

Name:

Enrolment No:



UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme Name: B. Sc. (H.) (Physics/Chemistry)

Semester : II

Course Name : Calculus

Time : 03 hrs

Course Code : MATH 1033

Max. Marks : 100

Nos. of page(s) : 2

SECTION A

(All questions are compulsory)

S. No.		Marks	CO
Q 1	What value should be assigned to $f(x) = \frac{1-x}{1-x^{1/3}}$, $x \neq 1$ at $x=1$, so that it is continuous at $x=1$	4	CO 1
Q 2	Investigate the continuity at $(0, 0)$ of $f(x, y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$	4	CO 2
Q 3	Find the equation of the envelope of the family of curves $x \cos \alpha + y \sin \alpha = p$ where p is a constant and α is a parameter.	4	CO 3
Q 4	Find the points of inflection of the curve $y = 3x^4 + 4x^3 + 6x^2 + 12x + 12$	4	CO 4
Q 5	Using the mean value theorem show that $ \cos b - \cos a \leq b - a $	4	CO 5

SECTION B

(All questions are compulsory, Question 9 has internal choices)

Q 6	Show that for all $x > 0$, $1-x < e^{-x} < 1-x + x^2/2$	10	CO 5
Q 7	Find the asymptotes of the curve $(2x + 3) y = (x-1)^2$	10	CO 3
Q 8	Trace the curve $x^3 + y^3 = 3axy$, where a is a constant.	10	CO 4
Q 9	Show that $\lim_{x \rightarrow 1} 2^{\frac{1}{x-1}}$ does not exist.	10	CO 1
OR			
Q 9	.Prove that, if f is derivable at c and $f'(c) \neq 0$ then the function $1/f$ is also derivable	10	CO 1

	thereat and $\left(\frac{1}{f}\right)'(c) = \frac{-f'(c)}{[f(c)]^2}$		
SECTION C (All questions are compulsory, Question 11 has internal choices)			
Q 10 (A)	Show that the normal to a given curve is a tangent to its evolute.	10	CO 3
Q 10 (B)	Sketch the graph of the curve $y = \frac{(x-1)(x-3)}{x^2}$	10	CO 4
Q 11 (A)	Show that the function $f(x, y) = \begin{cases} \frac{x^3+2y^3}{x^2+y^2}, & (x, y) \neq 0 \\ 0, & (x, y) = (0, 0) \end{cases}$ (i) is continuous at (0, 0) (ii) possesses partial derivatives $f_x(0, 0)$ and $f_y(0, 0)$ (iii) is not differentiable at (0, 0)	10	CO 2
Q 11 (B)	Find the n^{th} derivative of y where $y = e^{ax} \cdot \cos (bx+c)$	10	CO 2
OR			
Q 11 (A)	Determine $y_n(0)$ where $y = e^{m \cos^{-1} x}$	10	CO 2
Q 11 (B)	If $z = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$ then show that $\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial f}{\partial \theta}\right)^2$	10	CO 2

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SECTION A

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S. No.		Marks	CO
Q 1	Evaluate $\lim_{x \rightarrow \infty} x \tan \frac{1}{x}$	4	CO 1
Q 2	Investigate the continuity at (0, 0) of $f(x, y) = \begin{cases} \frac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$	4	CO 2
Q 3	Find the equation of the envelope of the family of straight lines $y = cx + c^2$, where c is a parameter.	4	CO 3
Q 4	Find the points of inflection of the curve $y = e^{-x^2}$.	4	CO 4
Q 5	Using the mean value theorem show that $ \cos b - \cos a \leq b - a $	4	CO 5

SECTION B

(All questions are compulsory, Question 9 has internal choices)

Q 6	Find the maximum/minimum values of the function $f(x) = \sin x(1 + \cos x), 0 \leq x \leq 2\pi$. Also find the greatest and smallest values	10	CO 5
Q 7	Find the asymptotes of the curve $(2x + 3)y = (x - 1)^2$	10	CO 3
Q 8	Trace the curve $y = \frac{x^2}{\sqrt{x^2 - 4}}$	10	CO 4
Q 9	Show that $\lim_{x \rightarrow 1} 2^{\frac{1}{x-1}}$ does not exist.	10	CO 1

	OR		
Q 9	Define uniform continuity and show that $f(x) = 1/x$ is not uniformly continuous on $(0, 1]$	10	CO 1
SECTION C (All questions are compulsory, Question 11 has internal choices)			
Q 10 (A)	Find the evolute of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	10	CO 3
Q 10 (B)	Sketch the graph of the curve $y = x + \frac{1}{x}$	10	CO 4
Q 11 (A)	If f is a homogeneous function of x and y of degree n then show that $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf$	10	CO 2
Q 11 (B)	Find the n^{th} derivative of y where $y = e^{ax} \cdot \sin (bx+c)$	10	CO 2
	OR		
Q 11 (A)	State and Prove Leibnitz's theorem of successive differentiation.	10	CO 2
Q 11 (B)	Find the total differentiation coefficient of x^2y with respect to x when x, y are connected by $x^2 + xy + y^2 = 1$.	10	CO 2