Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

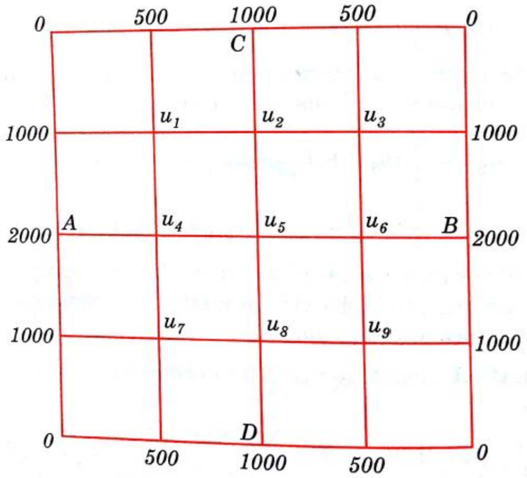
Course: M.Tech **Semester: I**
Programme: Petroleum Engineering
Time: 03 hrs. **Max. Marks: 100**
No. of Pages:03

Instructions: Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 10 marks)**.**

SECTION A

S. No.		**Marks**	**CO**
Q 1	An approximate value of π is given by 3.1428571 and its true value is 3.1415926. Find absolute and relative errors.	**4**	**CO1**
Q 2	Evaluate $\int_0^6 \frac{1}{1+x^2}dx$ by using Trapezoidal rule.	**4**	**CO2**
Q 3	Using Euler's method, find an approximate value of y corresponding to $x = 0.5$, given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$. (Take $h = 0.1$)	**4**	**CO5**
Q 4	Find by Taylor's series method, the values of y at $x = 0.1$ to four places of decimals from $\frac{dy}{dx} = x^2y - 1, y(0) = 1.$	**4**	**CO5**
Q 5	Define 5-Point finite difference approximation to partial derivatives.	**4**	**CO6**

SECTION B

Q 6	Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in figure.  	**10**	**CO6**

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Q 7</td><td>Four equidistant values u_{-1}, u_0 , u_1 and u_2 being given, a value is interpolated by Lagrange’s formula, show that it may be written in the form
$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_2$ where $x + y = 1$.</td><td>10</td><td>CO1</td></tr>
<tr><td>Q 8</td><td>Apply Runge-Kutta method to find approximate value of y for $x = 0.2$ in steps of 0.1, if $\frac{dy}{dx} = x + y^2$, given that $y = 1$ where $x = 0$.</td><td>10</td><td>CO5</td></tr>
<tr><td>Q 9</td><td>Apply Graeffe’s method to find all the roots of the equation $x^4 - 3x + 1 = 0$.
OR
Find the cube root of 30 correct to three decimal places, using Horner’s method.</td><td>10</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION-C</td></tr>
<tr><td>Q 10A</td><td>The population of a town in the decimal census was as given below. Estimate the population for the year 1895.
<table>
<tr><td>Year X:</td><td>1891</td><td>1901</td><td>1911</td><td>1921</td><td>1931</td></tr>
<tr><td>Population y (in thousands)</td><td>46</td><td>66</td><td>81</td><td>93</td><td>101</td></tr>
</table></td><td>10</td><td>CO1</td></tr>
<tr><td>Q 10B</td><td>Apply Gauss-Seidal iteration method to solve the equation $20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25$.</td><td>10</td><td>CO4</td></tr>
<tr><td>Q 11A</td><td>Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5, t \geq 0$ given that $u(x,0) = 20, u(0,t) = 0, u(5,t) = 100$. Compute u for the time step with $h = 1$ by Crank-Nicholson method.
OR
Find the values of $u(x,t)$ satisfying the parabolic equations $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ and the boundary conditions $u(0,t) = 0 = u(8,t)$ and $u(x,0) = 4x - \frac{1}{2}x^2$ at the points $x = i: i = 0, 1, 2, \dots \dots, 7$ and $t = \frac{1}{8}j: j = 0, 1, 2, \dots \dots, 5$.</td><td>10</td><td>CO5</td></tr>
<tr><td>Q 11B</td><td>Consider the following boundary value problem (BVP)
$$\frac{d^2y}{dx^2} - y = x^2, 0 \leq x \leq 1$$ $$y(0) = 1, y(1) = 0.$$ Find an approximate solution $\bar{y}(x) = a_1\phi_1(x) + a_2\phi_2(x)$ by Galerkin’s method. Consider the basis functions $\phi_1(x) = (1-x)$ and $\phi_2(x) = (1-x)^2$.
OR</td><td>[10]</td><td>CO6</td></tr>
</table>

