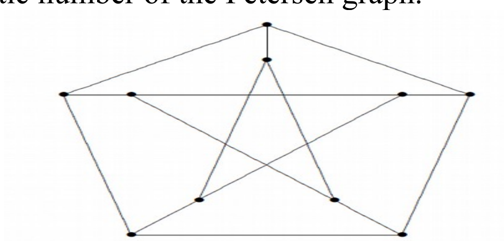
Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2019

Course:	**Discrete Mathematical Structures**	**Semester:**	**II**
Program:	**BCA**	**Time**	**03 hrs.**
Course Code:	**CSEG 2006**	**Max. Marks:**	**100**

Instructions: (i) Start answering a question on new page, (ii) All parts of a section should be answered together, (iii) Scattered part answers will not be evaluated, (iv) Exchange of mobile phone, calculator or any item is not allowed, (v) Exam is close book.

SECTION A

S. No.		Marks	CO
Q 1	Show that if graph 'G' has a circuit of odd length, then G is not bipartite.	**4**	**CO4**
Q 2	Show that every tree can be colored by using two colors.	**4**	**CO5**
Q 3	Determine the chromatic number of the Petersen graph. 	**4**	**CO4**
Q 4	Determine whether the vectors $\mathbf{v}_1 = (1, -1, 4)$, $\mathbf{v}_2 = (-2, 1, 3)$, and $\mathbf{v}_3 = (4, -3, 5)$ span R^3.	**4**	**CO3**
Q 5	What is a composite transformation? When doing a composite transformation, does the order in which you perform the transformations matter? Show with suitable example.	**4**	**CO3**

SECTION B

Q 6	Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is _______________.	**10**	**CO4, CO5**
Q 7	Find the shortest path from node 1 to all other nodes using Dijkstra's algorithm.	**10**	**CO4, CO5**

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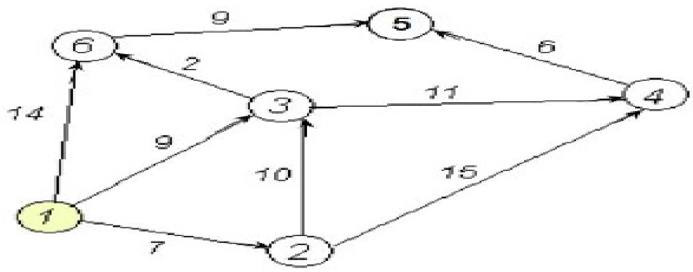

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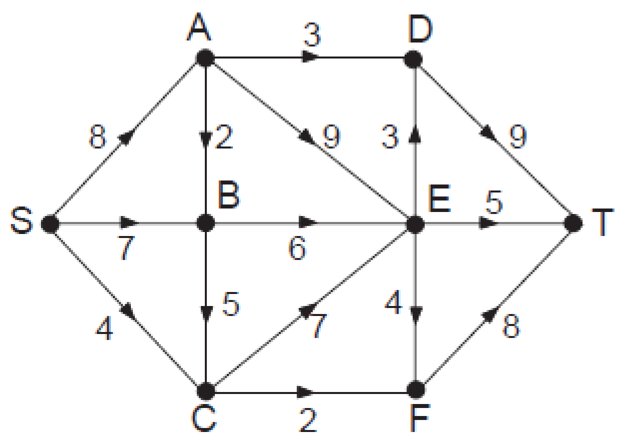
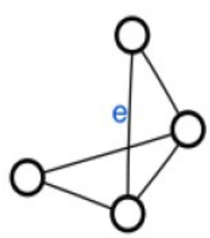
<tr><td>Q 8</td><td>
For the network shown below, find the value of the maximum flow and a cut which has capacity of the same value. Verify the maximum flow as well.



</td><td>10</td><td>CO4, CO5</td></tr>
<tr><td>Q 9</td><td>

1. Given any 4 sets A, B, C, and D, is it true that (A∪C) ∩ (A ∪D) ∩ (B∪ C) ∩ (B ∪ D) is always equal to (A ∩ B) ∪ (C ∩ D)? If true, use the distributive laws, the associative laws, and the idempotent laws to prove it. If not, give a counter example.
2. Let G be the graph below.



(a) Find its chromatic polynomial P(G) using deletion-contraction of the edge e.
(b) By using (a), compute χ(G).

OR

1. Let $f : A \to B$ and $g : B \to C$ be functions. Prove each of the following:

a) If f and g are one-to-one, then $g \, o \, f$ is one-to-one.
</td><td>5+5</td><td>CO 1, CO 4</td></tr>
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	b) If $g\ o\ f$ is one-to-one, then f is one-to-one. 2. Discuss the necessary and sufficient condition for Euler circuit with suitable example.		
	SECTION-C		
Q 10	Answer the following: a) A person deposits Rs. 500 in a saving account at a bank. The interest rate is 9% per year with interest compounded annually. Let ar be the total amount after r years. Find the recurrence relation for ar and solve it. How much amount the person will receive after 10 years. b) If the degree sequence of the simple graph G is 2, 2, 2, 1, 1, what is the degree sequence for complement of G? c) Find the generating functions for the following sequences. In each case, try to simplify the answer. I. 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, … II. 1, 1, 1, 1, 1, … III. 1, 3, 3, 1, 0, 0, 0, 0, … OR Write short notes on following: (a) Pigeonhole Principle (b) Mathematical Induction (c) Linear Homogeneous Recurrence Relation (d) Euler and Hamiltonian Graphs (e) Prim's and Kruskal's Algorithm	**20**	**CO1, CO2, CO4**
Q 11	I. Determine whether the following functions are linear transformations. If they are, prove it; if not, provide a counterexample to one of the properties: (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, with $$T\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x+y\\y\end{bmatrix}$$ (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$, with $$T\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x^2\\y^2\end{bmatrix}$$ II. Find the nullity of the following matrix $$A = \begin{pmatrix}1 & 1 & 1 & 1\\2 & 3 & 4 & 5\end{pmatrix}.$$ III. Find the rank of the matrix $$A = \begin{pmatrix}-1 & 0 & -1 & 2\\2 & 0 & 2 & 0\\1 & 0 & 1 & -1\end{pmatrix}.$$	**20**	**CO3**

