UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme: B. Tech. (ASE; ASE+AVE) | **Semester: VI**
Course Name: Applied Numerical Methods | **Max. Marks : 100**
Course Code: MATH 301 | **Duration : 3 Hrs**
No. of page/s: 02

Instructions:
Attempt all questions from **Section A** (each carrying 5 marks); all questions from **Section B** (each carrying 8 marks) and all questions from **Section C** (carrying 20 marks)**.**

Section A
(Attempt all questions)

1. Obtain the first term of the series whose second and subsequent terms are $8, 3, 0, -1, 0$. **[5]** **CO1**

2. Using Newton's divided difference formula, find the missing values from the table: **[5]** **CO1**

x	1	2	4	5	6
y	14	15	5		9

3. Find the real root of the equation $x = e^{-x}$ using Newton-Raphson method correct up to 4 decimal places. **[5]** **CO3**

4. If $\frac{dy}{dx} = \frac{y-x}{y+x}$, find the value of y at $x = 0.1$ using Picard's method. Given that $y(0) = 1$. **[5]** **CO5**

SECTION B
(Q5-Q8 are compulsory and Q9 has internal choice)

5. Given that **[8]** **CO1**

x	4	6	8	10	12	14
y	3.5460	5.0753	6.4632	7.7217	8.8633	9.8986

Apply Bessel's formula to find the value of y at $x = 9$.

6. Find all the roots of the equation $x^4 - 3x + 1 = 0$ by Graeffe's method. **[8]** **CO3**

<table>
<tr><td>7.</td><td>Solve equations $27x + 6y - z = 85;\ x + y + 54z = 110;\ 6x + 15y + 2z = 72$ using Gauss-Seidel method. Use only four iterations.</td><td>[8]</td><td>CO4</td></tr>
<tr><td>8.</td><td>If $f(x) = (2x+1)(2x+3)(2x+5) \dots \dots \dots (2x+15)$, find the value of $\Delta^4 f(x)$.</td><td>[8]</td><td>CO1</td></tr>
<tr><td>9.</td><td>Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = \log_{10}(x+y)$ with initial condition $y(0) = 1$ for $x = 0.2$ correct to four decimal places (take $h = 0.2$).
OR
Using Runge-Kutta method of fourth order, solve for y at $x = 0.2$ from $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given $y(0) = 1$ (take $h = 0.2$).</td><td>[8]</td><td>CO5</td></tr>
<tr><td colspan="4">SECTION C
(Q10 is compulsory and Q11A, Q11B have internal choices)</td></tr>
<tr><td>10.</td><td>Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0,\ y = 0,\ x = 3,\ y = 3$ with $u = 0$ on the boundary and mesh length is 1.</td><td>[20]</td><td>CO6</td></tr>
<tr><td>11.A</td><td>Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5,\ t \geq 0$ given that $u(x,\ 0) = 20,\ u(0,\ t) = 0,\ u(5,\ t) = 100$. Compute u for one time step with $h = 1$ by Crank-Nicolson method.
OR
Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = u(1,\ t) = 0$ and $u(x,\ 0) = \sin(\pi x),\ \ 0 \leq x \leq 1$ using Schmidt method for five steps in t direction (take $h = 0.2$ and $\lambda = \frac{1}{2}$).</td><td>[10]</td><td>CO6</td></tr>
<tr><td>11.B</td><td>The following date gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data:
<table><tr><td>Time t (Sec)</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td></tr><tr><td>Velocity $v(m/sec)$</td><td>0</td><td>3</td><td>14</td><td>69</td><td>228</td></tr></table>
OR
Compute the value of $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) dx$ using Simpsons's three eight rule (Take seven ordinates).</td><td>[10]</td><td>CO2</td></tr>
</table>

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Section A
(Attempt all questions)

1. If $\Delta f(x) = f(x+h) - f(x)$, then evaluate $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$ for $h = 10$. **[5]** **CO1**

2. Taking reference of the given table, find the value of $f(x)$ at point $x = 4$. **[5]** **CO1**

x	1.5	3	6
$f(x)$	-0.25	2	20

3. Using Netwon-Raphson method, derive the general formula for finding p^{th} root of a positive real number. **[5]** **CO3**

4. Using Picard's method, obtain the 2nd approximation, if $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. **[5]** **CO5**

SECTION B
(Q5-Q8 are compulsory and Q9 has internal choice)

5. Probability distribution function values of a normal distribution are given as follows: **[8]** **CO1**

x	0.2	0.6	1	1.4	1.8
$p(x)$	0.39104	0.33332	0.24197	0.14973	0.07895

Using Bessel's formula, find the value of $p(x)$ for $x = 1.2$.

6.	Using Horner's method, find the positive root of $x^3 + 9x^2 - 18 = 0$.	**[8]**	**CO3**
7.	Solve equations $27x + 6y - z = 85;\ x + y + 54z = 110;\ 6x + 15y + 2z = 72$ using Gauss-Seidel method. Use only four iterations.	**[8]**	**CO4**
8.	A second degree polynomial passes through $(0,\ 1),\ (1,\ 3),\ (2,\ 7), (3,\ 13)$. Find the polynomial.	**[8]**	**CO1**
9.	Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = x + \lvert\sqrt{y}\rvert$ with initial condition $y(0) = 1$ for $x = 0.2$ correct to three decimal places (take $h = 0.2$). **OR** Using Runge-Kutta method of fourth order, solve for y at $x = 1.2$ from $\frac{dy}{dx} = \frac{2xy+e^x}{x^2+xe^x}$ given $y(1) = 0$ (take $h = 0.2$).	**[8]**	**CO5**
SECTION C **(Q10 is compulsory and Q11A, Q11B have internal choices)**			
10.	Solve $u_{xx} + u_{yy} = 0$ in $0 \le x \le 4,\ 0 \le y \le 4$, given that $u(0,\ y) = 0$; $u(4,\ y) = 8 + 2y$; $u(x,\ 0) = \frac{1}{2}x^2$ and $u(x,\ 4) = x^2$. Take $h = k = 1$ and obtain the result using Liebmann's iteration formula (apply two iterations only).	**[20]**	**CO6**
11.A	Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5,\ t \ge 0$ given that $u(x,\ 0) = 20,\ u(0,\ t) = 0,\ u(5,\ t) = 100$. Compute u for one time step with $h = 1$ by Crank-Nicolson method. **OR** Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0,t) = u(1,\ t) = 0$ and $u(x,\ 0) = \sin(\pi x),\ 0 \le x \le 1$ using Schmidt method for five steps in t direction. (Take $h = 0.2$ and $\lambda = \frac{1}{2}$)	**[10]**	**CO6**
11.B	Using Newton forward interpolation formula, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 6$ given that <table><tr><td>x</td><td>4.5</td><td>5</td><td>5.5</td><td>6</td><td>6.5</td><td>7.0</td><td>7.5</td></tr><tr><td>y</td><td>9.69</td><td>12.90</td><td>16.71</td><td>21.18</td><td>26.37</td><td>32.34</td><td>39.15</td></tr></table> **OR** Evaluate $\int_0^1 \frac{dx}{1+x}$ by dividing the interval into 8 equal parts using Simpson's rule. Hence evaluate $\log_e 2$ approximately.	**[10]**	**CO2**