Find an approximate solution of the following problem by Subdomain (Partition) method diving the interval $0 \leq x \leq 1$ two equal subintervals and using the basis functions $\phi_1(x) = (1 - x)$ and $\phi_2(x) = (1 - x)^2$.

$$\frac{d^2y}{dx^2} - y = x, 0 \leq x \leq 1$$

$$y(0) = 1, y(1) = 0.$$

CONFIDENTIAL ✓

Name of Examination (Please tick, symbol is given)	:	**MID**		**END**	✓	**SUPPLE**	
Name of the School (Please tick, symbol is given)	:	**SOE**	✓	**SOCS**		**SOP**	
Programme	:	**M.Tech**					
Semester	:	**I**					
Name of the Course	:	**Petroleum Engineering**					
Course Code	:	**MATH-7001 (Applied Mathematics in Petroleum Engineering)**					
Name of Question Paper Setter	:	**Dr Reshu Gupta**					
Employee Code	:	**40001318**					
Mobile & Extension	:	**9456068062, 1577**					

Note: Please mention additional Stationery to be provided, during examination such as Table/Graph Sheet etc. else mention "NOT APPLICABLE":

FOR SRE DEPARTMENT

Date of Examination	:	
Time of Examination	:	
No. of Copies (for Print)	:	

Note: - Pl. start your question paper from next page

Model Question Paper (Blank) is on next page

Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2018

Course: M.Tech **Semester: I**
Programme: Petroleum Engineering
Time: 03 hrs. **Max. Marks: 100**
No. of Pages:03

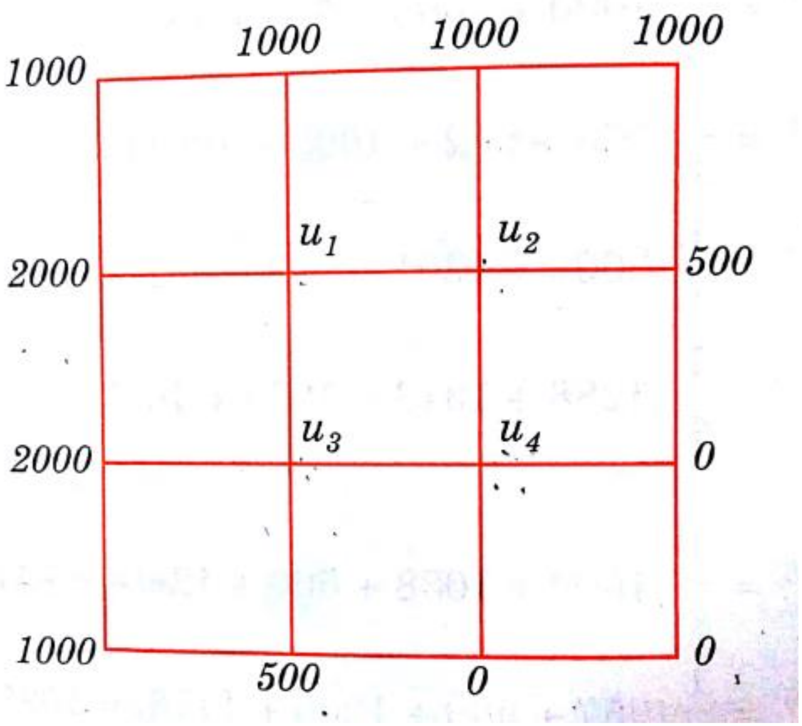
Instructions: Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 10 marks)**.**

