Boundary conditions are defined as $u(0,y)=0, u(3,y)=3+y$, $u(x,0)=x, u(x,3)=2x$. Assume step length as $h=1$.	**[08]**	**CO4**

8.	The area A inside the closed curve $x^2+y^2=cosx$ is given by $A=4\int_0^{\alpha}\sqrt{cosx-x^2}\,dx$ where α is a positive root of the equation $cosx=x^2$ (i) Compute α correct to three decimal places using Newton-Raphson method (ii) Compute area A using Trapezoidal rule by taking number of subintervals as 4.	**[08]**	**CO2**
9.	Evaluate L^{-1}¿using convolution theorem	**[08]**	**CO5**
10.	Show that $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}cosx$ **OR** Solve the Lagrange's equation $z(x+y)p+z(x-y)q=x^2+y^2$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.	**[08]**	**CO1**

SECTION C
(Q11 is compulsory and Q12A, Q12B have internal choice)

11. A	Solve the heat equation $u_t=u_{xx}, 0\le x\le 2$, subject to the initial and boundary conditions $u(x,0)=\sin(\pi x)+\sin(3\pi x), 0\le x\le 2$, $u(0,t)=0=u(2,t)$ using Crank-Nicolson method with $\Delta x=h=\frac{2}{3}, \Delta t=k=\frac{1}{9}$. Integrate for one time step.	**[10]**	**CO4**
11.B	Find the inverse Laplace transform of $\frac{s}{s^4+s^2+1}$.	**[10]**	**CO5**
12. A	Solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin(2x)$ **OR** Find a complete integral of $z^2=pqxy$ using Charpit's method where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.	**[10]**	**CO1**
12.B	Solve the differential equation $y''-3y'+2y=e^{3t}, y(0)=0, y'(0)=0$ by Laplace transform method. **OR** Evaluate $\int_0^{\infty} e^{-t}t\sin^2(3t)\,dt$ using Laplace transforms.	**[10]**	**CO5**

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