Roll No: ---------------------------

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme: B. Tech. (SOE)	**Semester – II**
Course Name: Mathematics II	**Max. Marks : 100**
Course Code: MATH 1004	**Duration : 3 Hrs**
No. of page/s: 3	

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

	Section A **(Attempt all questions)**		
1.	Solve the differential equation $2\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x$.	**[4]**	**CO1**
2.	Find the Laplace transform of $e^{-t}\cos t\cos 2t$.	**[4]**	**CO2**
3.	Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & \lvert x\rvert\le 1,\\ 0 & \lvert x\rvert>1\end{cases}$.	**[4]**	**CO2**
4.	A continuous random variable X has a pdf $f(x)=kx^2e^{-x}; x\ge 0$. Find k and mean.	**[4]**	**CO4**
5.	Find the complete solution of $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=8(x^2+e^{2x}+\sin 2x)$.	**[4]**	**CO1**
	SECTION B **(Q6-Q9 are compulsory and Q10 has internal choice)**		
6.	State convolution theorem and hence evaluate $L^{-1}\left(\frac{s}{(s^2+1)(s^2+4)}\right)$.	**[8]**	**CO2**

7.	Solve by removal of first order derivative $\frac{d^2y}{dx^2}+\frac{1}{x^{1/3}}\frac{dy}{dx}+\left(\frac{1}{4x^{2/3}}-\frac{1}{6x^{4/3}}-\frac{6}{x^2}\right)y=0$.	**[8]**	**CO1**
8.	Using Laplace transform, solve the differential equation $\frac{d^2x}{dt^2}+9x=\cos 2t$, if $x(0)=1, x\left(\frac{\pi}{2}\right)=-1$.	**[8]**	**CO2**
9.	Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.	**[8]**	**CO4**
10.	Apply Stoke's theorem to evaluate $\int_C\left[(x+y)\,dx+(2x-z)\,dy+(y+z)\,dz\right]$, where C is the boundary of the triangle with vertices (2,0,0), (0, 3, 0) and (0, 0, 6). **OR** Evaluate $\iint_S\left(yz\,\hat{i}+zx\,\hat{j}+xy\,\hat{k}\right)\bullet d\vec{s}$, where S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant.	**[8]**	**CO3**
SECTION C **(Q11 is compulsory and Q12 has internal choice)**			
11.	**A)** Prove that $div\left(grad\, r^n\right)=n\left(n+1\right)r^{n-2}$ where $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$. Hence show that $\nabla^2\left(\frac{1}{r}\right)=0$.	**[10]**	**CO3**
	B) Two lines of regression are given by $5y-8x+17=0$ and $2y-5x+14=0$. If $\sigma_y^2=16$, find i) the mean values of x and y ii) the coefficient of correlation between x and y.	**[10]**	**CO4**
12 A	The demand for a particular space part in a factory was found to vary from day to day. In a sample study, the following information was obtained: **Days**: **Mon** 1124, **Tue** 1125, **Wed** 1110, **Thu** 1120, **Fri** 1125, **Sat** 1116 (**No. of parts demanded**) Use chi-square to test the hypothesis that number of parts demanded does not depend on the day of the week at 5% level of significance. (Given $\chi^2_{5,0.05}=11.07$) **OR**	**[10]**	**CO4**

Table for Q12 A:

Days	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
No. of parts demanded	1124	1125	1110	1120	1125	1116

<table>
<tr>
<td></td>
<td>Ten students got the following percentage of marks in Economics and Statistics:
<table>
<tr><td>Roll No. :</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Marks in Eco. :</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr>
<tr><td>Marks in Stats.:</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr>
</table>
Calculate the coefficient of correlation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 B</td>
<td>Use divergence theorem to evaluate the integral $\iint_S \left(x\,dydz + y\,dzdx + z\,dxdy\right)$ where S is the portion of the plane $x+2y+3z=6$ which lies in the first octant.

OR

Apply Green's theorem to evaluate $\int_C \left[\left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy\right]$ where C is the boundary of the area enclosed by the X-axis and the upper half of the circle $x^2 + y^2 = a^2$.</td>
<td>[10]</td>
<td>CO3</td>
</tr>
</table>

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Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

	Section A **(Attempt all questions)**		
1.	Solve the differential equation $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{-2x} + e^{-3x}$.	**[4]**	**CO1**
2.	Find the Laplace transform $e^t\, t^{-\frac{1}{2}}$.	**[4]**	**CO2**
3.	Find the Fourier transform of $f(x) = \begin{cases} 1 & \lvert x\rvert < 1, \\ 0 & \lvert x\rvert > 1 \end{cases}$.	**[4]**	**CO2**
4.	A continuous random variable X has a pdf $f(x) = kxe^{-x}; x \geq 0$. Find k and mean.	**[4]**	**CO4**
5.	Find the complete solution of $\frac{d^2y}{dx^2} + y = \cos ec\, x$.	**[4]**	**CO1**
	SECTION B **(Q6-Q9 are compulsory and Q10 has internal choice)**		
6.	State convolution theorem and hence evaluate $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$.	**[8]**	**CO2**

7.	Solve by removal of first order derivative $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-1\right)y=-3e^{x^2}\sin 2x$.	**[8]**	**CO1**
8.	Using Laplace transform, solve the differential equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$, if $y(0)=2, y'(0)=0$.	**[8]**	**CO2**
9.	An irregular 6-faced dice is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets.	**[8]**	**CO4**
10.	Apply Stoke's theorem to evaluate $\iint_S(\nabla\times F)\bullet\hat{n}\,ds$, where S is the surface $x^2+y^2+z^2=4$ above the xy - plane and $\vec{F}=\left(x^2+y-4\right)\hat{i}+3xy\,\hat{j}+\left(2xz+z^2\right)\hat{k}$. **OR** Evaluate $\iint_S\left(yz\,\hat{i}+zx\,\hat{j}+xy\,\hat{k}\right)\bullet d\vec{s}$, where S is the surface of the sphere $x^2+y^2+z^2=16$ in the first octant.	**[8]**	**CO3**
SECTION C **(Q11 is compulsory and Q12 has internal choice)**			
11.	**A)** Show that the vector field $\vec{F}=\frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal. **B)** In a partially destroyed laboratory record of an analysis correlation data, the following results only are legible: Variance of $x=9$, Regression equations are $8x-10y+66=0;\ 40x-18y=214$. Find the coefficient of correlation between x and y.	**[10]** **[10]**	**CO3** **CO4**
12 A	The marks secured by recruits in the selection test (X) and in the proficiency test (Y) are given below: S. No.: 1 2 3 4 5 6 7 8 9 X : 10 15 12 17 13 16 24 14 22 Y : 30 42 45 46 33 34 40 35 39 Calculate the rank correlation coefficients.	**[10]** **[10]**	**CO4**

	OR A survey of 800 families having four children is as follows: No. of male births: 0 1 2 3 4 No. of female births: 4 3 2 1 0 No. of families: 32 178 290 236 64 Test whether the data is consistent with the hypothesis that the binomial law holds and the chance of male birth is equal to that of female birth. ($\chi^2_{4,\,0.05} = 9.49$).		
12 B	Use divergence theorem to evaluate the integral $\iint_S \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{1/2} ds$ where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$. **OR** Using Green's theorem to evaluate $\int_C \left[\left(y - \sin x\right)dx + \cos x\, dy\right]$ where C is the triangle formed by $y = 0, x = \frac{\pi}{2}, y = \frac{2}{\pi}x$.		**CO3**