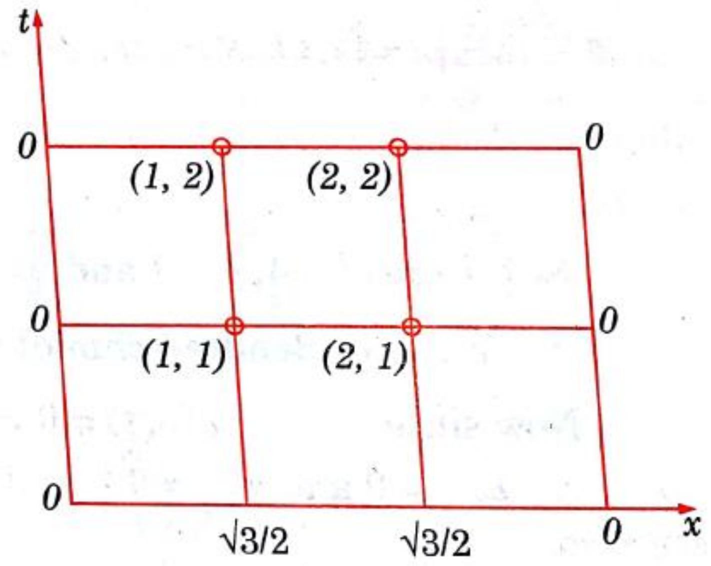
SECTION A

<table>
<tr><th>S. No.</th><th></th><th>Marks</th><th>CO</th></tr>
<tr><td>Q 1</td><td>Prove that, $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^2}{4}}$</td><td>4</td><td>CO1</td></tr>
<tr><td>Q 2</td><td>Find $\frac{dy}{dx}$ at $x = 0.1$ from the following table:
<table>
<tr><td>x:</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td></tr>
<tr><td>y:</td><td>0.9975</td><td>0.9900</td><td>0.9776</td><td>0.9604</td></tr>
</table></td><td>4</td><td>CO2</td></tr>
<tr><td>Q 3</td><td>Find by Taylor’s series method, the values of y at $x = 0.2$ for differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$</td><td>4</td><td>CO5</td></tr>
<tr><td>Q 4</td><td>Use Picard’s method to obtain y for $x = 0.2$. Given: $\frac{dy}{dx} = x - y$ with initial condition $y = 1$ when $x = 0$.</td><td>4</td><td>CO5</td></tr>
<tr><td>Q 5</td><td>Define Finite Difference approximations to partial derivatives in x direction.</td><td>4</td><td>CO6</td></tr>
</table>

SECTION B

Q 6	Solve the boundary value problem $u_t = u_{xx}$ under the conditions $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x,\ 0 \le x \le 1$ using Bendre-Schmidt method (Take $h = 0.2$ and $= \frac{1}{2}$).	**10**	**CO6**
Q 7	If p, q, r, s be the successive entries corresponding to the equidistant arguments in a table, show that when third differences are taken into account, the entry corresponding to the argument half way between the arguments at q and r is $A + \frac{B}{24}$, where A is the arithmetic mean of q and r and B is arithmetic mean of $3q - 2p - s$ and $3r - 2s - p$.	**10**	**CO1**

<table>
<tr><td>Q 8</td><td>Given that $\frac{dy}{dx} = \log_{10}(x+y)$ with the initial condition that $y = 1$ when $x = 0$. Find y for $x = 0.2$ and $x = 0.5$ using Euler's modified formula.</td><td>10</td><td>CO5</td></tr>
<tr><td>Q 9</td><td>Apply Graeffe's root squaring method to solve the equation
$$x^3 - 8x^2 + 17x - 10 = 0.$$
OR
Find by Horner's method, the positive root of the equation
$x^3 + x^2 + x - 100 = 0$ correct to three decimal places</td><td>10</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION-C</td></tr>
<tr><td>Q 10A</td><td>Given the values
<table>
<tr><td>x:</td><td>5</td><td>7</td><td>11</td><td>13</td><td>17</td></tr>
<tr><td>$f(x)$:</td><td>150</td><td>392</td><td>1452</td><td>2366</td><td>5202</td></tr>
</table>
Evaluate $f(9)$, using Lagrange's formula.</td><td>10</td><td>CO1</td></tr>
<tr><td>Q 10B</td><td>Solve the equations $27x + 6y - z = 85; x + y + 54z = 110; 6x + 15y + 2z = 72$ by Gauss-Jacobi method.</td><td>10</td><td>CO4</td></tr>
<tr><td>Q 11A</td><td>Given the values of $u(x,y)$ on the boundary of the square in the figure, evaluate the function $u(x,y)$ satisfying the Laplace equation $u_{xx} + u_{yy} = 0$ at the pivotal points of this figure by Liebmann's process of iteration.

 </td><td>10</td><td>CO5</td></tr>
</table>

	OR Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = \sin \pi x,\ 0 \le x \le 1;\ u(0,t) = u(1,t) = 0$ by using Crank Nicholson method. Carryout computations for two levels, taking $h = {}^1/_3\, , k = {}^1/_{36}$. 		
Q 11B	Use Galerkin's methods to solve the boundary value problem $y'' - y + x = 0, 0 \le x \le 1, y(0) = 0$ and $y(1) = 0$. OR Solve the equation $y'' + y = 3x^2$, with boundary points $(0,0)$ and $(2, 3.5)$ by using method of Point Collocation.	**[10]**	**CO6**