Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Course: MATHEMATICS III **Semester: III**

Programme: B. Tech. APUP,ASE,ASEA,ECE,EL,PSE,FSE,GI,GSE,MINING **Course Code: MATH 2001**

Time: 03 hrs. **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A

(Attempt all questions)

S. No.		Marks	CO
Q1	Solve $y_{n+2}-4y_{n+1}+4y_n=2^n$	**4**	**CO1**
Q2	Find the series solution of $y'-2xy=0$	**4**	**CO2**
Q3	Expand $\frac{1}{z^2-3z+2}$ for $0<\|z\|<1$.	**4**	**CO4**
Q4	If $f(z)$ and $f'(z)$ are both analytic then show that $f(z)$ is constant.	**4**	**CO3**
Q5	Evaluate $\int_c \frac{z-1}{(z-1)^2(z-2)}dz$, where c is $\|z\|=1$.	**4**	**CO5**

SECTION B

(Q6,Q7,Q8 are compulsory and Q9 and Q10 have internal choice)

Q6	Show that the transformation $w=\frac{5-4z}{4z-2}$ transforms the circle $\|z\|=1$ into a circle of radius unity in w-plane.	**8**	**CO3**
Q7	Prove the Rodrigues formula $P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$.	**8**	**CO2**
Q8	Solve the difference equation $y_{n+2}-5y_{n+1}+6y_n=2$ by the generating function method with initial conditions $y_0=1$ and $y_1=2$.	**8**	**CO1**
Q9	Evaluate using contour integration $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta$ **OR** $\int_0^{\infty}\frac{\sin x\,dx}{x(x^2+a^2)}dx$	**8**	**CO5**

Q10	Solve $\left(D^2+DD'-6D'^2\right)z=y\cos x$ **OR** $\left(D^2+2DD'+D'^2\right)z=2\cos y-x\sin y$	**8**	**CO6**
SECTION-C **(Q11a,Q11b are compulsory and Q12 has internal choice)**			
Q11 a	Expand $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ for i. $\lvert z\rvert<1$ ii $1<\lvert z\rvert<4$	**10**	**CO4**
Q11b	Apply the calculus of residues to evaluate the integral $\int_{-\infty}^{\infty}\frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$ where $(a>b>0)$.	**10**	**CO5**
Q12	A tightly stretched flexible string has its end fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $F(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t>0$. **OR** The ends A and B of a rod 20cm long have the temperature at 30℃ and 80℃ until steady state prevails. The temperature of the ends are changed to 40℃ and 60℃ respectively. Find the temperature distribution in the rod at time t.	**20**	**CO6**

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Instructions: Attempt all questions.

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q1	Solve $y_{n+2}-6y_{n+1}+9y_n=2^n$.	**4**	**CO1**
Q2	Find the ordinary and singular points of the equation $(1-x)^2 y''-6xy'-4y=0$.	**4**	**CO2**
Q3	Expand $\frac{1}{(z-2)(z-1)}$ for $0<\lvert z\rvert<1$.	**4**	**CO4**
Q4	Show that $f(z)=\log z$ is analytic everywhere except at the origin.	**4**	**CO3**
Q5	Evaluate $\oint_c \frac{4-3z}{z(z-1)(z-2)}dz$, where c is $\lvert z\rvert=\frac{3}{2}$.	**4**	**CO5**
SECTION B **(Q6,Q7,Q8 are compulsory and Q9 -Q10 have internal choice)**			
Q6	Show that the transformation $w=\frac{5-4z}{4z-2}$ transforms the circle $\lvert z\rvert=1$ into a circle of radius unity in w-plane.	**8**	**CO3**
Q7	Obtain the series solution of $2x(1-x)y''+(1-x)y'+3y=0$	**8**	**CO2**
Q8	Solve the difference equation $y_n-2y_{n-1}-3y_{n-2}=0$, $n\geq 2$ by the generating function method with initial conditions $y_0=3$ and $y_1=1$.	**8**	**CO1**
Q9	Evaluate using contour integration. $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta$ **OR**	**8**	**CO5**

	$\int_0^{\infty} \frac{\cos mx\, dx}{(x^2+1)^2} dx, m>0$		
Q10	Solve $(D+3D')(D-2D')z = y\cos x$ **OR** $(D+D')^2 z = 2\cos y - x\sin y$	**8**	**CO6**
SECTION-C **Q11a,Q11b are compulsory and Q12 has internal choice)**			
Q11 a	Expand $\frac{z^2-4}{(z+1)(z+4)}$ for i. $\|z\|<1$ ii $1<\|z\|<4$	**10**	**CO4**
Q11b	Apply the calculus of residues to evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$.	**10**	**CO5**
Q12	A tightly stretched string with fixed end points $x=0$ and $x=\pi$ is initially in a position given by $y=x, 0<x<\pi$. If it is released from rest from this position, find the displacement $y(x,t)$. **OR** The ends A and B of a rod 20cm long have the temperature at 30℃ and 80℃ until steady state prevails. The temperature of the ends are changed to 40℃ and 60℃ respectively. Find the temperature distribution in the rod at time t.	**20**	**CO6**