Name: Enrolment No:	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2019

Course: Mathematics II — **Semester: II**
Program: All SoCS Branches — **Time 03 hrs.**
Course Code: MATH 1005 — **Max. Marks: 100**

Instructions: Attempt all questions. Question 9 and Question 11 have internal choice attempt any one.

SECTION A

S. No.		**Marks**	**CO**
Q 1	Find the real root of the equation $x = e^{-x}$ in the interval $(0,1)$ using the newton-Raphson method. Perform three iterations.	**4**	**CO3**
Q 2	The speed, v metres per second, of a car, t seconds after it starts, is shown in the following table: t: 0, 12, 24, 36, 48, 60, 72 v: 0, 3.60, 10.08, 18.90, 21.60, 18.54, 10.26 Using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule, find the distance travelled by the car in 72 seconds.	**4**	**CO4**
Q 3	Prove that $\Delta = E\nabla = \nabla E = \delta E^{\frac{1}{2}}$ where Δ, ∇, δ and E are forward difference, backward difference, central difference and shift operators respectively.	**4**	**CO4**
Q 4	Use Euler's method to obtain approximate values of $y(0.1), y(0.2), y(0.3)$ and $y(0.4)$ for the differential equation $\frac{dy}{dx} = x + y, y(0) = 1$ with $h = 0.1$.	**4**	**CO3**
Q 5	In the poset $(\mathbb{R}, \leq)$ for the set $A = \{x \in \mathbb{R} : 1 < x < 2\}$, find (i) all the upper and lower bounds of the set A, (ii) greatest lower bound and least upper bound of set A. (Note: $\mathbb{R}$ represents set of real numbers)	**4**	**CO5**

SECTION B

Q 6	Let $X = \{1,2,3,4,5,6\}$, then divisibility relation $/$ is a partial order relation on X. Draw the Hasse diagram of the poset $(X, /)$. Hence, find greatest element, least element, minimal elements and maximal elements of $(X, /)$. (Note: x/y means "x divides y")	**10**	**CO5**
Q 7	Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls, (ii) at least one boy, (iii) no girl, (iv) at most two girls? Assume equal probabilities for boys and girls.	**10**	**CO2**

Q 8	Using the method of variation of parameters, solve the differential equation $$(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=2(x-1)^2e^{-x}, 0<x<1.$$	**10**	**CO1**
Q 9	Evaluate $$\int_0^6 \frac{dx}{1+x^2}$$ by using (i) Trapezoidal rule, (ii) by Simpson's one-third rule. (Take step size $h = 1$).	**10**	**CO4**
	OR		
Q 9	By means of Newton's divided difference formula, find the value of $y(8)$ from the following table: x: 4, 5, 7, 10, 11, 13 y: 48, 100, 294, 900, 1210, 2028	**10**	**CO4**
	SECTION-C		
Q 10 A	Use Gauss-Jacobi iterative method to solve the following system of simultaneous equations: $$9x+4y+z=-17$$ $$x+6y=4$$ $$x-2y-6z=14$$ Perform four iterations. Take initial approximation $x^{(0)}=y^{(0)}=z^{(0)}=0$.	**10**	**CO3**
Q 10 B	The table given below reveals the velocity 'v' of a body during the time 't' specified. Find its acceleration at $t = 1.0$ and $t = 1.1$. t: 1.0, 1.1, 1.2, 1.3, 1.4 v: 43.1, 47.7, 52.1, 56.4, 60.8	**10**	**CO4**
Q 11	(A) Solve the differential equation $$(D^2+5D+4)y = x^2+7x+9, \quad where\ D \equiv \frac{d}{dx}.$$	**10**	**CO1**
	(B) Find the value of $y(1.1)$ using Runge-Kutta method of fourth order, given that $$\frac{dy}{dx}=y^2+xy, \quad y(1)=1.0$$ Take $h = 0.05$.	**10**	**CO3**
	OR		
Q 11	(A) Solve the following differential equation: $$(D^2+4)y=\sin 3x+\cos 2x \quad where\ D\equiv\frac{d}{dx}.$$	**10**	**CO1**
	(B) Find the real root of the equation $xe^x=\cos x$ in the interval (0,1) using Regula-Falsi method correct to four decimal places.	**10**	**CO3**

Table for Q 9 (OR):

x:	4	5	7	10	11	13
y:	48	100	294	900	1210	2028

Table for Q 10 B:

t:	1.0	1.1	1.2	1.3	1.4
v:	43.1	47.7	52.1	56.4	60.8

Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2019

Course: Mathematics II **Semester: II**
Program: All SoCS Branches **Time 03 hrs.**
Course Code: MATH 1005 **Max. Marks: 100**

Instructions: Attempt all questions. Question 9 and Question 11 have internal choice attempt any one.