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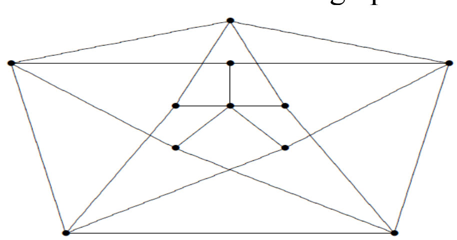
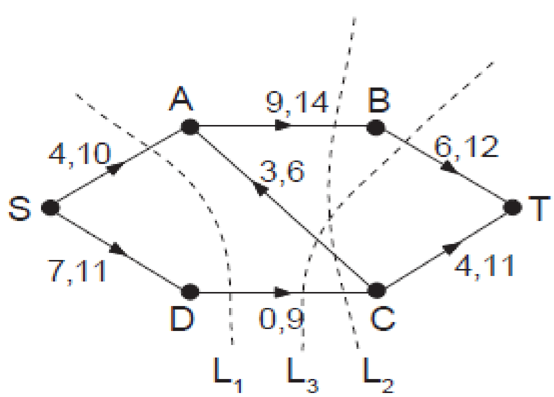
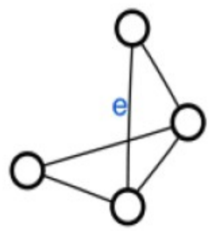
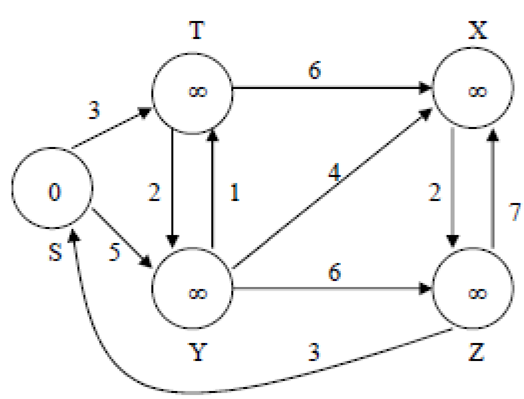
SECTION A

S. No.		**Marks**	**CO**
Q 1	Determine the chromatic number of the Grotzsch graph. 	4	CO 4
Q 2	Find the maximum flow of network shown below: 	4	CO 5

Q 3	The smallest subspace of R^3 containing the vectors (0, −3, 6) and (0, 1, −2) is the line whose equations are x = a and z = by where a = ? and b = ?.	4	CO 3
Q 4	Prove De Morgan's rule $(A\cup B)^c = A^c \cap B^c$ by considering an element *x* of both sides of the equation. Do not use a Venn diagram.	4	CO 1
Q 5	Show that if n is an integer and $n^3 +5$ is odd, then n is even using a) a proof by contraposition. b) a proof by contradiction.	4	CO 2
	SECTION B		
Q 6	Let V be the set of all real numbers. Define an operation of "addition" by $x \oplus y$ = the maximum of x and y for all x, y ∈ V . Define an operation of "scalar multiplication" by $\alpha \odot x = \alpha x$ for all α ∈ R and x ∈V. Under the operations ⊕ and ⊙ the set V is not a vector space. List the vector space axiom which fail to hold.	10	CO 3
Q 7	Determine whether the sequence $\{a_n\}$, where $a_n = 3n$ for every nonnegative integer n, is a solution of the recurrence relation $a_n = 2a_{n-1} — a_{n-2}$ for n = 2, 3,4, Answer the same question where $a_n = 2^n$ and where $a_n = 5$.	10	CO 2
Q 8	Draw an example graph for each of these a. A planar graph has 5 vertices and 3 faces. How many edges does it have? b. A planar graph has 7 edges and 5 faces. How many vertices does it have?	10	CO 4
Q 9	*Prove using mathematical induction that for all n ≥ 1,* $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$ OR There are 30 identical souvenirs, to be distributed among the 50 IMO trainees, and each trainee may get more than one souvenir. How many ways are there to distribute the 30 souvenirs among the 50 trainees?	10	CO 1

SECTION-C			
Q 10	a) A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month. Find a recurrence relation for the number of pairs of rabbits on the island after n months, assuming that no rabbits ever die. b) Let R = {(1, 1), (2, 1), (3, 2), (4, 3)}. Find the powers R^n, n = 2, 3,4,.... c) Let G be the graph below.  I. Find its chromatic polynomial P(G) using deletion-contraction of the edge e. II. By using (a), compute χ(G). OR Write short notes on following: (a) Pigeonhole Principle (b) Mathematical Induction (c) Linear Homogeneous Recurrence Relation (d) Euler and Hamiltonian Graphs (e) Prim's and Kruskal's Algorithm	20	CO 1, CO2, CO 4
Q 11	a. Prove that the chromatic polynomial of any tree with 's' vertices is: $k(k-1)^{s-1}$ b. Define Vector Space. List various properties of a vector space. c. Run Dijkstra's algorithm on the directed graph shown below. first using vertex 's' as the source and then using vertex 'z' as the source. 	20	CO 3, CO 4