SECTION A

S. No.		**Marks**	**CO**
Q 1	Perform five iterations of bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.	**4**	**CO3**
Q 2	A train is moving at the speed of 30 metres/second. Suddenly brakes are applied. The speed of the train per second after t seconds is given by Time t: 0, 5, 10, 15, 20, 25, 30 Speed v: 30, 24, 19, 16, 13, 11, 10 Apply Simpson one-third rule to determine the distance moved by the train in 30 seconds.	**4**	**CO4**
Q 3	Prove that $$\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$$ where Δ and ∇ are forward difference and backward difference operators respectively.	**4**	**CO4**
Q 4	For the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$. Calculate $y(0.2)$ by Taylor's series method retaining three non-zero terms only.	**4**	**CO3**
Q 5	In the poset $(\{1,2,3,4,6,9,12,18,36\}, \ /)$ find the greatest lower bound and least upper bound of the sets $\{6,18\}$ and $\{4,6,9\}$. (Note: x/y means "x divides y")	**4**	**CO5**

SECTION B

S. No.		**Marks**	**CO**
Q 6	The inclusion relation $\subseteq$ is partial order relation on the power set $P(S)$ of all subsets of S where $S = \{a, b, c\}$. Draw the Hasse diagram of the poset $(P(S), \ \subseteq)$. Hence, find greatest element, least element, minimal elements and maximal elements of $(P(S), \subseteq)$. (Note: $A \subseteq B$ means "A is subset of B")	**10**	**CO5**
Q 7	The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be (i) no accident, (ii) at least 2 accidents, (iii) at most 3 accidents, (iv) between 2 and 5 accidents.	**10**	**CO2**
Q 8	Solve by the method of variation of parameters the differential equation $$x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right).$$	**10**	**CO1**

<table>
<tr><td>Q 9</td><td>Evaluate integral

$$\int_0^6 \frac{e^x}{1+x}\,dx$$

.

Using (i) Trapezoidal rule, (ii) Simpson's $3/8^{th}$ rule, (Take step size $h = 1$).</td><td>10</td><td>CO4</td></tr>
<tr><td colspan="4">OR</td></tr>
<tr><td>Q 9</td><td>The population of a town in the decimal census was as given below. Estimate the population for the year 1895.
<table>
<tr><td>Year x:</td><td>1891</td><td>1901</td><td>1911</td><td>1921</td><td>1931</td></tr>
<tr><td>Population y:
(in thousands)</td><td>46</td><td>66</td><td>81</td><td>93</td><td>101</td></tr>
</table></td><td>10</td><td>CO4</td></tr>
<tr><td colspan="4">SECTION-C</td></tr>
<tr><td>Q 10 A</td><td>Solve the following system of equations by Gauss-Seidel iterative method:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Perform four iterations. Take initial approximation $x^{(0)} = y^{(0)} = z^{(0)} = 0$.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 10 B</td><td>The distance covered by an athlete for the 36.5 metre race is given in the following table:
<table>
<tr><td>Time (sec):</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Distance (metre):</td><td>0</td><td>2.5</td><td>8.5</td><td>15.5</td><td>24.5</td><td>36.5</td></tr>
</table>
Determine the speed of the athlete at $t = 4\ sec$ and $t = 5\ sec$.</td><td>10</td><td>CO4</td></tr>
<tr><td rowspan="2">Q 11</td><td>(A) Solve the differential equation:

$$(D^2 + D + 1)y = (1 + e^x)^2\ ;\ \ D \equiv \frac{d}{dx}.$$</td><td>10</td><td>CO1</td></tr>
<tr><td>(B) Using Picard's method of successive approximations, obtain a solution upto fifth approximation of the equation $\frac{dy}{dx} = y + x$ such that $y = 1$ when $x = 0$.</td><td>10</td><td>CO3</td></tr>
<tr><td colspan="4">OR</td></tr>
<tr><td rowspan="2">Q 11</td><td>(A) Obtain the general solution of the differential equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x \cos x.$$</td><td>10</td><td>CO1</td></tr>
<tr><td>(B) Find the real root of the equation $3x + \sin x - e^x = 0$ in the interval $(0,1)$ by the method of false position correct to four decimal places.</td><td>10</td><td>CO3</td></tr>
</table>